Three-Dimensional Geometric Analysis in Felid Limb-Bone Allometry: Equation list.

Number of thresholded pixels (n), pixel width (w), height (h) and voxel depth (d) are used throughout. All coordinates are given in mm.

Centroid (x_c, y_c)

$$(x_c, y_c) = \left(\sum_{i=1}^{n} x, \sum_{i=1}^{n} y \right)$$

Second moments of area relative to x and y axes

$$I_y = \sum_{i=1}^{n} \left(w \cdot h \cdot (x - x_c)^2 + \frac{w^3 \cdot h}{12} \right)$$

$$I_x = \sum_{i=1}^{n} \left(w \cdot h \cdot (y - y_c)^2 + \frac{h^3 \cdot w}{12} \right)$$

$$I_{xy} = \sum_{i=1}^{n} \left(w \cdot h \cdot (x - x_c) \cdot (y - y_c) \right)$$

Angle of principal axis (radians)

$$\theta = \arctan \left(\frac{I_x - I_y + \sqrt{(I_x - I_y)^2 + 4 \cdot I_{xy}^2}}{2 \cdot I_{xy}} \right)$$

Maximum ($I_{\max}$) and minimum ($I_{\min}$) second moments of area

$$I_{max} = \frac{I_x + I_y}{2} + \sqrt{\frac{(I_x - I_y)^2}{2} + I_{xy}^2}$$

$$I_{min} = \frac{I_x + I_y}{2} - \sqrt{\frac{(I_x - I_y)^2}{2} + I_{xy}^2}$$

Polar moment of inertia

$$J_z = I_{max} + I_{min}$$

Maximum distance of a thresholded pixel from principal axes

(x_c, y_c) = centroid
R_{max} = maximum distance from maximum principal axis
R_{min} = maximum distance from minimum principal axis

For all thresholded pixels (x, y):

$$R_{max} = max\{|(x - x_c) \cdot \cos\theta + (y - y_c) \cdot \sin\theta|\}$$

$$R_{min} = max\{|(y - y_c) \cdot \cos\theta - (x - x_c) \cdot \sin\theta|\}$$

Section moduli

$$Z_{max} = \frac{I_{max}}{R_{max}}$$

$$Z_{min} = \frac{I_{min}}{R_{min}}$$

3D centroid

$$(x_c, y_c, z_c) = \left(\sum_{i=1}^{n} x, \sum_{i=1}^{n} y, \sum_{i=1}^{n} z\right)$$

Inertia Tensor

$$M = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

Moments of inertia
(x_c, y_c, z_c) is the 3D centroid
ρ is density of whole object
V is volume of whole object

$$I_{xx} = \sum_{i=1}^{n} \left((y - y_c)^2 + (z - z_c)^2 + \frac{h^2 + d^2}{12}\right) \cdot \rho \cdot V$$
$$I_{yy} = \sum_{i=1}^{n} \left((x - x_c)^2 + (z - z_c)^2 + \frac{w^2 + d^2}{12}\right) \cdot \rho \cdot V$$
$$I_{zz} = \sum_{i=1}^{n} \left((y - y_c)^2 + (x - x_c)^2 + \frac{h^2 + w^2}{12}\right) \cdot \rho \cdot V$$

$$I_{xy} = -\sum_{i=1}^{n} \left((x - x_c) \cdot (y - y_c)\right) \cdot \rho \cdot V$$
$$I_{xz} = -\sum_{i=1}^{n} \left((x - x_c) \cdot (z - z_c)\right) \cdot \rho \cdot V$$
$$I_{zy} = -\sum_{i=1}^{n} \left((z - z_c) \cdot (y - y_c)\right) \cdot \rho \cdot V$$