Electronic supplementary material

Appendix A: Repeated n-player Prisoner's Dilemma for the smallest possible population size N ($N = n$) for general strategy

Each game involves all the N individuals in the population and consists of one or more rounds. We consider two strategies: ALLD and R. ALLD always defect no matter what other individuals do. R can be any strategy that cooperates at least once during each game when there are only one R and $N - 1$ ALLDs in the population. The payoffs, $V(\mathrm{C}|l)$ and $V(\mathrm{D}|l)$, have the same properties as above, given by (S1)-(S4).

First, let us consider whether strategy R is ESS_N or not. From (S1) and (S3), we have $V(\mathrm{D}|l) > V(\mathrm{C}|l)$. This inequality means that a single mutant ALLD in a population of strategy R does not have a lower fitness and thus the first condition for R to be ESS_N is violated. Therefore, we conclude strategy R that is ESS_N does not exist.

Second, we examine whether ALLD is ESS_N or not. Since $V(\mathrm{D}|1) > V(\mathrm{C}|1)$ and a mutant R cooperates at least once, selection opposes R invading ALLD, that is, the first condition for ALLD to be ESS_N is satisfied. The fixation probability of R is given by

$$\rho_{\mathrm{R}} = 1/\left(1 + \sum_{k=1}^{N-1}\prod_{i=1}^{k}\frac{1 - w + wGi}{1 - w + wFi}\right),$$

where i denotes the number of R individuals and F_i and G_i are the expected payoffs of R and ALLD, respectively (see main text). Since $F_1 < G_1$ and $F_i \le G_i$ ($2 \le i \le N - 1$) from the definition of R, the second condition for ALLD to be ESS_N is satisfied. Therefore, we can conclude that ALLD is ESS_N.

To sum up, even in repeated n-player Prisoner's Dilemma, cooperation cannot evolve when population size and group size are the same. Note that the argument in this subsection applies for any payoff functions that satisfy (S1)-(S4): we do not have to assume specific payoff functions such $V(\mathrm{C}|k) = bk/n - c$ and $V(\mathrm{D}|k) = bk/n$. Also, the argument does not presuppose weak selection.

Appendix B: One-shot n-player Prisoner's Dilemma for general population size

Groups of n individuals are sampled from the population of size N and a game is played within each group. A game consists of only one round, in each of which individuals either cooperate or defect. When some individuals cooperate, all the individuals in the group gain a benefit from it while only the cooperating individuals have to pay a cost. Let $V(\mathrm{C}|l)$ and $V(\mathrm{D}|l)$ be the payoffs to individuals choosing cooperation and defection given that l of n individuals in the group choose cooperation. The n-player Prisoner's Dilemma demands that these payoffs have the following properties:

$$V(\mathrm{D}|l) > V(\mathrm{C}|l + 1), \qquad \text{(S1)}$$

$$V(\mathrm{D}|l + 1) > V(\mathrm{D}|l), \qquad \text{(S2)}$$

$$V(\mathrm{C}|l + 1) > V(\mathrm{C}|l), \qquad \text{(S3)}$$

$$(l + 1)V(\mathrm{C}|l + 1)+(n - l - 1)V(\mathrm{D}|l + 1) > lV(\mathrm{C}|l)+(n - l)V(\mathrm{D}|l), \qquad \text{(S4)}$$

where $0 \le l \le n - 1$. Using (S1) and (S3), we can conclude that D is always traditional ESS while C never is. Now, let us examine for general N whether either or both C and D can be ESS_N. First, consider the case when a mutant D appears in a population of C. Using (S1) and (S3), we can obtain an inequality, that is,

$$(N - 1)V(\mathrm{D}|l) > (N - n)V(\mathrm{C}|l + 1) + (n - 1)V(\mathrm{C}|l).$$

This inequality implies that the first condition for C to be ESS_N is violated (let $a_k = V(\mathrm{D}|k - 1)$, $b_k = V(\mathrm{C}|k)$ and $l = n - 1$ in (5) in the main text), that is, a single mutant D in a population of C has a higher fitness. Therefore, we conclude that C cannot be ESS_N.

Second, suppose that a mutant C has appeared in a population of D. From (S1) and (S2), we obtain

$$(N-1)V(\mathrm{C}|l+1) < (N-n)V(\mathrm{D}|l) + (n-1)V(\mathrm{D}|l+1),$$

which suggests that the first condition for D to be ESS_N is satisfied (let $a_k = V(\mathrm{C}|n-k+1)$, $b_k = V(\mathrm{D}|n-k)$ and $l = 0$ in (5) in the main text). The fixation probability of C is given by

$$\rho_\mathrm{C} = 1/\left(1+\sum_{k=1}^{N-1}\prod_{i=1}^{k}\frac{1-w+wGi}{1-w+wFi}\right),$$

where i denotes the number of C individuals and F_i and G_i are the expected payoffs of C and D, respectively (see main text). From (S1) and (S2), we have

$$\sum_{k=1}^{n}\frac{\binom{i-1}{n-k}\binom{N-i}{k-1}}{\binom{N-1}{n-1}}V(C\,|\,n-k+1) < \sum_{k=1}^{n}\frac{\binom{i}{n-k}\binom{N-i-1}{k-1}}{\binom{N-1}{n-1}}V(D\,|\,n-k),$$

that is, $F_i < G_i$ ($1 \le i \le N-1$). Hence, $\rho_\mathrm{C} < 1/N$ and thus the second condition for D to be ESS_N is satisfied. Therefore, we can conclude that D is ESS_N.

To sum up, in nonrepeated n-player Prisoner's Dilemma, C is neither ESS_N nor traditional ESS, and D is not only ESS but also ESS_N. Hence, cooperation cannot emerge in populations consisting of defectors. Note that the argument in this subsection applies for any payoff functions that satisfy (S1)-(S4) and for any selection intensity.

Appendix C: For n=2, 3, no payoff matrix exists for which both A and B are traditional ESS but neither is ESS_N.

In two-player games,

$2a_1+a_2 < 2b_1+b_2$

$a_1+2a_2 > b_1+2b_2$

$a_1 > b_1$

$a_2 < b_2$

All these inequalities cannot be satisfied simultaneously. Hence, for n=2, no payoff matrix exists for which both A and B are traditional ESS but neither is ESS_N.

Similarly, in three player games,

$3a_1+2a_2+a_3 < 3b_1+2b_2+b_3$

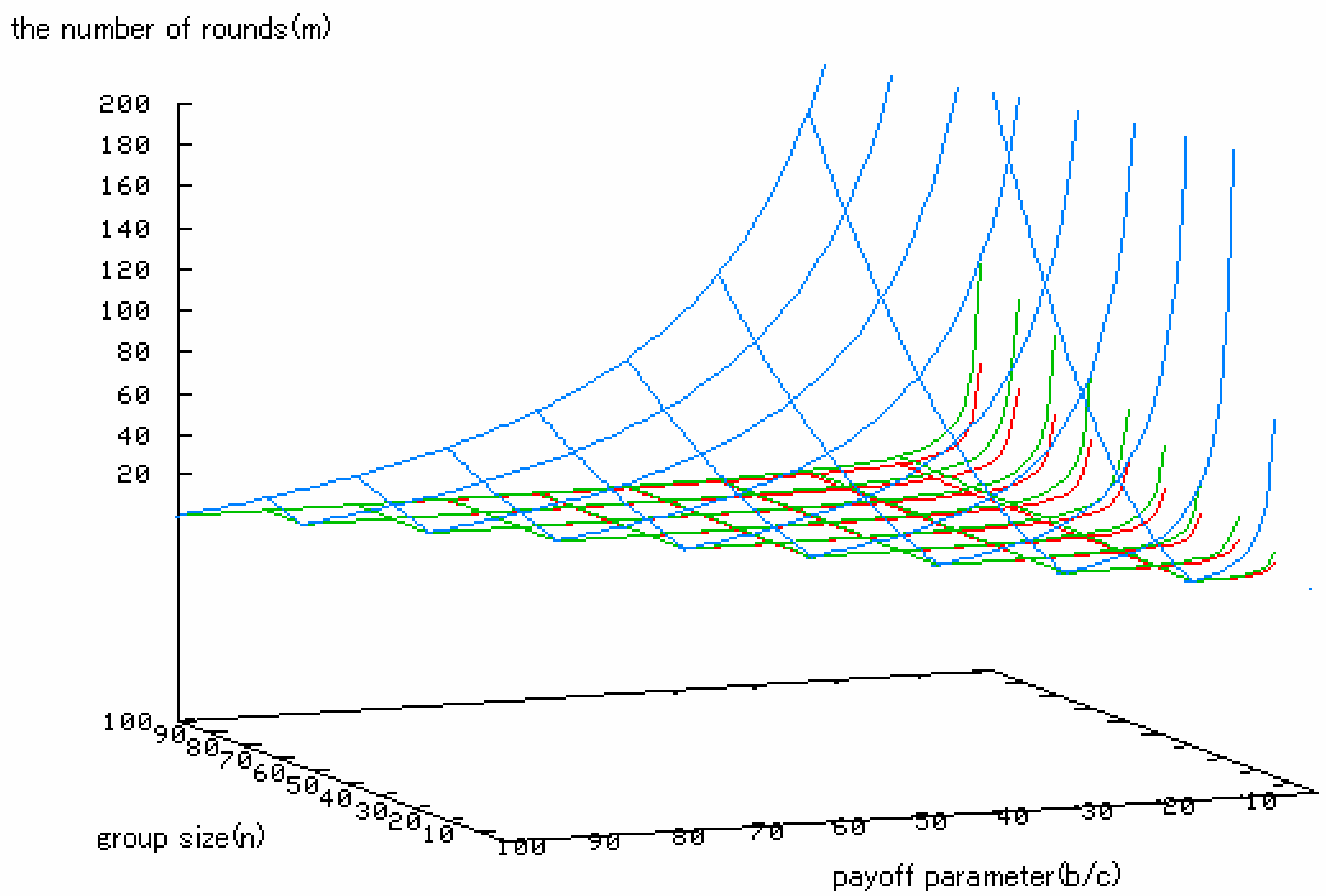
$a_1+2a_2+3a_3 > b_1+2b_2+3b_3$

$a_1 > b_1$

$a_3 < b_3$

All these inequalities cannot be met simultaneously. Hence, for n=3, no payoff matrix exists for which both A and B are traditional ESS but neither is ESS_N.

Appendix D: Figure illustrates the relationship between payoff parameter (b/c), group size (n) and the minimum number of rounds (m) required for $\rho_{ALLD} < 1/N$, $\rho_{TFTn\text{-}1} > \rho_{ALLD}$ or $\rho_{TFTn\text{-}1} > 1/N$ when N is large. The blue represents the number of rounds for which $\rho_{ALLD} = 1/N$, while the green represents that for which $\rho_{TFTn\text{-}1} = \rho_{ALLD}$ and the red represents that for which $\rho_{TFTn\text{-}1} = 1/N$. The parameter space is divided into the following four regions: (i) $TFT_{n\text{-}1}$ is ESS_N and ALLD is not ESS_N; (ii) both $TFT_{n\text{-}1}$ and ALLD are ESS_N and $\rho_{TFTn\text{-}1} > \rho_{ALLD}$; (iii) both $TFT_{n\text{-}1}$ and ALLD are ESS_N and $\rho_{TFTn\text{-}1} < \rho_{ALLD}$; and (iv) $TFT_{n\text{-}1}$ is not ESS_N and ALLD is ESS_N. This figure reduces to figure 1a when b/c = 1.5 and to figure 1b when $b/(nc)$ = 0.51.


the number of rounds(m)
200
180
160
140
120
100
80
60
40
20
100
90
80
70
60
50
40
30
20
10
group size(n)
100
90
80
70
60
50
40
30
20
10
payoff parameter(b/c)

Figure