Web-based Supplementary Materials for "Group Testing Regression Models with Fixed and Random Effects"

Peng Chen,[1] Joshua M. Tebbs,[1,*] and Christopher R. Bilder[2]

[1]Department of Statistics, University of South Carolina, Columbia, SC 29208, U.S.A.

[2]Department of Statistics, University of Nebraska, Lincoln, NE 68583, U.S.A.

[*]*email*: tebbs@stat.sc.edu

Web Appendix A: Description of the Metropolis-Hastings algorithm used within the MCEM algorithm in Section 2.2.

At the bth E-step iteration with parameter estimates $\boldsymbol{\theta}^{(b)} = (\boldsymbol{\beta}^{(b)}, \boldsymbol{\varphi}^{(b)})$, the random effect $\mathbf{u}_i$ is generated from the conditional distribution $f(\mathbf{u}_i|\mathbf{T};\boldsymbol{\theta}^{(b)})$ as follows:

1. Start with $\mathbf{u}_i^{(0)} = \mathbf{0}$.

2. Generate a candidate point $\mathbf{u}_i^{(*)}$ from the $\mathcal{N}_q(\mathbf{0}, \mathbf{D}(\boldsymbol{\varphi}^{(b)}))$ proposal density.

3. Accept $\mathbf{u}_i^{(*)}$ with probability

$$A(\mathbf{u}_i^{\text{old}}, \mathbf{u}_i^{(*)}) = \min\left\{1, \frac{\prod_{j=1}^{n_i} f_{ij}(c_{ij}, \mathbf{x}_{ij}, \mathbf{u}_i^{(*)}, \boldsymbol{\beta}^{(b)})}{\prod_{j=1}^{n_i} f_{ij}(c_{ij}, \mathbf{x}_{ij}, \mathbf{u}_i^{(\text{old})}, \boldsymbol{\beta}^{(b)})}\right\}.$$

4. Return to Step 2.

After a sufficiently long "burn-in" period, we collect random draws $\mathbf{u}_i^{(1)}, \mathbf{u}_i^{(2)}, ..., \mathbf{u}_i^{(M)}$ using this algorithm. To estimate the information matrix in Section 2.3, we replace $\boldsymbol{\beta}^{(b)}$ and $\boldsymbol{\varphi}^{(b)}$ with $\widehat{\boldsymbol{\beta}}$ and $\widehat{\boldsymbol{\varphi}}$, respectively, to sample from $P(\mathbf{u}|\mathbf{T};\widehat{\boldsymbol{\theta}})$.

Web Appendix B: Additional details on the derivation of the score statistic from Section 3.2.

Let $\gamma_{12} = 1 - \gamma_1 - \gamma_2$ and define

$$\mu_{ij}(\boldsymbol{\beta}, \alpha_i) = \mu_{ij} \equiv \Pr(T_{ij} = 1|\mathbf{x}_{ij}, v_i) = \gamma_1 + \gamma_{12}\prod_{k=1}^{c_{ij}}\left\{1 - g(\boldsymbol{\beta}'\mathbf{x}_{ijk} + \alpha_i)\right\}.$$

The derivatives of μ_{ij} are given by

$$\begin{aligned}
\boldsymbol{\mu}_{ij}^{(1,0)} \equiv \frac{\partial \mu_{ij}}{\partial \boldsymbol{\beta}} &= \gamma_{12} \prod_{k=1}^{c_{ij}} \{1 - g(\boldsymbol{\beta}'\mathbf{x}_{ijk} + \alpha_i)\} \sum_{k=1}^{c_{ij}} \left\{ \frac{-\partial g(\boldsymbol{\beta}'\mathbf{x}_{ijk} + \alpha_i)/\partial \boldsymbol{\beta}}{1 - g(\boldsymbol{\beta}'\mathbf{x}_{ijk} + \alpha_i)} \right\} \mathbf{x}_{ijk} \\
\mu_{ij}^{(0,1)} \equiv \frac{\partial \mu_{ij}}{\partial \alpha_i} &= \gamma_{12} \prod_{k=1}^{c_{ij}} \{1 - g(\boldsymbol{\beta}'\mathbf{x}_{ijk} + \alpha_i)\} \sum_{k=1}^{c_{ij}} \left\{ \frac{-\partial g(\boldsymbol{\beta}'\mathbf{x}_{ijk} + \alpha_i)/\partial \alpha_i}{1 - g(\boldsymbol{\beta}'\mathbf{x}_{ijk} + \alpha_i)} \right\} \\
\mu_{ij}^{(0,2)} \equiv \frac{\partial^2 \mu_{ij}}{\partial \alpha_i^2} &= \mu_{ij}^{(0,1)} \sum_{k=1}^{c_{ij}} \left\{ \frac{-\partial g(\boldsymbol{\beta}'\mathbf{x}_{ijk} + \alpha_i)/\partial \alpha_i}{1 - g(\boldsymbol{\beta}'\mathbf{x}_{ijk} + \alpha_i)} \right\} \\
&\quad + \gamma_{12} \prod_{k=1}^{c_{ij}} \{1 - g(\boldsymbol{\beta}'\mathbf{x}_{ijk} + \alpha_i)\} \sum_{k=1}^{c_{ij}} \frac{\partial}{\partial \alpha_i} \left\{ \frac{-\partial g(\boldsymbol{\beta}'\mathbf{x}_{ijk} + \alpha_i)/\partial \alpha_i}{1 - g(\boldsymbol{\beta}'\mathbf{x}_{ijk} + \alpha_i)} \right\}.
\end{aligned}$$

Define

$$W_{ij}(\boldsymbol{\beta}, \alpha_i) \equiv W_{ij} = \frac{T_{ij} - \mu_{ij}}{\mu_{ij}(1 - \mu_{ij})}.$$

It follows from the chain rule that

$$W_{ij}^{(0,1)} \equiv \frac{\partial W_{ij}}{\partial \alpha_i} = \frac{\mu_{ij}^{(0,1)}(-\mu_{ij}^2 - T_{ij} + 2\mu_{ij}T_{ij})}{\mu_{ij}^2(1 - \mu_{ij})^2}.$$

For $i = 1, 2, ..., l$, we define the $n_i \times 1$ vectors

$$\begin{aligned}
\mathbf{T}_i &= (T_{i1}, T_{i2}, ..., T_{in_i})' \\
\mathbf{W}_i &= (W_{i1}, W_{i2}, ..., W_{in_i})',
\end{aligned}$$

the $n_i \times 1$ vectors

$$\begin{aligned}
\boldsymbol{\mu}_{i(0,1)} &= (\mu_{i1}^{(0,1)}, \mu_{i2}^{(0,1)}, ..., \mu_{in_i}^{(0,1)})' \\
\boldsymbol{\mu}_{i(0,2)} &= (\mu_{i1}^{(0,2)}, \mu_{i2}^{(0,2)}, ..., \mu_{in_i}^{(0,2)})' \\
\mathbf{W}_{i(0,1)} &= (W_{i1}^{(0,1)}, W_{i2}^{(0,1)}, ..., W_{in_i}^{(0,1)})',
\end{aligned}$$

and the $n_i \times p$ matrix $\boldsymbol{\mu}_{i(1,0)} = (\boldsymbol{\mu}_{i1}^{(1,0)'}, \boldsymbol{\mu}_{i2}^{(1,0)'}, ..., \boldsymbol{\mu}_{in_i}^{(1,0)'})'$. With $f_i = f_i(\mathbf{x}_i, 0, \boldsymbol{\beta})$, it follows that $(\partial/\partial\alpha_i)\log f_i = \mathbf{W}_i'\boldsymbol{\mu}_{i(0,1)}$, $(\partial^2/\partial\alpha_i^2)\log f_i = \mathbf{W}_{i(0,1)}'\boldsymbol{\mu}_{i(0,1)} + \mathbf{W}_i'\boldsymbol{\mu}_{i(0,2)}$, and $(\partial/\partial\boldsymbol{\beta})\log f_i = \boldsymbol{\mu}_{i(1,0)}'\mathbf{W}_i$. Thus,

$$\begin{aligned}
S(\boldsymbol{\beta}) &= \frac{1}{2} \sum_{i=1}^{l} \left\{ \left(\frac{\partial \log f_i}{\partial \alpha_i} \right)^2 + \frac{\partial^2 \log f_i}{\partial \alpha_i^2} \right\} \\
&= \frac{1}{2} \sum_{i=1}^{l} \left\{ \left(\mathbf{W}_i'\boldsymbol{\mu}_{i(0,1)} \right)^2 + \mathbf{W}_{i(0,1)}'\boldsymbol{\mu}_{i(0,1)} + \mathbf{W}_i'\boldsymbol{\mu}_{i(0,2)} \right\}.
\end{aligned}$$

The expectations presented in Section 3.2 are

$$
\begin{aligned}
I_{\tau\tau} &= \frac{1}{4}\sum_{i=1}^{l} E\left[\left\{\left(\mathbf{W}_i'\boldsymbol{\mu}_{i(0,1)}\right)^2 + \mathbf{W}_{i(0,1)}'\boldsymbol{\mu}_{i(0,1)} + \mathbf{W}_i'\boldsymbol{\mu}_{i(0,2)}\right\}^2\right] \\
\mathbf{I}_{\tau\boldsymbol{\beta}} &= \frac{1}{2}\sum_{i=1}^{l} E\left[\left\{\left(\mathbf{W}_i'\boldsymbol{\mu}_{i(0,1)}\right)^2 + \mathbf{W}_{i(0,1)}'\boldsymbol{\mu}_{i(0,1)} + \mathbf{W}_i'\boldsymbol{\mu}_{i(0,2)}\right\}\boldsymbol{\mu}_{i(1,0)}'\mathbf{W}_i\right] \\
\mathbf{I}_{\boldsymbol{\beta}\boldsymbol{\beta}} &= \sum_{i=1}^{l} E\left(\boldsymbol{\mu}_{i(1,0)}'\mathbf{W}_i\mathbf{W}_i'\boldsymbol{\mu}_{i(1,0)}\right) = \sum_{i=1}^{l}\boldsymbol{\mu}_{i(1,0)}'\mathbf{B}_i^{-1}\boldsymbol{\mu}_{i(1,0)},
\end{aligned}
$$

where the $n_i \times n_i$ matrix $\mathbf{B}_i = \text{diag}[\mu_{ij}(1-\mu_{ij})]$. The formula for $\mathbf{I}_{\boldsymbol{\beta}\boldsymbol{\beta}}$ has a closed-form representation, because $\mathbf{W}_i$ has mean zero so that $E(\mathbf{W}_i\mathbf{W}_i') = \text{cov}(\mathbf{W}_i)$. Expectations $I_{\tau\tau}$ and $\mathbf{I}_{\tau\boldsymbol{\beta}}$ are not available in closed form, so we approximate their values under H_0 as described in Section 3.2.