

Table S1. Mitochondrial energetics and ROS-induced-ROS-release model equations.

S1.1. Mitochondrial membrane potential ($\Delta\Psi_m$)

$$\frac{d\Delta\Psi_m}{dt} = \frac{V_{He} + V_{He(F)} - V_{Hu} - V_{ANT} - V_{HLeak} - V_{NaCa} - 2V_{uni} - V_{IMAC}}{C_{mito}} \quad \text{E1}$$

S1.2. Energy metabolites mass balance equations

$\frac{d[\text{ADP}]_m}{dt} = V_{\text{ANT}} - V_{\text{ATPase}} - V_{\text{SL}}$	E2
$[\text{ATP}]_m = C_A - [\text{ADP}]_m$	E3
$\frac{d[\text{NADH}]}{dt} = -V_{\text{O}_2} + V_{\text{IDH}} + V_{\text{KGDH}} + V_{\text{MDH}}$	E4
$\frac{d[\text{ISOC}]}{dt} = V_{\text{ACO}} - V_{\text{IDH}}$	E5
$\frac{d[\alpha\text{KG}]}{dt} = V_{\text{IDH}} - V_{\text{KGDH}} + V_{\text{AAT}}$	E6
$\frac{d[\text{SCoA}]}{dt} = V_{\text{KGDH}} - V_{\text{SL}}$	E7
$\frac{d[\text{Suc}]}{dt} = V_{\text{SL}} - V_{\text{SDH}}$	E8
$\frac{d[\text{FUM}]}{dt} = V_{\text{SDH}} - V_{\text{FH}}$	E9
$\frac{d[\text{MAL}]}{dt} = V_{\text{FH}} - V_{\text{MDH}}$	E10
$\frac{d[\text{OAA}]}{dt} = V_{\text{MDH}} - V_{\text{CS}} - V_{\text{AAT}}$	E11
$[\text{CIT}] = C_{\text{Kint}} - \left(\begin{array}{l} [\text{ISOC}] + [\alpha\text{KG}] + [\text{SCoA}] + [\text{Suc}] \\ + [\text{FUM}] + [\text{MAL}] + [\text{OAA}] \end{array} \right)$	E12
$\frac{d[\text{O}_2^-]_m}{dt} = \text{shunt} \cdot V_{\text{O}_2} - V_{\text{ROS}}^{\text{Tr}}$	E13
$\frac{d[\text{O}_2^-]_i}{dt} = V_{\text{ROS}}^{\text{Tr}} - V_{\text{SOD}}$	E14
$\frac{d[\text{H}_2\text{O}_2]}{dt} = V_{\text{SOD}} - V_{\text{CAT}} - V_{\text{GPX}}$	E15
$\frac{d[\text{GSH}]}{dt} = V_{\text{GR}} - V_{\text{GPX}}$	E16
$\frac{d[\text{Ca}^{2+}]_m}{dt} = \delta(V_{\text{uni}} - V_{\text{NaCa}})$	E51

S1.3. Tricarboxylic acid cycle reaction rates

$$V_{CS} = k_{cat}^{CS} E_T^{CS} \left(1 + \frac{K_M^{AcCoA}}{[AcCoA]} + \frac{K_M^{OAA}}{[OAA]} + \frac{K_M^{AcCoA}}{[AcCoA]} \frac{K_M^{OAA}}{[OAA]} \right)^{-1} \quad E17$$

$$V_{ACO} = k_f^{ACO} \left([CIT] - \frac{[ISOC]}{K_E^{ACO}} \right) \quad E18$$

$$f_a^{IDH} = \left[\left(1 + \frac{[ADP]_m}{K_{ADP}^a} \right) \left(1 + \frac{[Ca^{2+}]_m}{K_{Ca}^a} \right) \right]^{-1} \quad E19$$

$$f_i^{IDH} = \left(1 + \frac{[NADH]}{K_{i,NADH}} \right) \quad E20$$

$$V_{IDH} = k_{cat}^{IDH} E_T^{IDH} \left[\left(1 + \frac{[H^+]}{k_{h,1}} + \frac{k_{h,2}}{[H^+]} \right) + f_i^{IDH} \left(\frac{K_M^{NAD}}{[NAD]} \right) \right. \\ \left. + f_a^{IDH} \left(\frac{K_M^{ISOC}}{[ISOC]} \right)^{ni} + f_a^{IDH} f_i^{IDH} \left(\frac{K_M^{ISOC}}{[ISOC]} \right)^{ni} \left(\frac{K_M^{NAD}}{[NAD]} \right) \right]^{-1} \quad E21$$

$$f_a^{KGDH} = \left[\left(1 + \frac{[Mg^{2+}]}{K_D^{Mg^{2+}}} \right) \left(1 + \frac{[Ca^{2+}]_m}{K_D^{Ca^{2+}}} \right) \right]^{-1} \quad E22$$

$$V_{KGDH} = \frac{k_{cat}^{KGDH} E_T^{KGDH}}{1 + f_a^{KGDH} \frac{K_M^{\alpha KG}}{[\alpha KG]} + f_a^{KGDH} \left(\frac{K_M^{NAD}}{[NAD]} \right)^{n_{\alpha KG}}} \quad E23$$

$$V_{SL} = k_f^{SL} \left([SCoA][ADP]_m - \frac{[Suc][ATP]_m[CoA]}{K_E^{SL}} \right) \quad E24$$

$$V_{SDH} = \frac{k_{cat}^{SDH} E_T^{SDH}}{1 + \left(\frac{K_M^{Suc}}{[Suc]} \right) \left(1 + \frac{[OAA]}{K_{i,SDH}^{OAA}} \right) \left(1 + \frac{[FUM]}{K_i^{FUM}} \right)} \quad E25$$

$$V_{FH} = k_f^{FH} \left([FUM] - \frac{[MAL]}{K_E^{FH}} \right) \quad E26$$

$$f_{h,a} = \left(1 + \frac{[H^+]}{k_{h1}} + \frac{[H^+]^2}{k_{h1} k_{h2}} \right)^{-1} + k_{offset} \quad E27$$

$$f_{h,i} = \left(1 + \frac{k_{h3}}{[H^+]} + \frac{k_{h3} k_{h4}}{[H^+]^2} \right)^{-2} \quad \text{E28}$$

$$V_{MDH} = \frac{k_{cat}^{MDH} E_T^{MDH} f_{h,a} f_{h,i}}{1 + \frac{K_M^{MAL}}{[MAL]} \left(1 + \frac{[OAA]}{K_i^{OAA}} \right) + \frac{K_M^{NAD}}{[NAD]} + \frac{K_M^{MAL}}{[MAL]} \left(1 + \frac{[OAA]}{K_i^{OAA}} \right) \frac{K_M^{NAD}}{[NAD]}} \quad \text{E29}$$

$$V_{AAT} = k_f^{AAT} [OAA][GLU] \frac{k_{ASP} K_E^{AAT}}{\left(k_{ASP} K_E^{AAT} + [\alpha KG] k_f^{AAT} \right)} \quad \text{E30}$$

S1.4. Oxidative phosphorylation reaction rates

$$V_{O_2} = 0.5 \rho^{\text{res}} \frac{\left(r_a + r_{c1} e^{\left(\frac{6F \Delta \Psi_B}{RT} \right)} \right) e^{\left(\frac{A_{\text{res}} F}{RT} \right)} - r_a e^{\left(\frac{g 6F \Delta \mu_H}{RT} \right)} + r_{c2} e^{\left(\frac{A_{\text{res}} F}{RT} \right)} e^{\left(\frac{g 6F \Delta \mu_H}{RT} \right)}}{\left(1 + r_1 e^{\left(\frac{F A_{\text{res}}}{RT} \right)} \right) e^{\left(\frac{6F \Delta \Psi_B}{RT} \right)} + \left(r_2 + r_3 e^{\left(\frac{F A_{\text{res}}}{RT} \right)} \right) e^{\left(\frac{g 6F \Delta \mu_H}{RT} \right)}} \quad \text{E31}$$

$$V_{He} = 6 \rho^{\text{res}} \frac{\left(r_a e^{\left(\frac{A_{\text{res}} F}{RT} \right)} - (r_a + r_b) e^{\left(\frac{g 6F \Delta \mu_H}{RT} \right)} \right)}{\left(1 + r_1 e^{\left(\frac{F A_{\text{res}}}{RT} \right)} \right) e^{\left(\frac{6F \Delta \Psi_B}{RT} \right)} + \left(r_2 + r_3 e^{\left(\frac{F A_{\text{res}}}{RT} \right)} \right) e^{\left(\frac{g 6F \Delta \mu_H}{RT} \right)}} \quad \text{E32}$$

$$A_{\text{res}} = \frac{RT}{F} \ln \left(K_{\text{res}} \sqrt{\frac{[\text{NADH}]}{[\text{NAD}^+]}} \right) \quad \text{E33}$$

$$[\text{NAD}^+] = C_{\text{PN}} - [\text{NADH}] \quad \text{E34}$$

$$V_{\text{He(F)}} = \frac{4 \rho^{\text{res(F)}} \times \left(r_a e^{\left(\frac{A_{\text{res(F)}} F}{RT} \right)} - (r_a + r_b) e^{\left(\frac{g 6F \Delta \mu_H}{RT} \right)} \right)}{\left(1 + r_1 e^{\left(\frac{F A_{\text{res(F)}}}{RT} \right)} \right) e^{\left(\frac{6F \Delta \Psi_B}{RT} \right)} + \left(r_2 + r_3 e^{\left(\frac{F A_{\text{res(F)}}}{RT} \right)} \right) e^{\left(\frac{g 6F \Delta \mu_H}{RT} \right)}} \quad \text{E35}$$

$$A_{\text{res(F)}} = \frac{RT}{F} \ln \left(K_{\text{res(F)}} \sqrt{\frac{[\text{FADH}_2]}{[\text{FAD}]}} \right) \quad \text{E36}$$

$$V_{\text{ATPase}} = -\rho^{\text{Fl}} \frac{\left(10^2 p_a + p_{c1} e^{\left(\frac{3F \Delta \Psi_B}{RT} \right)} \right) e^{\left(\frac{A_{\text{Fl}} F}{RT} \right)} - \left(p_a e^{\left(\frac{3F \Delta \mu_H}{RT} \right)} + p_{c2} e^{\left(\frac{A_{\text{Fl}} F}{RT} \right)} e^{\left(\frac{3F \Delta \mu_H}{RT} \right)} \right)}{\left(1 + p_1 e^{\left(\frac{F A_{\text{Fl}}}{RT} \right)} \right) e^{\left(\frac{3F \Delta \Psi_B}{RT} \right)} + \left(p_2 + p_3 e^{\left(\frac{F A_{\text{Fl}}}{RT} \right)} \right) e^{\left(\frac{3F \Delta \mu_H}{RT} \right)}} \quad \text{E37}$$

$$V_{\text{Hu}} = -3 \rho^{\text{Fl}} \frac{10^2 p_a \left(1 + e^{\left(\frac{F A_{\text{Fl}}}{RT} \right)} \right) - (p_a + p_b) e^{\left(\frac{3F \Delta \mu_H}{RT} \right)}}{\left(1 + p_1 e^{\left(\frac{F A_{\text{Fl}}}{RT} \right)} \right) e^{\left(\frac{3F \Delta \Psi_B}{RT} \right)} + \left(p_2 + p_3 e^{\left(\frac{F A_{\text{Fl}}}{RT} \right)} \right) e^{\left(\frac{3F \Delta \mu_H}{RT} \right)}} \quad \text{E38}$$

$$A_{F1} = \frac{R T}{F} \ln \left(K_{F1} \frac{[ATP]_m}{[ADP]_m Pi} \right) \quad E39$$

$$V_{Hleak} = g_H \Delta\mu_H \quad E40$$

$$\Delta\mu_H = -2.303 \frac{R T}{F} \Delta pH + \Delta\Psi_m \quad E41$$

S1.5. Mitochondrial Ca^{2+} uniporter and Ca^{2+} - Na^+ exchanger rates

$$V_{\text{uni}} = V_{\text{max}}^{\text{uni}} \frac{\frac{[\text{Ca}^{2+}]_i}{K_{\text{trans}}} \left(1 + \frac{[\text{Ca}^{2+}]_i}{K_{\text{trans}}}\right)^3 \frac{2 F (\Delta\Psi_m - \Delta\Psi^0)}{R T}}{\left(1 + \frac{[\text{Ca}^{2+}]_i}{K_{\text{trans}}}\right)^4 + \frac{L}{\left(1 + \frac{[\text{Ca}^{2+}]_i}{K_{\text{act}}}\right)^{n_a}} \left(1 - e^{\left\{\frac{-2 F (\Delta\Psi_m - \Delta\Psi^0)}{R T}\right\}}\right)}$$
E42

$$V_{\text{NaCa}} = V_{\text{max}}^{\text{NaCa}} \frac{e^{\left(\frac{b F (\Delta\Psi_m - \Delta\Psi^0)}{R T}\right)} \left(\frac{\ln[\text{Ca}^{2+}]_m}{[\text{Ca}^{2+}]_i}\right)}{\left(1 + \frac{K_{\text{Na}}}{[\text{Na}^+]_i}\right)^n \left(1 + \frac{K_{\text{Ca}}}{[\text{Ca}^{2+}]_m}\right)}$$
E43

S1.6. ROS-induced-ROS-release rates

$$V_{SOD} = \frac{2 k_{SOD}^1 k_{SOD}^5 \left(k_{SOD}^1 + k_{SOD}^3 \left(1 + \frac{[H_2O_2]}{K_i^{H2O2}} \right) \right) E_{SOD}^T [O_2^{\cdot-}]}{k_{SOD}^5 \left(2 k_{SOD}^1 + k_{SOD}^3 \left(1 + \frac{[H_2O_2]}{K_i^{H2O2}} \right) \right) + [O_2^{\cdot-}] k_{SOD}^1 k_{SOD}^3 \left(1 + \frac{[H_2O_2]}{K_i^{H2O2}} \right)}$$

E44

$$V_{CAT} = 2 k_{CAT}^1 E_{CAT}^T [H_2O_2] e^{-fr [H_2O_2]}$$

E45

$$V_{GPX} = \frac{E_{GPX}^T [H_2O_2][GSH]}{\Phi_1 [GSH] + \Phi_2 [H_2O_2]}$$

E46

$$V_{GR} = \frac{k_{GR}^1 E_{GR}^T}{1 + \frac{K_M^{GSSG}}{[GSSG]} + \frac{K_M^{NADPH}}{[NADPH]} + \frac{K_M^{GSSG}}{[GSSG]} \frac{K_M^{NADPH}}{[NADPH]}}$$

E47

$$V_{ROS}^{Tr} = j \cdot \frac{V_{IMAC}}{\Delta\Psi_m} \left(\Delta\Psi_m - \frac{RT}{F} \log \left(\frac{[O_2^{\cdot-}]_m}{[O_2^{\cdot-}]_i} \right) \right)$$

E48

$$V_{IMAC} = \left(a + \frac{b}{1 + \frac{K_{cc}}{[O_2^{\cdot-}]_i}} \right) \left(G_L + \frac{G_{max}}{1 + e^{\left(\kappa (\Delta\Psi_m^b - \Delta\Psi_m) \right)}} \right) \Delta\Psi_m$$

E49

$$G_T = [GSH] + 2 \times [GSSG]$$

E50
