Supplementary material for: Temporal evolution of ‘automatic gain scaling’.

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This document details the modelling work on activation dependent stiffness undertaken for the associated paper.

Basic approach

To examine how changes in muscle activation contribute to joint stiffness we: (a) built a simple model of the musculoskeletal system, (b) held the activation of the muscle constant at different levels, (c) applied perturbations and, (d) observed what effect the different levels of muscle activation had on the induced kinematics.

Specification of the model

The examined system is a single segment with a revolute joint controlled by a single lumped muscle (Supplemental Figure 1). The single joint - which is made to emulate the elbow joint in the related experiments - is simulated as a rigid segment with an inertia of $I = 0.07$ kilograms·metres^2, similar to that of the human forearm and hand taken together [8]. The dynamics for the single joint are thus given by the equation:

$$\ddot{\theta} = \frac{\tau(t)}{I} \tag{1}$$

where, $\ddot{\theta}$, is the angular acceleration of the joint and, $\tau(t)$, is the total torque applied at the joint at time, t. The total torque, $\tau(t)$, is net of the three

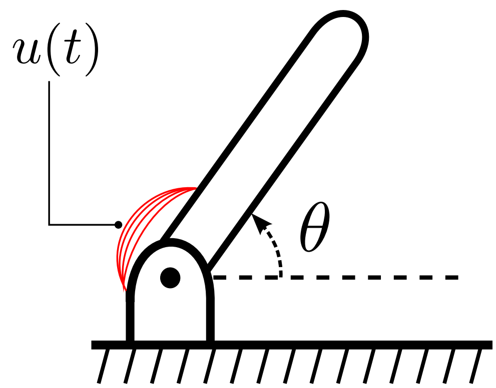

Figure 1: A schematic of the system used for the simulations described in this document. A single joint arm is controlled by a single flexor muscle (in red). The angular position of the joint, θ, is measured in radians from the horizontal dashed line. The motor command, $u(t)$ drives the muscle which acts on the arm with torque, $\tau_{\mathrm{m}}(t)$.

contributing torques as given by:

$$\tau(t) = \tau_{\mathrm{m}}(t) + \tau_{\mathrm{bias}}(t) + \tau_{\mathrm{pert}}(t) \tag{2}$$

where, τ_{m}, is the torque produced by the muscle, τ_{bias}, is the background load and, τ_{pert}, is the contribution from the perturbation applied to the joint during the trial. The joint angle, θ, and velocity, $\dot{\theta}$, evolve according to the following equation:

$$\frac{d}{dt}\begin{pmatrix} \theta \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \dot{\theta} \\ \ddot{\theta} \end{pmatrix} \tag{3}$$

In practice, simple Euler integration with, $\Delta t = 0.005$ seconds, is used to solve this equation through time. The joint is driven by a single monoarticular lumped flexor muscle with a physiological cross-sectional area of, $PCSA = 23$ centimetres^2 [5] (18 centimetres^2 from the monoarticulars and 5 centimetres^2 from the biarticulars). The maximum force production for the muscle is taken to be 31.8 Newton/centimetres^2 [7]. The moment arm of the muscle is taken as, $M = 0.04$ metres [6, 5], and the optimal muscle length L_0, occurs at the joint angle $\frac{\pi}{2}$.

The muscle model was composed of three discernable components: a force-length curve, a force-velocity curve, and a component which models the dependence of force on length and activation together [1]. The model is a pared back version of the Virtual Muscle published by Cheng et al. (2000). The unitless tension, T, which is produced via the product of the three modelled components, is given by the equation:

$$\mathrm{T}(\mathrm{FL}(L), \mathrm{FV}(V, L), \mathrm{Af}(a, L_{\mathrm{eff}})) = \mathrm{FL}(L) \cdot \mathrm{FV}(V, L) \cdot \mathrm{Af}(a, L_{\mathrm{eff}}) \qquad (4)$$

The unitless tension from the above equation is then used to calculate the torque generated by the muscle, τ_m, according to the following equation:

$$\tau_m(t) = M \cdot \mathrm{T}(\mathrm{FL}(L), \mathrm{FV}(V, L), \mathrm{Af}(a, L_{\mathrm{eff}})) \cdot PCSA \cdot 31.8 \qquad (5)$$

where, M, is the moment arm for the muscle; L, L_{eff}, and V are the length, effective length, and velocity of the muscle in units of L_0, L_0, and L_0/second respectively; and finally, a, is the activation of the muscle in units of $f_{0.5}$. The dependence of force on the length of the muscle (in normalized units of L_0) is given by the equation [1]:

$$\mathrm{FL}(L) = \exp\left(-\mathrm{abs}\left(\frac{(L^{\beta} - 1)}{\omega}\right)^{\rho}\right) \qquad (6)$$

The dependence of force on the length and velocity of the muscle (in normalized units of L_0 and L_0/second respectively) is given by the equation [1]:

$$\mathrm{FV}(V, L) = \begin{cases} \dfrac{V_{\max} - V}{V_{\max} + (c_{V0} + c_{V1}L)V}, & V \leq 0 \\ \dfrac{b_V - (a_{V0} + a_{V1}L + a_{V2}L^2)V}{b_V + V}, & V > 0 \end{cases} \qquad (7)$$

The dependence of the muscle force on muscle activation, a, and the effective length of the muscle, L_{eff}, is captured by the following equations [1]:

$$\mathrm{Af}(a, L_{\mathrm{eff}}) = 1 - \exp\left[-\left(\frac{a}{a_f n_f}\right)^{n_f}\right] \qquad (8)$$

$$n_f = n_{f0} + n_{f1}\left(\frac{1}{L_{\mathrm{eff}}} - 1\right) \qquad (9)$$

The effective length of the muscle, which acts like a low-pass filter of the actual muscle length, evolves according to the differential equation [1]:

$$\dot{L}_{\text{eff}}(L, \text{Af}) = \frac{[L - L_{\text{eff}}]^3}{T_L\,(1 - \text{Af})} \tag{10}$$

Since, the effective length is a low pass filtered version of the actual muscle length, the stiffness like properties induced by equation 8 will tend to enter slowly after a perturbation. It should be noted that, even if the actual length is used in place of the effective length, the results described here are not significantly effected.

Muscle activation, a (in units of $f_{0.5}$), is generated by passing the neural command, u, through a filter which approximates calcium dynamics [2, 5]. Here, we used the simplified version of this filter employed by Li and Todorov (2005).

$$\dot{a} = \frac{u(t)}{\tau_{\text{calcium}}(u, a)} \tag{11}$$

where,

$$\tau_{\text{calcium}}(u, a) = \begin{cases} \tau_{\text{deact}} + u(t)(\tau_{\text{deact}} - \tau_{\text{act}}), & u > a \\ \tau_{deact}, & u \leq a \end{cases} \tag{12}$$

Constants used to parameterize the muscle property equations are given in the following table and are based on those used in [1, 5].

Parameter	Parameter Value
β	1.55
ω	0.81
ρ	2.12
$V_{\max}$	-7.39
a_{V0}	-3.12
a_{V1}	4.21
a_{V2}	-2.67
b_V	0.62
c_{V0}	-3.21
c_{V1}	4.17
a_f	0.56
n_{f0}	2.11
n_{f1}	3.31
T_L	0.088
τ_{act}	0.05
τ_{deact}	0.066

The sequence of events for a single 'trial'

Each 'trial' is similar to the corresponding experimental work. The sequence of events for each trial runs as follows:

1. A PID control law is used to move the arm to the same position, $\theta = \frac{\pi}{2}$, for each trial.
2. A hand picked muscle stimulus frequency is kept constant throughout the trial. The muscle stimulus frequency, $u(t) = C$, is chosen so that when the PID control law stabalizes the joint it has to use either 1, 2, or 3 Newton·metres of force to maintain the posture. These are the background loads as described in the accompanying paper (i.e. the PID control law was outputing -1, -2, or -3 Newton·metres respectively).
3. The PID control is given enough time to stabilize the joint position. Then, its output is held constant for the remainder of the trial; i.e. it's output is clamped and plays the roll of the background load for the remainder of the trial.

4. Step perturbations are added in on top of the constant background load. The perturbation, depending on the trial, is -1.25 or -2.5 Newton·metres. Since there are two perturbations, and three background load, there are $2 \times 3 = 6$ different trials. Perturbations are held on for 2 seconds, but we are only really interested in the first 250ms of movement.

Simulation code

These simulations were written in the Matlab Language (written in Version 7.6.0, R2008a). The mfiles for the simulations can be found online with the rest of the supplementary material for the associated paper. The following files are required to reproduce the simulations: `mainscript.m`, `stiffness.m`, `muscleinit.m`, `muscleconfig.m`, `muscleconfigscript.m`, `muscleconfig_userdef.m`, `muscleforwardeff.m`, `pidcontrol.m`. To reproduce the simulations, make sure that all of these files are in your Matlab path and then run `mainscript.m`.

Acknowledgements

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References

[1] Ian E. Brown, Ernest J. Cheng, and Gerald E. Loeb. Measured and modeled properties of mammalian skeletal muscle. ii. the effects of stimulus frequency on force-length and force-velocity relationships. *Journal of Muscle Research and Cell Motility*, 20:627–643, 1999.

[2] Ian E. Brown and Gerald E. Loeb. Measured and modeled properties of mammalian skeletal muscle. iv. the effects of stimulus frequency on stretch-induced force enhancement and shortening-induced force depression. *Journal of Muscle Research and Cell Motility*, 21:21–31, 2000.

[3] Ernest J. Cheng, Ian E. Brown, and Gerald E. Loeb. Virtual muscle: a computational approach to understanding the effects of muscle properties on motor control. *Journal of Neuroscience Methods*, 101:117–130, 2000.

[4] Kirsten M. Graham and Stephen H. Scott. Morphometry of *macaca mulatta* forelimb. iii. moment arm of shoulder and elbow muscles. *Journal of Morphology*, 255:301–314, 2003.

[5] Weiwei Li and Emanuel Todorov. Iterative linear quadratic regulator design for nonlinear biological movement systems. *1st International Conference on Informatics in Control, Automation and Robotics*, 1:222–229, 2004.

[6] Pascale Pigeon, L'Hocine Yahia, and Anatol G. Feldman. Moment arms and lengths of human upper limb muscles as functions of joint angles. *Journal of Biomechanics*, 29(10):1365–1370, 1996.

[7] Stephen H. Scott, Ian E. Brown, and Gerald E. Loeb. Mechanics of feline soleus: I. effect of fascicle length and velocity and force output. *Journal of Muscle Research and Cell Motility*, 17:207–219, 1996.

[8] David A. Winter. *Biomechanics of Human Movement*. John Wiley & Sons, New York, 1979.