

Supporting Text S1

Stochastic context-free grammars (SCFGs) employed in study. SCFGs are provided in Chomsky normal form. Different models of recruitment using the same grammar are defined by alternative parameterizations, *i.e.*, assignment of rule probabilities.

1. Uniform recruitment models:

Rule	Multinomial (\mathcal{M}_1)	Binomial (\mathcal{M}_1^b)
$S \rightarrow \mathbf{bR}$	1	
$\mathbf{b} \rightarrow ($	1	
$\mathbf{R} \rightarrow \mathbf{Gr}$	1	
$\mathbf{r} \rightarrow)$	1	
$\mathbf{G} \rightarrow \mathbf{p}$	p_1	p_1^3
$\mathbf{p} \rightarrow \cdot$	1	
$\mathbf{G} \rightarrow \mathbf{pI}$	p_2	$3p_1^2(1 - p_1)$
$\mathbf{I} \rightarrow \mathbf{bJ}$	1	
$\mathbf{J} \rightarrow \mathbf{Gr}$	1	
$\mathbf{G} \rightarrow \mathbf{pV}$	p_3	$3p_1(1 - p_1)^2$
$\mathbf{V} \rightarrow \mathbf{bU}$	1	
$\mathbf{U} \rightarrow \mathbf{GJ}$	1	
$\mathbf{G} \rightarrow \mathbf{pW}$	$1 - p_1 - p_2 - p_3$	$(1 - p_1)^3$
$\mathbf{W} \rightarrow \mathbf{bX}$	1	
$\mathbf{X} \rightarrow \mathbf{GU}$	1	
$\mathbf{G} \rightarrow :$	(1)	

Each model parameter is denoted by p_i where $max(i)$ corresponds to the degrees of freedom in the model. Note that the probabilities of rules for replacing the non-terminal symbol G sum to greater than unity because of an additional rule for censoring respondents (each encoded in the RDS string as ‘:’); this rule is disregarded during model inference. Chomsky normal form (CNF) of a grammar can be interpreted as an expansion of rules from their original form, as demonstrated in the following example: $\mathbf{G} \rightarrow \mathbf{pI} \rightarrow \cdot \mathbf{bJ} \rightarrow \cdot (\mathbf{Gr} \rightarrow \cdot (\mathbf{G})$

2. Hidden state recruitment model ($\mathcal{M}_{1 \times 2}$), no visible differentiation among respondents:

Rule		Rule	
S → bR	1		
b → (1		
R → Gr	p_1		
R → Hr	$1 - p_1$		
r →)	1		
G → d	p_2	H → d	p_6
d → .	1		
G → pI	p_3	H → pi	p_7
I → bJ	1	i → pj	1
J → Gr	p_4	j → Gr	p_8
J → Hr	$1 - p_4$	j → Hr	$1 - p_8$
G → pV	p_5	H → pv	p_9
V → bU	1	v → bu	1
U → GJ	p_4	u → Gj	p_8
U → HJ	$1 - p_4$	u → Hj	$1 - p_8$
G → pW	$1 - p_2 - p_3 - p_5$	H → pw	$1 - p_6 - p_7 - p_9$
W → bX	1	w → bx	1
X → GU	p_4	x → Gu	p_8
X → HU	$1 - p_4$	x → Hu	$1 - p_8$
G → :	(1)	H → :	(1)

Entries are left blank to indicate that the rule assumes the same probability parameterization as the preceding rule on the same row.

3. Binary visible state recruitment models (\mathcal{M}_2 family):

Rule	Full model	Constrain number of recruits (\mathcal{M}_2)	Constrain mixing rates (\mathcal{M}_2^C)	Mixing dependent on recruitment (\mathcal{M}_2^D)
S → bR	1			
b → (1			
R → Gr	p_1			
R → Hr	$1 - p_1$			
r →)	1			
G → d	p_2			
d → A	1			
G → dI	p_3			
I → bJ	1			
J → Gr	p_4			
J → Hr	$1 - p_4$			
G → dV	p_5			
V → bU	1			
U → GJ	p_4			$p_4 p_7 / (p_4 p_7 + 1 - p_4)$
U → HJ	$1 - p_4$			$(1 - p_4) / (p_4 p_7 + 1 - p_4)$
G → dW	$1 - p_2 - p_3 - p_5$			
W → bX	1			
X → GU	p_4			$p_4 p_7^2 / (p_4 p_7^2 + 1 - p_4)$
X → HU	$1 - p_4$			$(1 - p_4) / (p_4 p_7^2 + 1 - p_4)$
G → :	(1)			
H → e	p_6	p_2	p_6	p_2
e → B	1			
H → ei	p_7	p_3	p_7	p_3
i → bj	1			
j → Gr	p_8	p_6	p_4	p_6
j → Hr	$1 - p_8$	$1 - p_6$	p_4	$1 - p_6$
H → ev	p_9	p_5	p_8	p_5
v → bu	1			
u → Gj	p_8	p_6	p_4	$p_6 / (1 - p_6 p_7 + p_6)$
u → Hj	$1 - p_8$	$1 - p_6$	$1 - p_4$	$(1 - p_6) p_7 / (1 - p_6 p_7 + p_6)$
H → ew	$1 - p_6 - p_7 - p_9$	$1 - p_2 - p_3 - p_5$	$1 - p_6 - p_7 - p_8$	$1 - p_2 - p_3 - p_5$
w → bx	1			
x → Gu	p_8	p_6	p_4	$p_6 / (1 - p_6 p_7^2 + p_6)$
x → Hu	$1 - p_8$	$1 - p_6$	$1 - p_4$	$(1 - p_6) p_7^2 / (1 - p_6 p_7^2 + p_6)$
H → :	(1)			

In constraining the number of recruits, we assume that the number of peers recruited by a respondent follows the same distribution regardless of the respondent's visible state. Similarly, in constraining the mixing rates, we assume that the probability that a respondent recruits peers in a given visible state is independent of the respondent's visible state.

4. Binary visible state model with mixing on latent variables ($\mathcal{M}_{2 \times 2}^L$) or hidden states ($\mathcal{M}_{2 \times 2}^H$), and constrained numbers of recruits:

Rule	Latent mixing	Hidden	Rule	Latent mixing	Hidden
S → bR	1				
b → (1				
R → Gr	p_1	p_1			
R → Hr	p_2	$1 - p_1$			
R → gr	p_3	0			
R → hr	$1 - p_1 - p_2 - p_3$	0			
r →)	1				
G → d	p_4	p_2	g → d	p_4	p_9
d → A	1				
G → dI	p_5	p_3	g → dI'	p_5	p_{10}
I → bJ	1		I' → bJ'	1	
J → Gr	$p_6 p_7$	$p_4 p_5$	J' → Gr	$(1 - p_9) p_7$	0
J → Hr	$p_6(1 - p_7)$	$p_4(1 - p_5)$	J' → Hr	$(1 - p_9)(1 - p_7)$	0
J → gr	$(1 - p_6) p_8$	$(1 - p_4) p_5$	J' → gr	$p_9 p_8$	p_{11}
J → hr	$(1 - p_6)(1 - p_8)$	$(1 - p_4)(1 - p_5)$	J' → hr	$p_9(1 - p_8)$	$1 - p_{11}$
G → dV	p_{10}	p_6	g → dV'	p_{10}	p_{12}
V → bU	1		V' → bU'	1	
U → GJ	$p_6 p_7$	$p_4 p_5$	U' → GJ'	$(1 - p_9) p_7$	0
U → HJ	$p_6(1 - p_7)$	$p_4(1 - p_5)$	U' → HJ'	$(1 - p_9)(1 - p_7)$	0
U → gJ	$(1 - p_6) p_8$	$(1 - p_4) p_5$	U' → gJ'	$p_9 p_8$	p_{11}
U → hJ	$(1 - p_6)(1 - p_8)$	$(1 - p_4)(1 - p_5)$	U' → hJ'	$p_9(1 - p_8)$	$1 - p_{11}$
G → dW	$1 - p_4 - p_5 - p_{10}$	$1 - p_2 - p_3 - p_6$	g → dW'	$1 - p_4 - p_5 - p_{10}$	$1 - p_9 - p_{10} - p_{12}$
W → bX	1		W' → bX'	1	
X → GU	$p_6 p_7$	$p_4 p_5$	X' → GU'	$(1 - p_9) p_7$	0
X → HU	$p_6(1 - p_7)$	$p_4(1 - p_5)$	X' → HU'	$(1 - p_9)(1 - p_7)$	0
X → gU	$(1 - p_6) p_8$	$(1 - p_4) p_5$	X' → gU'	$p_9 p_8$	p_{11}
X → hU	$(1 - p_6)(1 - p_8)$	$(1 - p_4)(1 - p_5)$	X' → hU'	$p_9(1 - p_8)$	$1 - p_{11}$
G → a	(1)		g → a	(1)	
H → e		p_2	h → d		p_9
e → B		1			

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Rules	Latent mixing	Hidden	Rules	Latent mixing	Hidden
$H \rightarrow ei$		p_3	$h \rightarrow di'$		p_{10}
$i \rightarrow bj$	1		$i' \rightarrow bj'$	1	
$j \rightarrow Gr$	$p_6 p_7$	$p_7 p_8$	$j' \rightarrow Gr$	$(1 - p_9) p_7$	0
$j \rightarrow Hr$	$p_6(1 - p_7)$	$p_7(1 - p_8)$	$j' \rightarrow Hr$	$(1 - p_9)(1 - p_7)$	0
$j \rightarrow gr$	$(1 - p_6) p_8$	$(1 - p_7) p_8$	$j' \rightarrow gr$	$p_9 p_8$	p_{11}
$j \rightarrow hr$	$(1 - p_6)(1 - p_8)$	$(1 - p_7)(1 - p_8)$	$j' \rightarrow hr$	$p_9(1 - p_8)$	$1 - p_{11}$
$H \rightarrow ev$		p_6	$h \rightarrow dv'$		p_{12}
$v \rightarrow bu$	1		$v' \rightarrow bu'$	1	
$u \rightarrow Gj$	$p_6 p_7$	p_6	$u' \rightarrow Gj'$	$(1 - p_9) p_7$	0
$u \rightarrow Hj$	$p_6(1 - p_7)$	$1 - p_6$	$u' \rightarrow Gj'$	$(1 - p_9)(1 - p_7)$	0
$u \rightarrow gj$	$(1 - p_6) p_8$	$(1 - p_7) p_8$	$u' \rightarrow gr$	$p_9 p_8$	p_{11}
$u \rightarrow hj$	$(1 - p_6)(1 - p_8)$	$(1 - p_7)(1 - p_8)$	$u' \rightarrow hr$	$p_9(1 - p_8)$	$1 - p_{11}$
$H \rightarrow ew$		$1 - p_2 - p_3 - p_6$	$h \rightarrow dw'$		$1 - p_9 - p_{10} - p_{12}$
$w \rightarrow bx$	1		$w' \rightarrow bx'$	1	
$x \rightarrow Gu$	$p_6 p_7$	p_6	$x' \rightarrow Gu'$	$(1 - p_9) p_7$	0
$x \rightarrow Hu$	$p_6(1 - p_7)$	$1 - p_6$	$x' \rightarrow Gu'$	$(1 - p_9)(1 - p_7)$	0
$x \rightarrow gu$	$(1 - p_6) p_8$	$(1 - p_7) p_8$	$x' \rightarrow gr$	$p_9 p_8$	p_{11}
$x \rightarrow gu$	$(1 - p_6)(1 - p_8)$	$(1 - p_7)(1 - p_8)$	$x' \rightarrow hr$	$p_9(1 - p_8)$	$1 - p_{11}$
$H \rightarrow b$	(1)		$h \rightarrow b$	(1)	

Our choice of this grammar is motivated by results from binary visible state models without hidden states, is supported by model selection procedures (Akaike's information criterion), and obtains similar parameter estimates to the full binary visible state model with hidden states.