Power law model of microsphere size as a function of pH

The reaction of vinylsulfone groups with amines will be primarily due to the more nucleophilic [-NH_2] groups instead of the [-NH_3^+] groups.

$$[-NH_2] + [-VS] \xrightarrow{k_f} [NH - VS]$$

The rate of the irreversible reaction is dominated by:

$$\frac{d[NH - VS]}{dt} = k_f[-NH_2][-VS]$$

From the Henderson-Hasselbach equation and the conservation law:

$$[-NH_2] = \frac{10^{pH-pKa}}{1 + 10^{pH-pKa}}[-NH]_{tot} = \frac{1}{1 + 10^{pKa-pH}}[-NH]_{tot}$$

pH is held constant by buffering and thus the fraction of total amines as –NH_2 groups is constant throughout the reaction. Note that $[-NH]_{tot}$ is not $[-NH]_{tot,0}$, which is the initial concentration of amine groups. The reaction rate is:

$$\frac{d[NH - VS]}{dt} = \frac{k_f}{1 + 10^{pKa-pH}}[-NH]_{tot}[-VS]$$

$[-NH]_{tot}$ was equal to [VS] due to 1:1 ratio of amines to vinyl sulfone groups in the reaction:

$$\frac{d[NH - VS]}{dt} = \frac{k_f}{1 + 10^{pKa-pH}}\left([-NH]_{tot,0} - [NH - VS]\right)^2$$

[NH-VS] is initially zero:

$$\frac{[NH - VS]}{\left([-NH]_{tot,0}\right)\left([-NH]_{tot,0} - [NH - VS]\right)} = \frac{k_f t}{1 + 10^{pKa-pH}}$$

The degree of conversion is χ = [NH-VS]/$[-NH]_{tot,0}$:

$$\frac{\chi}{(1 - \chi)} = \frac{[-NH]_{tot,0} k_f t_\chi}{1 + 10^{pKa-pH}}$$

The time to reach a certain extent of reaction is:

$$t_\chi = \frac{\chi}{(1-\chi)} \frac{1+10^{pKa-pH}}{[-NH]_{tot,0} k_f} \propto 1+10^{pKa-pH}$$

Gelation should occur at the same degree of conversion regardless of pH, although this degree of conversion at which coarsening stops is unknown.

For coarsening, the average diameter of the PEG-rich domains will grow as the cube root of time. The mean diameter of PEG microspheres, $d_{\mu sphere}$ should scale as:

$$d_{\mu sphere} \propto \sqrt[3]{t_\chi} \propto \sqrt[3]{1+10^{pKa-pH}}$$