Plus-end entrainment can lead to self-organization of cortical microtubules in plants

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SUPPLEMENTAL MATERIAL

Three-state MTs considering only dynamic instability

In the absence of interactions, the length distribution of a population can be modeled using a partial differential equation [3]. For the two-state model,

$$\frac{\partial}{\partial t}\begin{bmatrix} N_g \\ N_s \end{bmatrix} = A\begin{bmatrix} N_g \\ N_s \end{bmatrix} + \frac{\partial}{\partial l}\left(V\begin{bmatrix} N_g \\ N_s \end{bmatrix}\right) \tag{1}$$

where $N_g(l,t)$ and $N_s(l,t)$ represent the density of growing and shrinking MTs of length l, respectively, and

$$A = \begin{bmatrix} -f_{gs} & f_{sg} \\ f_{gs} & -f_{sg} \end{bmatrix}, \qquad V = \begin{bmatrix} -v_g & 0 \\ 0 & +v_s \end{bmatrix} \tag{2}$$

represent transitions between states and advection, respectively. If new MTs are nucleated with zero length and in the growing state at rate k, the boundary conditions are $v_g N_g(0,t) = k$ and $N_s(l,t) \to 0$ as $l \to \infty$. This leads to a unique steady-state $N_i = \alpha_i \exp\left(-l/\bar{l}\right)$ where

$$\bar{l} = \frac{v_g v_s}{f_{gs} v_s - f_{sg} v_g} \tag{3}$$

as long as the denominator is positive [3]. The mean lifetime can be found by assuming the system is in steady-state, when nucleation must balance a constant death rate τ^{-1},

$$k = \frac{1}{\tau}\int_0^l N_g + N_s dl \tag{4}$$

where τ is the mean lifetime. This gives

$$\tau = \frac{v_g + v_s}{f_{gs} v_s - f_{sg} v_g} \tag{5}$$

in agreement with [4]. For the three-state model, the partial differential equations now involve $N_i(l,t)$, $i = (g,p,s)$ and the matrices become

$$A = \begin{bmatrix} -(f_{gp} + f_{gs}) & f_{pg} & f_{sg} \\ f_{gp} & -(f_{pg} + f_{ps}) & f_{sp} \\ f_{gs} & f_{ps} & -(f_{sg} + f_{sp}) \end{bmatrix} \tag{6}$$

and

$$V = \begin{bmatrix} -v_g & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & +v_s \end{bmatrix}. \tag{7}$$

The mean length and mean lifetime can be found as above,

$$\bar{l} = \frac{v_g v_s (f_{pg} + f_{ps})}{D} \tag{8}$$

$$\tau = \frac{v_s(f_{gp} + f_{pg} + f_{ps}) + v_g(f_{sp} + f_{pg} + f_{ps})}{D} \tag{9}$$

where the denominator

$$D = v_s(f_{gp}f_{ps} + f_{gs}f_{pg} + f_{gs}f_{ps}) - v_g(f_{pg}f_{sg} + f_{pg}f_{sp} + f_{ps}f_{sg}) \tag{10}$$

is the threshold quantity: if it is negative, the mean length and lifetime are infinite. In both the two-state and three-state models, if the minus-end shrinks at a constant velocity, we make the coordinate transformation

$$v_g = v_g^p - v_s^m \tag{11}$$

$$v_s = v_s^p + v_s^m. \tag{12}$$

Relationship between two-state models and Baulin et al. [1]

To understand the difference between catastrophe-inducing collisions and pause-inducing collisions, we consider the two-state model, which has five parameters, v_g^p, v_s^p, f_{gs} and f_{sg} which all pertain to the plus-end, and v_s^m, which pertains to the minus-end. In addition, the rate of nucleation, k_0, provides an additional time scale. However, if we rescale time to be measured in units of $T \equiv (v_g^p)^{-2/3} k_0^{-1/3}$ and length $L \equiv (v_g^p/k_0)^{1/3}$, then the two-state model is described by four parameters,

$$\alpha = v_s^p / v_g^p \tag{13}$$

$$\beta = f_{gs} T \tag{14}$$

$$\gamma = 1/(f_{sg} T) \tag{15}$$

$$\delta = v_s^m / v_g^p. \tag{16}$$

(Note that scaling by $\bar{l}$ and τ is not appropriate here, since we are sometimes in the infinite-growth regime.) To ensure MT nucleation can occur, $\delta < 1$. In this parametrization, the model of Baulin et al. [1] corresponds to $\alpha, \beta, \gamma \to 0$ and it completely described by one parameter, δ (related to their α, which they set in $[0.17, 1.5]$). The two-state parameters reported in [2] (Table 1) give $\alpha = 1.8$, $\beta = 0.16$, $\gamma = 3.1$ and either $\delta = 0$ (since they did not study minus-end dynamics) or $\gamma = 0.09$ (using v_s^m from [5]).

Effects of a catastrophe-inducing boundary in the absence of MT-MT interactions

Even in the absence of any MT-MT interactions, MTs randomly nucleated on a cylindrical cortex can lead to a transverse ordering if collisions with the boundaries induce catastrophe. A MT plus-end a distance y from the boundary making an angle θ, measured from transverse the axis of the cylinder, can grow to a maximum length $L = (L_C - y)/\sin\theta$ where L_C is the cell length. In this case, the right boundary condition on the system of partial differential equations in Eq. 1 is $v_s N_s(L,t) = v_g N_g(L,t)$. The solution is still exponential with decay length $\bar{l}$ but is truncated. The average length of MTs of angle θ at height y is

$$\langle l \rangle \propto \left(1 - e^{-y/(\bar{l}\sin\theta)}\right)\left(1 - e^{-(L_C - y)/(\bar{l}\sin\theta)}\right). \quad (17)$$

From this it is straightforward to compute the order parameter S. We can also compute a *local* order parameter $S(y)$ that takes into account all MTs passing through a given y value (a given circumference of the cylinder). Although $S(y)$ has no closed form, it can be computed numerically. We find that this boundary-induced ordering decays away from the boundaries towards midcell, with a decay length scale of roughly $\bar{l}$ (data not shown).

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