Bayesian state-space modeling for neuronal activity

We outline in this section a method for estimating the rate and change-point in neural activity. For consistency and comparison with results from the behavioral analysis, we selected a model for neural activity very similar in structure to our previous Bayesian model for behavioral data (Smith et al., 2007).

Assume the neural count observations in each period are represented by $\{n_1, n_2, n_3, ..., n_K\}$ where K is the total number of trials. We assume that the neural activity has an underlying state process defined by

$$x_k = x_{k-1} + \varepsilon_k \tag{A1}$$

where x_k is the unknown state at trial k and ε_k is a Gaussian random variable with mean zero and variance σ_ε^2. We assume further that x_0 is unknown and assign a broad uninformative Gaussian prior with mean 0 and variance 100.

Model 1. In our initial analysis we assumed that the count observations were Poisson-distributed as follows:

$$\Pr(n_k \mid x_k) = \exp\{n_k \log \lambda(k \mid x_k) - \lambda(k \mid x_k)\} \tag{A2}$$

where the rate at trial k, $\lambda(k \mid x_k)$, is related to the state by a log link function as follows

$$\lambda(k \mid x_k) = \exp(x_k)\,. \tag{A3}$$

We fitted this model using a Monte Carlo Markov chain approach (WinBUGS, Lunn et al., 2000) outlined for behavioral data in Smith et al. (2007).

Model 2. Our results from this analysis technique indicated that in some cases the model appeared to over-fit the data and that the variance of the data was so high in many cases that a Poisson-based model might not provide the best fit to the data. As a result, we used a mixture model approach designed to account for Poisson overdispersion outlined in Congdon (p157, 2005) and Durham et al. (2004). Essentially this allowed us to add an extra parameter that enabled the model no longer to be constrained by the Poisson assumption of equal mean and variance. To implement this we replaced Eqs. A2 and A3 with the following 4 equations:

$$\Pr(n_k \mid x_k) = \exp\{n_k \log \lambda^1(k \mid x_k) - \lambda^1(k \mid x_k)\} \tag{A4}$$

$$\lambda^1(k \mid x_k) = r_k \ \lambda(k \mid x_k) \tag{A5}$$

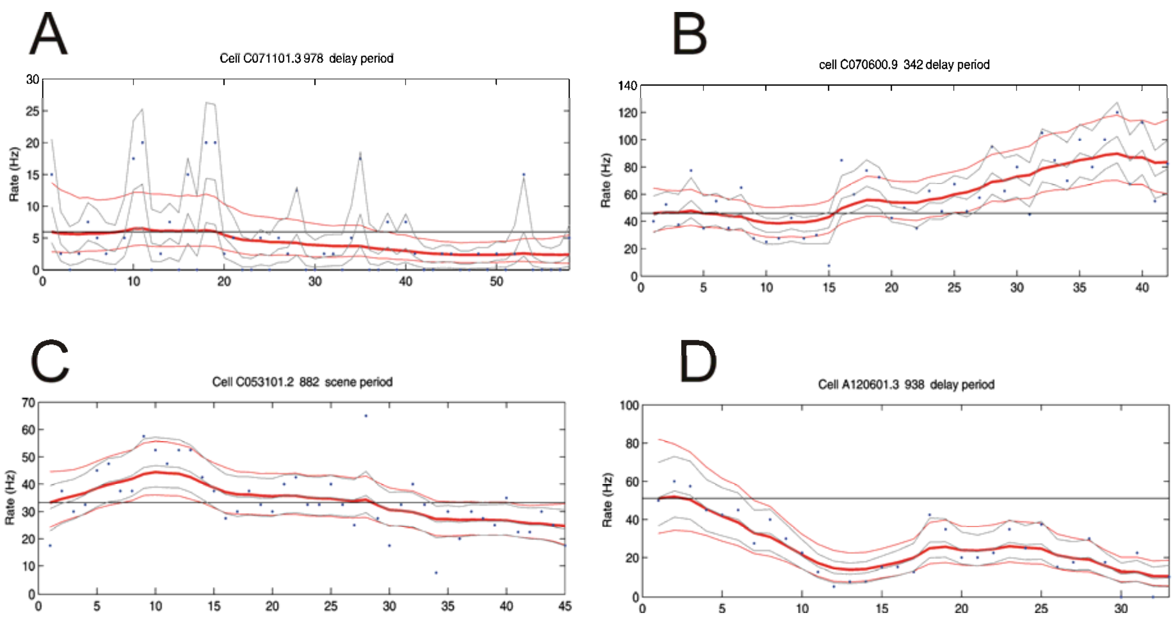
$$P(r_k) = \frac{\alpha_2^{\alpha_1} r_k^{\alpha_1 - 1} e^{-\alpha_2 x}}{\Gamma(\alpha_1)} \tag{A6}$$

and

$$\lambda(k \mid x_k) = \exp(x_k)\,. \tag{A7}$$

By adding the gamma-distributed random variable r_k (Eqs. A5 and A6), the model is better able to accommodate data with large variances. For identifiability we assumed the gamma distribution parameters α_1 and α_2 were equal (Congdon, 2005) and had priors that were exponential transforms of the normal distribution as used in Durham et al. (2004).

We demonstrate below 4 example data sets. The raw neural firing rate is indicated by blue dots and the ratio of the raw data mean to variance is 0.34, 0.24, 0.64 and 0.28 for data in Panels A, B, C and D, respectively. For each data set, we fitted the Poisson model (Model 1; gray lines indicate median and 95% credible intervals) and the gamma-Poisson model (Model 2; red lines indicate median and 95% credible intervals). For the data sets in Panels A and B, Model 2 provides a smoother model fit compared to Model 1. In Panels C and D, both models give similar results. Since Model 2 is a more general version of Model 1, we chose to estimate the neural firing for all data sets using Model 2.



References

Congdon, P. (2005). Bayesian Models for Categorical Data. Wiley: London, UK.

Durham, C. A., I. Pardoe, and E. Vega (2004). A methodology for evaluating how product characteristics impact choice in retail settings with many zero observations: an application to restaurant wine purchase. *Journal of Agricultural and Resource Economics 29(1),* 112-131.