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Supporting Material

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Table S1 Variables and constants

Diffusion and aster aggregation with the karyosome	
r_k	Radius of karyosome
r_{gv}	Radius of germinal vesicle
ρ	Distance of a given point from the karyosome (i.e., radial coordinate in the spherical coordinate system)
S	Distance between the center of the germinal vesicle and the karyosome
θ	Colatitude in the spherical coordinate system (i.e., angle defined by the given point, the karyosome, and the center of the germinal vesicle)
Capture of asters by the karyosome	
k^*_{on}	Pseudo-first order association rate constant for asters binding cytoskeletal filaments
k_{off}	Dissociation rate constant for asters from cytoskeletal filaments
C_{eq}	Fraction of bound asters at equilibrium
F_{eq}	Fraction of free asters at equilibrium, $F_{eq}+ C_{eq}=1$
K_b	Strength of binding constant
f	A function of space and time representing the number of asters per unit volume
D	Diffusion coefficient

v	Transport velocity
C_1	1st constant of integration from solving Eq. 5 at steady state
C_2	2nd constant of integration from solving Eq. 5 at steady state
Q	v/DK_b (quantifies importance of transport relative to diffusion)
R	Distance of a given point from the center of a capturing object (here the karyosome)
J_{in}	Flux of asters toward the karyosome
J_{out}	Flux of asters outward from the karyosome
J_ρ	Flux of asters along the radial dimension
Ei	Exponential integral function
C_{init}	Fraction of asters initially bound
Aster capture by the karyosome in live cells	
M_r	Relative molecular mass
Spindle elongation and the onset of bipolarity	
x	Spindle axis coordinate, distance to chromosomes
ρ_1	Density of unaligned spindle microtubules
ρ_2	Density of spindle axis-aligned microtubules
ρ_3	Density of cross-linking spindle microtubules
k_a	Rate constant of aligning microtubules
k_d	Rate constant of losing microtubule alignment
k_c	Rate constant of cross-linking microtubules attached to different chromosomes
k_{dc}	Rate constant of breaking cross-links between microtubules attached to different chromosomes
k_e	Rate constant quantifying elongation for spindle axis-aligned microtubules (defined by Eq. 13)

k_r	Rate constant quantifying elongation for other microtubules (defined by Eq. 16)
L	Spindle length

Capture of asters by the karyosome

An expanded form of Eq. 11 is shown below:

$$P_{tot} = \frac{3}{4\pi(r_{gv}{}^3 - r_k{}^3)}\left(\begin{array}{l}\int_0^{2\pi}\int_{r_k}^{r_{gv}-S}\int_0^{\pi} P(\rho)\rho^2 \sin\theta d\theta d\rho d\phi \\ +\int_0^{2\pi}\int_{r_{gv}-S}^{r_{gv}+S}\int_0^{\cos^{-1}\left(\frac{S^2+\rho^2-r_{gv}{}^2}{2S\rho}\right)} P(\rho)\rho^2 \sin\theta d\theta d\rho d\phi\end{array}\right) =$$

$$\frac{3}{2(r_{gv}{}^3 - r_k{}^3)\left(1-e^{-Q/r_k}\right)}\left(\begin{array}{l}\frac{1}{3}\left[e^{-Q/r_m}(-2r_m{}^3 + Qr_m{}^2 - Q^2 r_m) + 2r_m{}^3 - Q^3 Ei(-Q/r_m)\right] \\ -\frac{1}{3}\left[e^{-Q/r_k}(-2r_k{}^3 + Qr_k{}^2 - Q^2 r_k) + 2r_k{}^3 - Q^3 Ei(-Q/r_k)\right] \\ +\frac{1}{24}\left[\begin{array}{l}\left(\frac{-Q^4}{2S} - 4Q^3 + 6WQ^2\right)Ei(-Q/r_p) + \\ r_p e^{-Q/r_p}\left(\begin{array}{l}Q^2\left(\frac{r_p}{2S} - 4\right) + 2Q\left(3W - \frac{r_p{}^2}{2S} + 2r_p\right) \\ +2r_p(e^{Q/r_p} - 1)\left(3W - \frac{3r_p{}^2}{2S} + 4r_p\right) - \frac{Q^3}{2S}\end{array}\right)\end{array}\right] \\ -\frac{1}{24}\left[\begin{array}{l}\left(\frac{-Q^4}{2S} - 4Q^3 + 6WQ^2\right)Ei(-Q/r_m) + \\ r_m e^{-Q/r_m}\left(\begin{array}{l}Q^2\left(\frac{r_m}{2S} - 4\right) + 2Q\left(3W - \frac{r_m{}^2}{2S} + 2r_m\right) \\ +2r_m(e^{Q/r_m} - 1)\left(3W - \frac{3r_m{}^2}{2S} + 4r_m\right) - \frac{Q^3}{2S}\end{array}\right)\end{array}\right]\end{array}\right)$$

$$r_p = r_{gv} + S$$

$$r_m = r_{gv} - S$$

$$W = \frac{r_{gv}{}^2 - S^2}{S}$$

where *Ei* is an exponential integral function. Note that the total captured fraction does not depend on *v*, K_b, or *D* explicitly, but only on *Q*.

Eq. 12 can be expanded as shown:

$$C_{init} = \frac{3}{4\pi(r_{gv}{}^3 - r_k{}^3)}\left(\begin{array}{l}\int_0^{2\pi}\int_{r_k}^{r_{gv}-S}\int_0^{\pi}\frac{1}{1+K_b\rho^2}\rho^2\sin\theta d\theta d\rho d\phi \\ +\int_0^{2\pi}\int_{r_{gv}-S}^{r_{gv}+S}\int_0^{\cos^{-1}\left(\frac{S^2+\rho^2-r_{gv}{}^2}{2S\rho}\right)}\frac{1}{1+K_b\rho^2}\rho^2\sin\theta d\theta d\rho d\phi\end{array}\right) =$$

$$\frac{3}{2(r_{gv}{}^3 - r_k{}^3)}\left(\begin{array}{l}2\left(\frac{r_m}{K_b} - \frac{\tan^{-1}(r_m\sqrt{K_b})}{K_b{}^{3/2}} - \frac{r_k}{K_b} + \frac{\tan^{-1}(r_k\sqrt{K_b})}{K_b{}^{3/2}}\right) \\ +\frac{1}{4K_b{}^2 S}\left(\begin{array}{l}K_b r_p(4S - r_p) - K_b r_m(4S - r_m) \\ +4\sqrt{K_b}S\left(\tan^{-1}(\sqrt{K_b}r_m) - \tan^{-1}(\sqrt{K_b}r_p)\right) \\ +(K_b SW + 1)\ln\left(\frac{K_b r_p{}^2 + 1}{K_b r_m{}^2 + 1}\right)\end{array}\right)\end{array}\right)$$