The following text is included for refereeing purpose only. It will not be part of the paper.

Proof of (A1). It can be shown by following the same steps as the proof of (A3) with $\zeta = 0$.

Proof of (A2). For convenience, drop the subscript i and define

$$\begin{aligned} A &= g(X); \\ Z_k &= A^{1/2}(\epsilon_k - \bar{\epsilon}.); \\ V &= \bar{Y}. + (\zeta/m)^{1/2} \sum_{j=1}^{m} c_{b,j} Y_j = X + A^{1/2}\{\bar{\epsilon}. + (\zeta/m)^{1/2} \sum_{j=1}^{m} c_{b,j}\epsilon_j\}. \end{aligned}$$

Note that

$$\begin{aligned} \text{cov}\{\bar{\epsilon}., \bar{\epsilon}. + (\zeta/m)^{1/2} \sum_{j=1}^{m} c_{b,j}\epsilon_j\} &= 1/m; \\ \text{cov}\{\epsilon_k, \bar{\epsilon}. + (\zeta/m)^{1/2} \sum_{j=1}^{m} c_{b,j}\epsilon_j\} &= (1/m) + (\zeta/m)^{1/2} c_{b,k}; \\ \text{var}(\epsilon_k - \bar{\epsilon}.) &= (m-1)/m. \end{aligned}$$

Now calculate some moments:

$$\begin{aligned} \text{var}(Z_k|X) &= A(m-1)/m; \\ \text{var}(V|X) &= A(1+\zeta)/m; \\ \text{cov}(Z_k, V|X) &= A(\zeta/m)^{1/2} c_{b,k}. \end{aligned}$$

Note that Z_k and V are jointly normally distributed. We thus have

$$\begin{aligned} E(Z_k|V, X) &= \frac{\text{cov}(Z_k, V)}{\text{var}(V)}(V - X) = \frac{(\zeta/m)^{1/2} c_{b,k}}{(1+\zeta)/m}(V - X); \\ \text{var}(Z_k|V, X) &= \text{var}(Z_k) - \text{cov}^2(Z_k, V)/\text{var}(V) = A\left\{\frac{m-1}{m} - \frac{(\zeta/m)c_{b,k}^2}{(1+\zeta)/m}\right\}. \end{aligned}$$

We have thus shown that

$$E(Z_k^2|V, X) = A(m-1)/m + \frac{(\zeta/m)c_{b,k}^2}{(1+\zeta)/m}\left\{\frac{(V-X)^2}{(1+\zeta)/m} - A\right\}.$$

Note that $S = (m-1)^{-1}\sum_{k=1}^{m} Z_k^2$ and $\sum_k c_{b,k}^2 = 1$. Now putting the subscript i back in and remembering that $A = g(X_i)$, we have shown that

$$\begin{aligned} E\{S_i|X_i, W_{b,i}(\zeta)\} &= g(X_i) + \frac{\{1/(m-1)\}(\zeta/m)}{(1+\zeta)/m}\left[\frac{\{W_{b,i}(\zeta)-X_i\}^2}{(1+\zeta)/m} - g(X_i)\right] \\ &= g(X_i) + s(\zeta)\left[\frac{\{W_{b,i}(\zeta)-X_i\}^2}{(1+\zeta)/m} - g(X_i)\right]. \end{aligned}$$

Proof of (A3). Because $[W_{b,i}(\zeta)|X_i] \sim N\{X_i, g(X_i)(1+\zeta)/m\}$, we have

$$\begin{aligned} &E\left[K_h\left\{W_{b,i}(\zeta)-x_0\right\}g(X_i)\right] \\ &= \int\int \frac{1}{\{(1+\zeta)g(x)/m\}^{1/2}} K_h(w-x_0)g(x)\phi\left[\frac{x-w}{\{(1+\zeta)g(x)/m\}^{1/2}}\right] f_X(x)dwdx. \end{aligned}$$

Now make the change of variables:

$$z = (w-x_0)/h \rightsquigarrow w = x_0 + zh,$$

so that

$$\begin{aligned} &E\left[K_h\left\{W_{b,i}(\zeta)-x_0\right\}g(X_i)\right] \\ &= \int\int \frac{1}{\{(1+\zeta)g(x)/m\}^{1/2}} K(z)g(x)\phi\left[\frac{x-x_0-zh}{\{(1+\zeta)g(x)/m\}^{1/2}}\right] f_X(x)dzdx. \end{aligned}$$

Make the further change of variables

$$s = \frac{x-x_0-zh}{\{(1+\zeta)/m\}^{1/2}} \rightsquigarrow x = x_0 + zh + s\{(1+\zeta)/m\}^{1/2},$$

so that

$$\begin{aligned} &E\left[K_h\left\{W_{b,i}(\zeta)-x_0\right\}g(X_i)\right] \\ &= \int\int \frac{1}{g^{1/2}[x_0+zh+s\{(1+\zeta)/m\}^{1/2}]} K(z)g\left[x_0+zh+s\{(1+\zeta)/m\}^{1/2}\right] \\ &\quad \times\phi\left(s/g^{1/2}[x_0+zh+s\{(1+\zeta)/m\}^{1/2}]\right) f_X\left[x_0+zh+s\{(1+\zeta)/m\}^{1/2}\right] dzds. \end{aligned}$$

As $h \to 0$, because $\int K(z)dz = 1$, the right hand side of the above equation converges to

$$\begin{aligned} &\int \frac{1}{g^{1/2}\{x_0+s\{(1+\zeta)/m\}^{1/2}\}} g\left[x_0+s\{(1+\zeta)/m\}^{1/2}\right] \\ &\quad \times\phi\left(s/g^{1/2}[x_0+s\{(1+\zeta)/m\}^{1/2}]\right) f_X\left[x_0+s\{(1+\zeta)/m\}^{1/2}\right] ds. \end{aligned}$$

Then note that

$$\begin{aligned}
&\lim_{\zeta\to -1}\int \frac{1}{g^{1/2}[x_0+s\{(1+\zeta)/m\}^{1/2}]} g\left[x_0+s\{(1+\zeta)/m\}^{1/2}\right] \\
&\quad\times\phi\left(s/g^{1/2}[x_0+s\{(1+\zeta)/m\}^{1/2}]\right) f_X\left[x_0+s\{(1+\zeta)/m\}^{1/2}\right] ds \\
&= g(x_0)f_X(x_0)\int \frac{1}{g^{1/2}(x_0)}\phi\{s/g^{1/2}(x_0)\}ds \\
&= g(x_0)f_X(x_0),
\end{aligned}$$

as claimed.

Proof of (A4). The term in question is

$$\begin{aligned}
\mathcal{C}(h,\zeta) &= s(\zeta)\int\int K_h(w-x_0)f_X(x)g(x)\frac{1}{\{g(x)(1+\zeta)/m\}^{1/2}}\phi\left[\frac{w-x}{\{g(x)(1+\zeta)/m\}^{1/2}}\right] \\
&\quad\times\left(\left[\frac{w-x}{\{g(x)(1+\zeta)/m\}^{1/2}}\right]^2-1\right)dwdx.
\end{aligned}$$

Making the change of variables $z=(w-x_0)/h$ and letting $h\to 0$, we have

$$\mathcal{C}(h,\zeta)\to\frac{s(\zeta)}{(1+\zeta)/m}\int f_X(x)g^{1/2}(x)\phi\left[\frac{x_0-x}{\{g(x)(1+\zeta)/m\}^{1/2}}\right]\left(\left[\frac{x_0-x}{\{g(x)(1+\zeta)/m\}^{1/2}}\right]^2-1\right)dx.$$

Then by making the change of variables, $x=x_0+s\{(1+\zeta)/m\}^{1/2}$, it is seen that (A4) equals

$$\lim_{\zeta\to -1}\frac{\zeta/(m-1)}{1+\zeta}\int H_s[x_0+s\{(1+\zeta)/m\}^{1/2}]ds.$$

Note that $\int H_s(x_0)ds=0$, so we can apply L'Hospital's rule so that (A4) equals

$$\lim_{\zeta\to -1}\frac{\zeta/(m-1)}{2(1+\zeta)^{1/2}}\int\frac{s}{m^{1/2}}H_s^{(1)}[x_0+s\{(1+\zeta)/m\}^{1/2}]ds.$$

Note that $\int sH_s^{(1)}(x_0)ds=0$, so we can apply L'Hospital's rule again so that (A4) equals

$$-\frac{1}{2m(m-1)}\int s^2H_s^{(2)}(x_0)ds.$$

Proof of (A9).

$$
\begin{aligned}
& [E\{\widehat{\Lambda}(x_0)|\widetilde{Y}\}]^2 \\
= \;& (4n)^{-1}E\left(\sum_{s=\pm 1}\sum_{k=0}^{K}\{d_k/v(x_0,\zeta_k)\}K_h\{W_i(s,\zeta_k)-x_0\}Q\left[Y_i,W_i(s,\zeta_k),g\{W_i(s,\zeta_k),\zeta_k\}\right]\right)^2 \\
= \;& (4n)^{-1}\sum_{s_1=\pm 1}\sum_{s_2=\pm 1}\sum_{k_1=0}^{K}\sum_{k_2=0}^{K}\{d_{k_1}/v(x_0,\zeta_{k_1})\}\{d_{k_2}/v(x_0,\zeta_{k_2})\} \\
& \times E\left(K_h\{W_i(s_1,\zeta_{k_1})-x_0\}K_h\{W_i(s_2,\zeta_{k_2})-x_0\}\right. \\
& \quad \left.\times Q\left[Y_i,W_i(s_1,\zeta_{k_1}),g\{W_i(s_1,\zeta_{k_1}),\zeta_{k_1}\}\right]Q\left[Y_i,W_i(s_2,\zeta_{k_2}),g\{W_i(s_2,\zeta_{k_2}),\zeta_{k_2}\}\right]\right) \\
= \;& (n)^{-1}\{d_0/v(x_0,0)\}^2K_h^2\{W_i(1,0)-x_0\}Q^2\left[Y_i,W_i(1,0),g\{W_i(s,0),0\}\right] \\
& +(4n)^{-1}\sum_{s_1=\pm 1}\sum_{s_2=\pm 1}\sum_{k_1=1}^{K}\sum_{k_2=1}^{K}\{d_{k_1}/v(x_0,\zeta_{k_1})\}\{d_{k_2}/v(x_0,\zeta_{k_2})\} \\
& \quad \times E\left(K_h\{W_i(s_1,\zeta_{k_1})-x_0\}K_h\{W_i(s_2,\zeta_{k_2})-x_0\}\right. \\
& \qquad \left.\times Q\left[Y_i,W_i(s_1,\zeta_{k_1}),g\{W_i(s_1,\zeta_{k_1}),\zeta_{k_1}\}\right]Q\left[Y_i,W_i(s_2,\zeta_{k_2}),g\{W_i(s_2,\zeta_{k_2}),\zeta_{k_2}\}\right]\right) \\
& +(2n)^{-1}\sum_{s_1=\pm 1}\sum_{s_2=\pm 1}\sum_{k=1}^{K}\{d_0/v(x_0,0)\}\{d_k/v(x_0,\zeta_k)\} \\
& \quad \times E\left(K_h\{W_i(s_1,0)-x_0\}K_h\{W_i(s_2,\zeta_k)-x_0\}\right. \\
& \qquad \left.\times Q\left[Y_i,W_i(s_1,0),g\{W_i(s_1,0),0\}\right]Q\left[Y_i,W_i(s_2,\zeta_k),g\{W_i(s_2,\zeta_k),\zeta_k\}\right]\right) \\
= \;& A_1+A_2+A_3,
\end{aligned}
$$

where the third equality follows because $W_i(s,0)$ does not depend on s.

By standard calculations, we obtain

$$
\begin{aligned}
E(A_1) &= E\left[E\{A_1|W_i(1,0)\}\right] \\
&= E\left[n^{-1}\{d_0/v(x_0,0)\}^2K_h^2\{W_i(1,0)-x_0\}\beta\{W_i(1,0),0\}\right] \\
&= \frac{1}{nh}\{d_0/v(x_0,0)\}^2\int K^2(z)f_W(x_0+hz,0)\beta(x_0+hz,0)dz \\
&= \frac{\gamma}{nh}\{d_0/v(x_0,0)\}^2f_W(x_0,0)\beta(x_0,0)\left\{1+o(1)\right\}.
\end{aligned}
$$

Similarly, for $E(A_2)$, all the terms are $O(n^{-1})$ except when $k_1=k_2$ and $s_1=s_2$, and we then have

$$
E(A_2)=\frac{\gamma}{2hn}\sum_{k=1}^{K}\left\{d_k/v(x_0,\zeta_k)\right\}^2f_W(x_0,\zeta_k)\beta(x_0,\zeta_k)\left\{1+o(1)\right\}.
$$

It is an easy calculation to see that

$$E(A_3) = O(n^{-1}).$$

Putting all this together, we have

$$\begin{aligned} \mathrm{var}\left[E\left\{\widehat{\Lambda}(x_0)|\widetilde{Y}\right\}\right] &= \frac{\gamma f_W(x_0,0)\beta(x_0,0)d_0^2}{v^2(x_0,0)nh} \\ &\quad + \frac{1}{2}\sum_{k=1}^{K}\frac{\gamma f_W(x_0,\zeta_k)\beta(x_0,\zeta_k)d_0^2}{v^2(x_0,\zeta_k)nh} + o\{h^4 + (nh)^{-1}\}. \end{aligned}$$