Temporal-difference reinforcement learning with distributed representations

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Appendix S1

Hyperbolic discounting from a sum of exponentials. The summed effect of these exponential discounting functions provides the overall agent with hyperbolic discounting:

$$\int_0^1 \gamma^x d\gamma = \frac{1}{1+x} \tag{1}$$

By the standard integration power law,

$$\int_0^1 \gamma^x d\gamma = \lim_{\gamma \to +0} \frac{-(\gamma^{x+1} - 1)}{x+1} \tag{2}$$

which, if $x > 0$, approaches $1/(1+x)$ as γ approaches 0 from $\gamma > 0$.