

# 1 Supplementary Materials

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## 3 **Estimation of trial-to-trial generalization under noise influence**

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5 Here we provide supplementary simulation results for evaluating the performance of the  
6 parameter estimation method (the prediction-error method) in the presence of process and  
7 observation noises. Because human adaptation processes are inherently noisy, it is necessary to  
8 assess how robust the estimation method is. Noise terms are included to the single-state  
9 state-space model as

$$10 \quad \begin{cases} \Phi_{k+1} = \Phi_k + BH_k^T \Delta\phi_k + w_k \\ \phi_k = H_k \Phi_k + v_k \end{cases} \quad (S1)$$

11 where  $w$  and  $v$  are process and observation noises with zero means and covariances

12  $\sigma_{proc}^2 \cdot I$  and  $\sigma_{obs}^2$ , respectively. Using various representative values of  $\sigma_{proc}$  and  $\sigma_{obs}$ , the  
13 state-space model generated 200 independent realizations of artificial error data for randomized  
14 target orders. In the simulations shown below three values of generalization widths ( $1^\circ$ ,  $30^\circ$   
15 and  $60^\circ$ ) were used to define the matrix  $B$  as in Fig.1 of the main text. The prediction-error  
16 method was then applied to the artificial error data to estimate the eight parameters of  $B$  matrix.

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18 First, we examined the individual effect of process and observation noises separately ( $(\sigma_{proc},$   
19  $\sigma_{obs})=(0.4, 0.0)$  in Fig.S1 and  $(\sigma_{proc}, \sigma_{obs})=(0.0, 3.0)$  in Fig.S2). The simulation results  
20 demonstrate that the prediction-error method essentially reproduced the shape of generalization  
21 functions. Note that we needed to use a smaller value of  $\sigma_{proc}$  than that of  $\sigma_{obs}$  because the  
22 process noise has an accumulative influence over error time courses. Next, we investigated the  
23 combined effects of process and observation noises ( $(\sigma_{proc}, \sigma_{obs})=(0.2, 2.)$  in Fig.S3,  $(\sigma_{proc},$   
24  $\sigma_{obs})=(0.4, 2.)$  in Fig.S4, and  $(\sigma_{proc}, \sigma_{obs})=(0.2, 4.)$  in Fig.S1). Again, the prediction-error method  
25 could recover the generalization functions reasonably well.

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27 Besides the simulation results demonstrated above, various combinations of noise levels and  
28 generalization function width were tested and confirmed that the prediction-error method was  
29 reasonably robust for a wide range of noise values. To conclude, the above numerical results  
30 guarantee that the prediction-error method is able to reliably reproduce the generalization width  
31 for a variety of process and observation noise strengths and may safely be applied to human

32 psychophysical error data.

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35 **Fitting of the state-space model to individual data**

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37 Fig S6 shows three individual error data together with the state-space model fitting (left column) and  
38 the derived individual trial-to-trial generalization functions (right column), using the same format of  
39 Fig. 6A and 6B. Each  $R^2$  value of fitness is included below the trial-to-trial error data. Each  
40 generalization function, though noisy, exhibits essentially a narrow single-peaked shape that was  
41 found in the group average (Fig. 6B).

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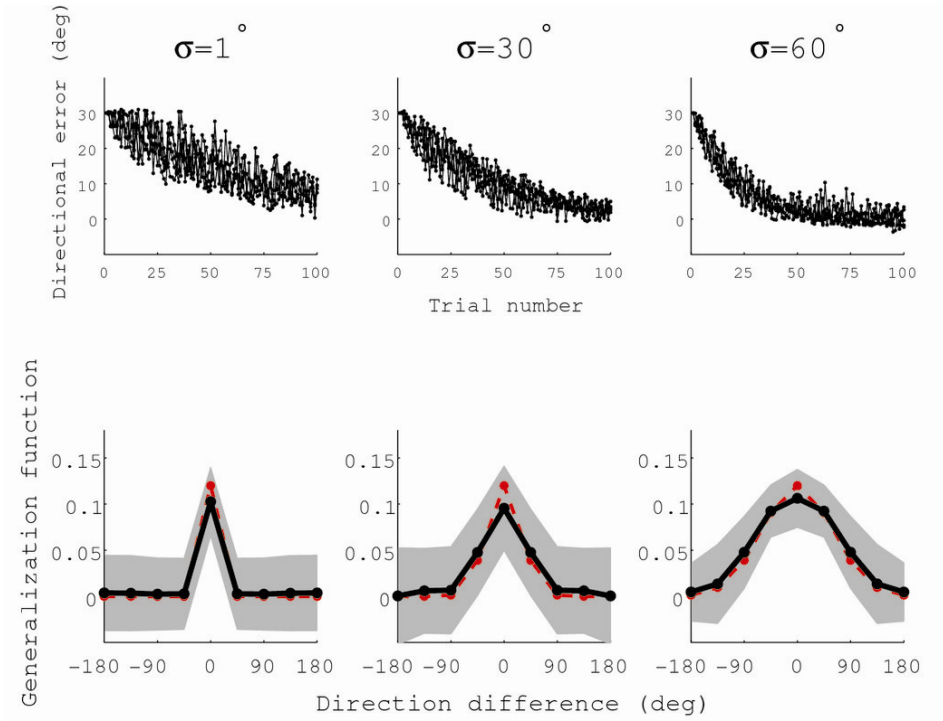


Figure S1: Estimation of generalization functions using the least-squares method. Left, middle, right columns are for generalization widths  $\sigma = 1^\circ, 30^\circ, 60^\circ$ . Panels in top rows are five typical error time courses, and panels in bottom are estimated generalization functions. Red dash lines represent true values.  $\sigma_{proc} = 0.4$  and  $\sigma_{obs} = 0$ .

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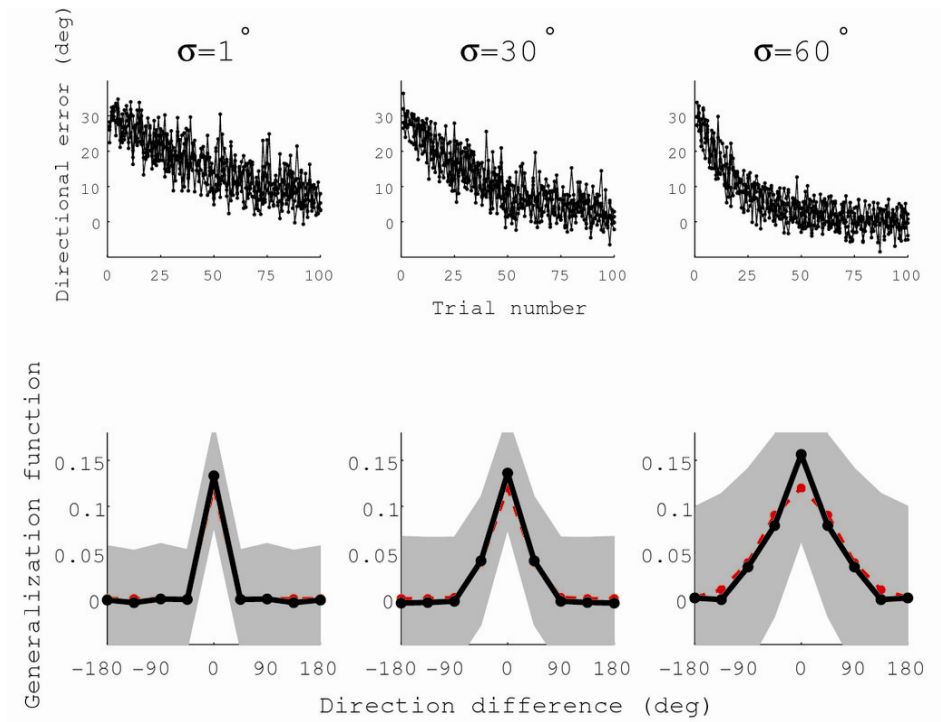


Figure S2:  $\sigma_{proc} = 0.0$  and  $\sigma_{obs} = 3.0$ .

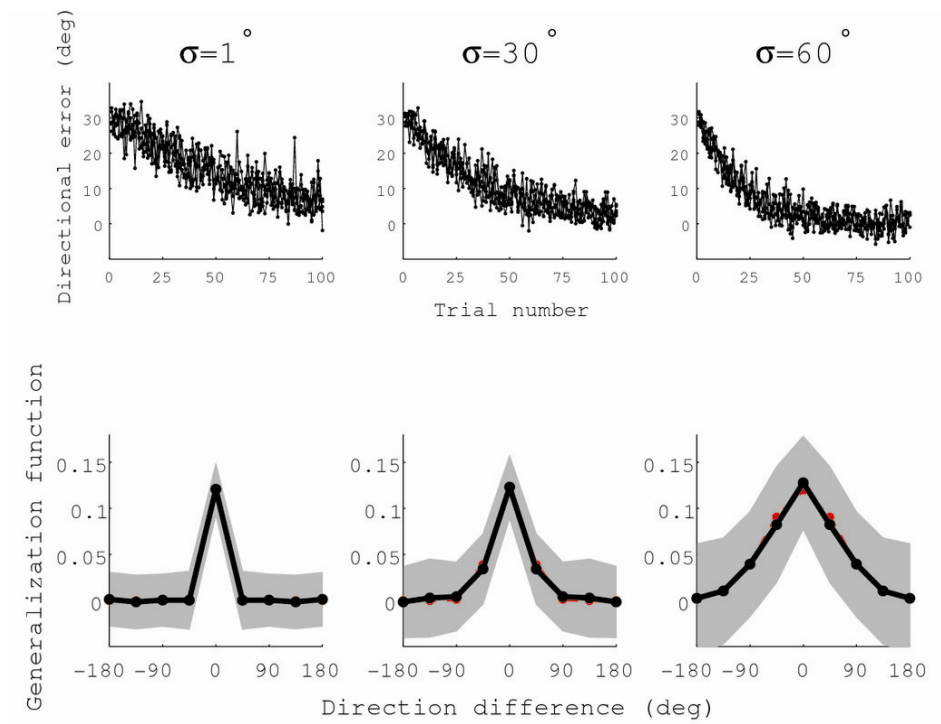
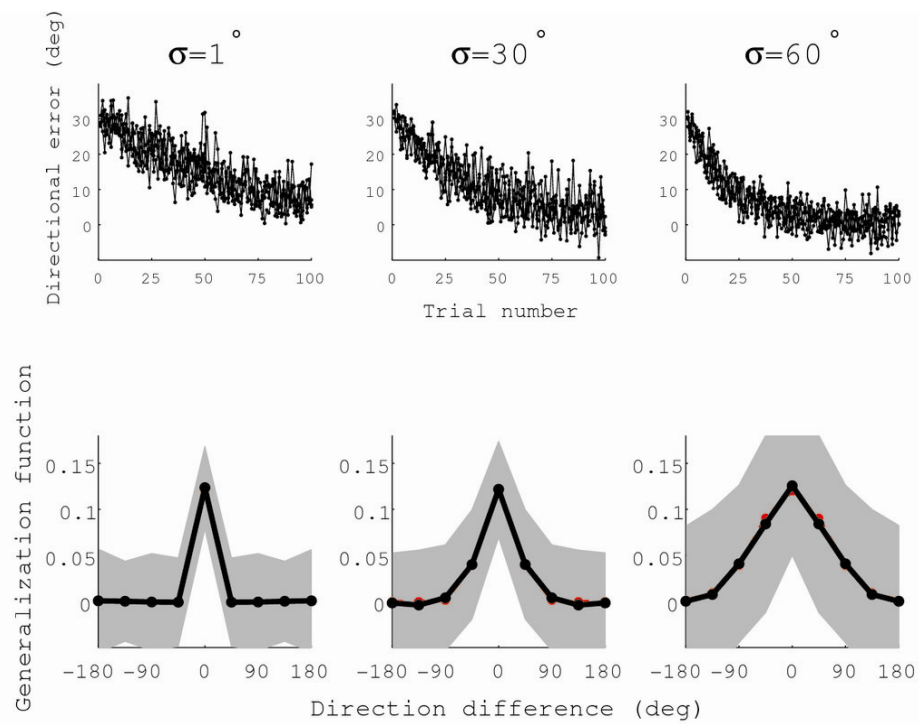


Figure S3:  $\sigma_{proc} = 0.2$  and  $\sigma_{obs} = 2.0$ .



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Figure S4:  $\sigma_{proc} = 0.4$  and  $\sigma_{obs} = 2.0$ .

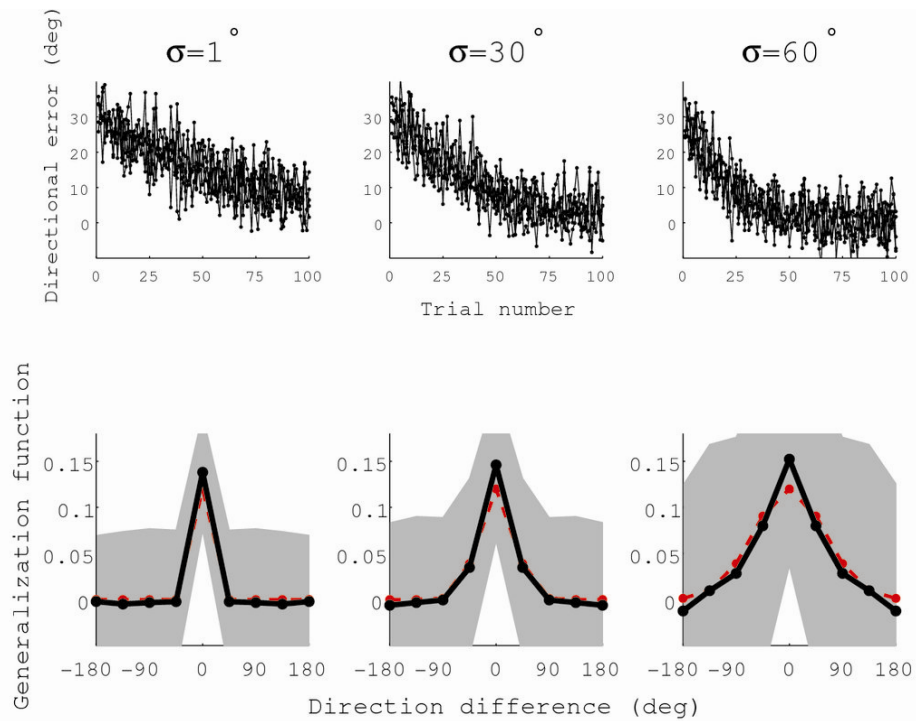


Figure S5:  $\sigma_{proc} = 0.2$  and  $\sigma_{obs} = 4.0$ .

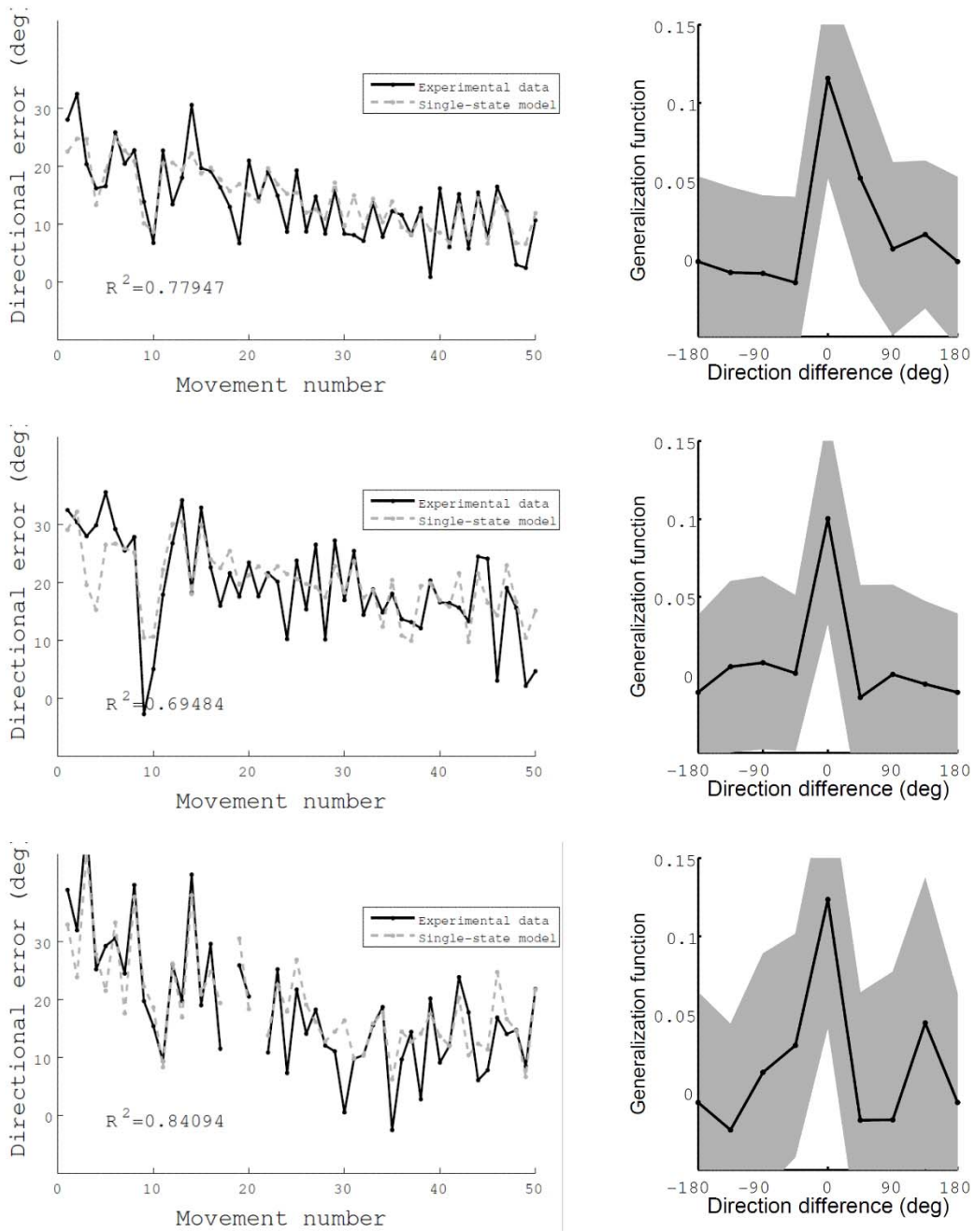


Figure S6: Experimental error data and the state-space model fitting (left column) and corresponding trial-to-trial generalization functions (right column) from three typical participants.