Supplementary Figures for "Using Wavelet-Based Functional Mixed Models to Characterize Population Heterogeneity in Accelerometer Profiles: A Case Study"

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Figure 1: *Summary of missingness in the data.* The top curve summarizes the proportion of daily profiles observed (i.e., nonmissing) as a function of the time of day. The bottom is a histogram summarizing the proportion of time between 9am and 8pm that is observed for each profile.



Figure 2: Posterior mean and 90% pointwise band for child-to-child standard deviation function, $f(t) = \sqrt{Q(t,t)}$



Figure 3: Posterior mean and 90% pointwise band for day-to-day standard deviation function, $f(t) = \sqrt{S(t,t)}$



Figure 4: Posterior mean and 90% pointwise band for proportion of variability from day-to-day as function of time, $f(t) = S(t,t)/\{Q(t,t) + S(t,t)\}$



Figure 5: Within-Scale Decorrelation. Heat maps of empirical estimates of the wavelet-space correlation matrices corresponding to Q^* and S^* for wavelets at the three finest scales, j=1,2, and 3 for the complete case data. Note how these plots are dominated by the diagonal, so it appears that the DWT did a reasonable job of decorrelation within the wavelet scales for our example.



Figure 6: Correlation Matrix for White Noise and AR(1). Plot of empirically estimated correlation matrix for white noise and AR(1) process with $\rho = 0.8$, with sample sizes equivalent to the plots in Figure 5. The purpose of this plot is to serve as a comparison for Figure 5.



Figure 7: Simulated Data. We randomly generated 200 realizations from a Gaussian process with mean $\mu(t)$ and covariance $S(t_1, t_2)$ on an equally-spaced grid of length 256 on (0, 1). From top to bottom, column (a) contains the true mean function $\mu(t)$, the true variance function v(t) = diag(S), and the true autocorrelation surface $\rho_S(t_1, t_2) = v^{-1/2}Sv^{-1/2}$. Columns (b) and (c) contain the posterior mean estimates of these quantities using wavelet-based methods. Both assume independence across wavelet coefficients, but (b) allows the wavelet-space variance components to vary across scale j and location k as in Morris and Carroll (2004), and (c) only allows them to vary across j, as assumed in Morris, et al. (2003a) and other work involving wavelet regression. Note that the framework used in (b) is sufficiently flexible to pick up on the nonstationary features of S, while (c) is not. Specifically, it is able to model the increasing variance in t, the extra variance near the peak at 0.5, the different degrees of smoothness in the region (0,0.4) and (0.6,1), and the extra autocorrelation from the peak at 0.5. Also note it appears to have done a marginally better job of denoising the estimate of the mean function. These same principles apply to the covariance across random effect functions.