Appendix S1

Measure definitions

Let $D = (V, E)$ be a directed graph with the set of nodes V and the set of oriented edges E. Assume the edges $e = (u, v)$ have weights $w(u, v) > 0$, $(u, v) \in E$ and $w(u, v)) = 0$ for $(u, v) \notin E$. For unweighted graphs $w(u, v) = 1$, $e(u, v) \in E$.

A undirected graph $G = (V, E)$ can be constructed from D by replacing every oriented edge (u, v) by an undirected edge (u, v) with weight $\hat{w}(u, v) = [w(u, v) + w(v, u)]/2$. For unweighted graphs this amounts to replacing each directed link by an undirected link.

We use the following definitions of network measures:

Degree Centrality

The degree centrality of a node u is defined as the sum of weights of edges incident to u

$$C_D(u) = \sum_{(u,v)\in E} w(u, v)\,. \tag{1}$$

For directed graphs there is an indegree

$$C_{D_{in}}(u) = \sum_{(v,u)\in E} w(v, u)\,, \tag{2}$$

and outdegree

$$C_{D_{out}}(u) = \sum_{(u,v)\in E} w(u, v)\,. \tag{3}$$

Closeness Centrality

Let $d_G(s, t)$ be the shortest distance between nodes s and t in G (the sum of path weight is minimum).

The closeness centrality [1] of a node u is

$$C_C(u) = \frac{1}{\sum_{t\in V} d_G(u, t)}. \tag{4}$$

We use the algorithm from Ref. [2].

Betweenness Centrality

Let σ_{st} be the number of weighted shortest paths between nodes s and t in the graph and $\sigma_{st}(u)$ be the number of those shortest paths that pass through node u. The betweenness centrality [3] is

$$C_B(u) = \sum_{s\neq u\neq t\in V} \frac{\sigma_{st}(u)}{\sigma_{st}}. \tag{5}$$

We use the algorithm from Ref. [2].

PageRank

The PageRank [4] of node v is

$$PR(u) = (1 - d) + d \sum_{(v,u)\in E} w(v, u)\frac{PR(v)}{C_{D_{out}}(v)}\,. \tag{6}$$

See the review [5] for implementation details.

Measure definitions

For a directed graph $D = (V, E)$ with weights $w(u, v)$

Directed weighted

- Directed Weighted In-degree Centrality: Eq. (2)
- Directed Weighted Out-degree Centrality: Eq. (3)
- Directed Weighted Pagerank: Eq. (6)

Directed Unweighted

With $w(u, v) = 1$ for $(u, v) \in E$.

- Directed Unweighted In-degree Centrality: Eq. (2)
- Directed Unweighted Out-degree Centrality: Eq. (3)
- Directed Unweighted Pagerank: Eq. (6)

Undirected weighted

Replacing every oriented edge $(u, v) \in E$ by an undirected edge (u, v) with weight $\hat{w}(u, v) = [w(u, v) + w(v, u)]/2$.

- Undirected Weighted LCC* Betweenness: Eq. (5)
- Undirected Weighted LCC Closeness: Eq. (4)
- Undirected Weighted LCC Out-Degree Centrality: Eq. (1)
- Undirected Weighted LCC Pagerank: Eq. (6)

$\star$: Largest Connected Component

Undirected unweighted

Replacing every oriented edge $(u, v) \in E$ by an undirected edge (u, v) with weight $\hat{w}(u, v) = 1$.

- Undirected Unweighted LCC Betweenness: Eq. (5)
- Undirected Unweighted LCC Closeness: Eq. (4)
- Undirected Unweighted LCC Out-degree Centrality: Eq. (1)
- Undirected Unweighted LCC Pagerank: Eq. (6)

References

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