## Supplementary Figures for "Using Wavelet-Based Functional Mixed Models to Characterize Population Heterogeneity in Accelerometer Profiles: A Case Study"

## List of Figures





Figure 1: Summary of missingness in the data. The top curve summarizes the proportion of daily profiles observed (i.e., nonmissing) as a function of the time of day. The bottom is a histogram summarizing the proportion of time between 9am and 8pm that is observed for each profile.



Figure 2: Posterior mean and 90% pointwise band for child-to-child standard deviation function,  $f(t) = \sqrt{Q(t,t)}$ 



Figure 3: Posterior mean and 90% pointwise band for day-to-day standard deviation function,  $f(t) = \sqrt{S(t,t)}$ 



Figure 4: Posterior mean and 90% pointwise band for proportion of variability from day-to-day as function of time,  $f(t) = S(t,t)/\{Q(t,t) + S(t,t)\}$ 



Figure 5: Within-Scale Decorrelation. Heat maps of empirical estimates of the wavelet-space correlation matrices corresponding to  $Q^*$  and  $S^*$  for wavelets at the three finest scales,  $j=1,2$ , and 3 for the complete case data. Note how these plots are dominated by the diagonal, so it appears that the DWT did a reasonable job of decorrelation within the wavelet scales for our example.



Figure 6: Correlation Matrix for White Noise and  $AR(1)$ . Plot of empirically estimated correlation matrix for white noise and and  $AR(1)$  process with  $\rho = 0.8$ , with sample sizes equivalent to the plots in Figure 5. The purpose of this plot is to serve as a comparison for Figure 5.



Figure 7: Simulated Data. We randomly generated 200 realizations from a Gaussian process with mean  $\mu(t)$  and covariance  $S(t_1, t_2)$  on an equally-spaced grid of length 256 on  $(0, 1)$ . From top to bottom, column (a) contains the true mean function  $\mu(t)$ , the true variance function  $v(t) = \text{diag}(S)$ , and the true autocorrelation surface  $\rho_S(t_1, t_2) = v^{-1/2} S v^{-1/2}$ . Columns (b) and (c) contain the posterior mean estimates of these quantities using wavelet-based methods. Both assume independence across wavelet coefficients, but (b) allows the wavelet-space variance components to vary across scale j and location k as in Morris and Carroll (2004), and (c) only allows them to vary across  $j$ , as assumed in Morris, et al.  $(2003a)$  and other work involving wavelet regression. Note that the framework used in (b) is sufficiently flexible to pick up on the nonstationary features of  $S$ , while (c) is not. Specifically, it is able to model the increasing variance in  $t$ , the extra variance near the peak at 0.5, the different degrees of smoothness in the region  $(0,0.4)$  and  $(0.6,1)$ , and the extra autocorrelation from the peak at 0.5. Also note it appears to have done a marginally better job of denoising the estimate of the mean function. These same principles apply to the covariance across random effect functions.