Web-based Supplementary Materials for " Estimated Pseudo-Partial-Likelihood Method for Correlated Failure Time Data with Auxiliary Covariates" by Liu, Zhou and Cai.

1 Proofs for the Asymptotic Results

1.1 Proof of consistency

Note that $\hat{\beta}_E$ solves $n^{-1}\hat{U}(\beta) = 0$. Follow closely the arguments of Foutz(1977), one can show that $\hat{\beta}_E$ is consistent for β_0, provided:

(I) $n^{-1}\partial\hat{U}(\beta)/\partial\beta$ exists and is continuous in an open neighborhood B of β_0;

(II) $n^{-1}\partial\hat{U}(\beta_0)/\partial\beta_0$ is negative definite with probability going to 1;

(III) $n^{-1}\partial\hat{U}(\beta)/\partial\beta$ converges in probability to a fixed function, say, $\Sigma(\beta)$, uniformly in an open neighborhood of β_0;

(IV) $n^{-1}\hat{U}(\beta_0) \to 0$ in probability.

Let

$$\hat{S}_k^{(d)}(\beta, t) = n^{-1}\sum_{i=1}^{n} Y_{ik}\hat{r}_{ik}^{(d)}(\beta, t)\, (d = 0, 1, 2)$$

$$\hat{e}_{1k}(\beta, t) = n^{-1}\sum_{i=1}^{n} Y_{ik}(t)\left(\frac{\hat{r}_{ik}^{(1)}(\beta, t)}{\hat{r}_{ik}(\beta, t)}\right)^{\otimes 2} r_{ik}(\beta_0, t)$$

$$\hat{e}_{2k}(\beta, t) = n^{-1}\sum_{i=1}^{n} Y_{ik}(t)\frac{\hat{r}_{ik}^{(2)}(\beta, t)}{\hat{r}_{ik}(\beta, t)} r_{ik}(\beta_0, t)$$

Specifically, one can write

$$\begin{aligned} & n^{-1}\frac{\partial\hat{U}(\beta)}{\partial\beta} \\ &= n^{-1}\sum_{k=1}^{K}\sum_{i=1}^{n}\int_0^{\tau}\left[\frac{\hat{r}_{ik}^{(2)}(\beta, t)}{\hat{r}_{ik}(\beta, t)} - \left(\frac{\hat{r}_{ik}^{(1)}(\beta, t)}{\hat{r}_{ik}(\beta, t)}\right)^{\otimes 2} - \frac{\hat{S}_k^{(2)}(\beta, t)}{\hat{S}_k^{(0)}(\beta, t)} + \left(\frac{\hat{S}_k^{(1)}(\beta, t)}{\hat{S}_k^{(0)}(\beta, t)}\right)^{\otimes 2}\right] dM_{ik}(t) \\ & +A(\beta) \end{aligned} \quad (1)$$

where

$$A(\beta) = \frac{1}{n}\sum_{k=1}^{K}\sum_{i=1}^{n}\int_0^\tau \left[\frac{\hat{r}_{ik}^{(2)}(\beta,t)}{\hat{r}_{ik}(\beta,t)} - (\frac{\hat{r}_{ik}^{(1)}(\beta,t)}{\hat{r}_{ik}(\beta,t)})^{\otimes 2} - \frac{\hat{S}_k^{(2)}(\beta,t)}{\hat{S}_k^{(0)}(\beta,t)} + (\frac{\hat{S}_k^{(1)}(\beta,t)}{\hat{S}_k^{(0)}(\beta,t)})^{\otimes 2}\right] Y_{ik}(t) r_{ik}(\beta_0,t)\lambda_{0k}(t)dt$$

Clearly (I) is satisfied. For (II) and (III), we can prove

$$\sup_{B\times[0,\tau]} \|\hat{S}_k^{(d)}(\beta,t) - s_k^{(d)}(\beta,t)\| \to 0 \quad \text{a.s. for } d = 0,1,2;$$

and

$$\sup_{B\times[0,\tau]} \|\hat{e}_{jk}(\beta,t) - e_{jk}(\beta,t)\| \to 0 \quad \text{a.s. for } j = 1,2;$$

Note that

$$\frac{\hat{r}_{ik}^{(2)}(\beta,t)}{\hat{r}_{ik}(\beta,t)} - (\frac{\hat{r}_{ik}^{(1)}(\beta,t)}{\hat{r}_{ik}(\beta,t)})^{\otimes 2} = \begin{pmatrix} \frac{\hat{R}_{ik}^{(2)}(\beta_1,t)}{\hat{R}_{ik}(\beta_1,t)} - (\frac{\hat{R}_{ik}^{(1)}(\beta_1,t)}{\hat{R}_{ik}(\beta_1,t)})^{\otimes 2} & 0 \\ 0 & 0 \end{pmatrix}.$$

By assumptions (i) and (iii) and the Lenglart inequality, we can prove $n^{-1}\sum_{i=1}^{n}\int_0^\tau \left(\frac{\hat{R}_{ik}^{(2)}(\beta_1,t)}{\hat{R}_{ik}(\beta_1,t)}\right) dM_{ik}(t)$ and $n^{-1}\sum_{i=1}^{n}\int_0^\tau \left(\frac{\hat{R}_{ik}^{(1)}(\beta_1,t)}{\hat{R}_{ik}(\beta_1,t)}\right)^{\otimes 2} dM_{ik}(t)$ converge to 0 in probability. Therefore,

$$n^{-1}\sum_{i=1}^{n}\int_0^\tau \left(\frac{\hat{r}_{ik}^{(2)}(\beta,t)}{\hat{r}_{ik}(\beta,t)} - (\frac{\hat{r}_{ik}^{(1)}(\beta,t)}{\hat{r}_{ik}(\beta,t)})^{\otimes 2}\right) dM_{ik}(t) \to 0 \quad \text{in probability.}$$

It can be shown that $n^{-1}\sum_{i=1}^{n}\sum_{k=1}^{K}\int_0^\tau \left[\frac{\hat{S}_k^{(2)}(\beta,t)}{\hat{S}_k^{(0)}(\beta,t)} - (\frac{\hat{S}_k^{(1)}(\beta,t)}{\hat{S}_k^{(0)}(\beta,t)})^{\otimes 2}\right] dM_{ik}(t)$ is asymptotically equivalent to a local square integrable martingale, hence by the Lenglart inequality (e.g.Andersen and Gill, 1982), $n^{-1}\sum_{i=1}^{n}\sum_{k=1}^{K}\int_0^\tau \left[\frac{\hat{S}_k^{(2)}(\beta,t)}{\hat{S}_k^{(0)}(\beta,t)} - (\frac{\hat{S}_k^{(1)}(\beta,t)}{\hat{S}_k^{(0)}(\beta,t)})^{\otimes 2}\right] dM_{ik}(t)$ converges to zero in probability, uniformly in B. For $A(\beta)$, we have

$$A(\beta) = \int_0^\tau \sum_{k=1}^{K}[\hat{e}_{2k}(\beta,t) - \frac{\hat{S}_k^{(2)}(\beta,t)}{\hat{S}_k^{(0)}(\beta,t)} S_k^{(0)}(\beta_0,t) - \hat{e}_{1k}(\beta,t) + (\frac{\hat{S}_k^{(1)}(\beta,t)}{\hat{S}_k^{(0)}(\beta,t)})^{\otimes 2} S_k^{(0)}(\beta_0,t)]\lambda_{0k}(t)dt,$$

which can be shown to converge to

$$\int_0^\tau \sum_{k=1}^{K}\left[e_{2k}(\beta,t) - \frac{s_k^{(2)}(\beta,t)}{s_k^{(0)}(\beta,t)} s_k^{(0)}(\beta_0,t) - s_{1k}(\beta,t) + (\frac{s_k^{(1)}(\beta,t)}{s_k^{(0)}(\beta,t)})^{\otimes 2} s_k^{(0)}(\beta_0,t)\right]\lambda_{0k}(t)dt$$

in probability. Based on the above results, (III) is satisfied.

When $\beta = \beta_0$, we have $e_{1k}(\beta_0, t) = s_k^{(2)}(\beta_0, t)$, hence the above limit equals

$$-\Sigma(\beta_0) = \sum_{k=1}^{K} \int_0^{\tau} \left[(\frac{s_k^{(1)}(\beta_0, t)}{s_k^{(0)}(\beta_0, t)})^{\otimes 2} s_k^{(0)}(\beta_0, t) - e_{1k}(\beta_0, t) \right] \lambda_{0k}(t) dt.$$

Based on condition (iv), (II) is satisfied.

For (IV), using similar arguments as above, $n^{-1}\hat{U}(\beta_0)$ converges to the same limit as

$$B(\beta_0) = \frac{1}{n} \sum_{k=1}^{K} \sum_{i=1}^{n} \int_0^{\tau} [\frac{\hat{r}_{ik}^{(1)}(\beta_0, t)}{\hat{r}_{ik}(\beta_0, t)} - \frac{\hat{S}_k^{(1)}(\beta_0, t)}{\hat{S}_k^{(0)}(\beta_0, t)}] Y_{ik}(t) r_{ik}(\beta_0, t) \lambda_{0k}(t) dt.$$

Since $\frac{\hat{r}_{ik}^{(1)}(\beta_0,t)}{\hat{r}_{ik}(\beta_0,t)} = \left(\frac{\hat{R}_{ik}^{(1)}(\beta_{10},t)}{\hat{R}_{ik}(\beta_{10},t)}, Z'_{ik}(t) \right)'$ and

$$\begin{aligned} &\frac{1}{n} \sum_{i=1}^{n} \frac{\hat{R}_{ik}^{(1)}(\beta_{10}, t)}{\hat{R}_{ik}(\beta_{10}, t)} Y_{ik}(t) R_{ik}(\beta_{10}, t) e^{\beta_2' Z_{ik}} \\ &= \frac{1}{n} \sum_{i=1}^{n_v} E_{ik} Y_{ik}(t) e^{\beta_{10}' E_{ik}^*(t) + \beta_{20}' Z_{ik}(t)} + \frac{1}{n} \sum_{i=1}^{\overline{n}_v} \frac{\hat{\phi}_{ik}^{(1)}(\beta_{10}, t)}{\hat{\phi}_{ik}(\beta_{10}, t)} Y_{ik}(t) \phi_{ik}(\beta_{10}, t) e^{\beta_{20}' Z_{ik}(t)}, \end{aligned}$$

where $\overline{n}_v = n - n_v$ is the size of the non-validation set. Let L_{1k} and L_{2k} denote the first and second term of the above expression and let $\hat{\phi}_{a_m k}(\beta_1, t)$ be the $\hat{\phi}_{ik}(\beta_1, t)$ when $A_{ik}^*(t) = a_m$, and

$$\phi_{a_m k}(\beta_1, t) = E\{e^{\beta_1' E_{ik}^*(t)} | Y_{ik}(t) = 1, A_{ik}^*(t) = a_m\}.$$

By assumption (v) and the law of large numbers, we have

$$\begin{aligned} L_{2k} &= \frac{\overline{n}_v}{n} \frac{1}{\overline{n}_v} \sum_{i=1}^{\overline{n}_v} Y_{ik}(t) e^{\beta_{20}' Z_{ik}(t)} \sum_{m=1}^{L} \frac{\hat{\phi}_{a_m k}^{(1)}(\beta_{10}, t)}{\hat{\phi}_{a_m k}(\beta_{10}, t)} \phi_{a_m k}(\beta_{10}, t) I(A_{ik}^*(t) = a_m) \\ &\to (1 - q) \sum_{m=1}^{L} \phi_{a_m k}^{(1)}(\beta_{10}, t) E[Y_{ik}(t) e^{\beta_{20}' Z_{ik}(t)} I(A_{ik}^*(t) = a_m)] \quad \text{in probability.} \end{aligned}$$

By the law of large numbers, $L_{1k} \to qE(Y_{ik}(t) E_{ik} e^{\beta_{10} E_{ik}(t) + \beta_{20}' Z_{ik}(t)})$ in probability. Therefore, by assumption (v), $L_{1k} + L_{2k} \to E(Y_{ik}(t) R_{ik}^{(1)}(\beta_{10}, t) e^{\beta_{20}' Z_{ik}(t)})$ in probability.

Since

$$\frac{1}{n} \sum_{i=1}^{n} Z_{ik}(t) Y_{ik}(t) R_{ik}(\beta_{10}, t) e^{\beta_{20}' Z_{ik}(t)} \to E(Y_{ik}(t) R_{ik}(\beta_{10}, t) Z_{ik}(t) e^{\beta_{20}' Z_{ik}(t)}) \quad \text{in probability,}$$

it can be proved that

$$\frac{1}{n}\sum_{i=1}^{n}\frac{\hat{r}_{ik}^{(1)}(\beta_0,t)}{\hat{r}_{ik}(\beta_0,t)}Y_{ik}(t)r_{ik}(\beta_0,t)\rightarrow E\{Y_{ik}(t)r_{ik}^{(1)}(\beta_0,t)\}=s_k^{(1)}(\beta_0,t)\quad \text{in probability.}$$

It follows that

$$B(\beta_0)\rightarrow\sum_{k=1}^{K}\int_0^{\tau}(s_k^{(1)}(\beta_0,t)-s_k^{(1)}(\beta_0,t))\lambda_{0k}(t)dt=0\quad \text{in probability,}$$

and (IV) is satisfied. Therefore we have shown that $\hat{\beta}_E$ converges in probability to β_0.

1.2 Proof of normality

It can be shown that the score function $n^{-1/2}(\partial/\partial\beta)\{\log EPPL(\beta)\}$, can be expressed as

$$\begin{aligned} n^{-1/2}\hat{U}(\beta) &= n^{-1/2}\sum_{k=1}^{K}\sum_{i=1}^{n}\int_0^{\tau}\left[\frac{\hat{r}_{ik}^{(1)}(\beta,t)}{\hat{r}_{ik}(\beta,t)}-\frac{\sum_{l=1}^{n}Y_{lk}(t)\hat{r}_{lk}^{(1)}(\beta,t)}{\sum_{l=1}^{n}Y_{lk}(t)\hat{r}_{lk}(\beta,t)}\right]dM_{ik}(t) \\ &\quad +n^{-1/2}\sum_{k=1}^{K}\sum_{i=1}^{n}\int_0^{\tau}\left[\frac{\hat{r}_{ik}^{(1)}(\beta,t)}{\hat{r}_{ik}(\beta,t)}-\frac{\sum_{l=1}^{n}Y_{lk}(t)\hat{r}_{lk}^{(1)}(\beta,t)}{\sum_{l=1}^{n}Y_{lk}(t)\hat{r}_{lk}(\beta,t)}\right]r_{ik}(\beta_0,t)Y_{ik}(t)\lambda_{0k}(t)dt. \end{aligned} \tag{2}$$

By Taylor expansion of $\hat{U}(\beta_0)$, we have

$$n^{-\frac{1}{2}}\hat{U}(\beta_0)=\{-n^{-1}\frac{\partial}{\partial\beta^T}\hat{U}(\beta_*)\}n^{\frac{1}{2}}(\hat{\beta}_E-\beta_0), \tag{3}$$

where β_* is between $\hat{\beta}_E$ and β_0. To prove the asymptotic normality, it suffices to prove that $n^{-\frac{1}{2}}\hat{U}(\beta_0)$ converges to a normal random variable in distribution and that $n^{-1}\frac{\partial}{\partial\beta^T}\hat{U}(\beta_*)$ converges to an invertible matrix. By consistency of $\hat{\beta}_E$ and the convergence proof of $n^{-1}\frac{\partial\hat{U}(\beta)}{\partial\beta^T}$ for (III), it can be shown that $n^{-1}\frac{\partial}{\partial\beta^T}\hat{U}(\beta_*)$ converges to the invertible matrix $\Sigma(\beta_0)$.

For proving the asymptotic distribution of $n^{-\frac{1}{2}}\hat{U}(\beta_0)$, apply the first order expansion $x/y=x_0/y_0+(x-x_0)/y_0-(y-y_0)x_0/y_0^2+o\{(x-x_0)^2+(y-y_0)^2\}$ to $\hat{r}_{ik}^{(1)}/\hat{r}_{ik}$ and $\hat{S}_k^{(1)}/\hat{S}_k^{(0)}$ at $r_{ik}^{(1)}/r_{ik}$ and $S_k^{(1)}/S_k^{(0)}$, respectively, we can show that the second term of (2) evaluated at β_0 can be written as:

$$
\begin{aligned}
& -n^{-\frac{1}{2}}\sum_{k=1}^{K}\sum_{i=1}^{n}\int_0^\tau \left[\frac{r_{ik}^{(1)}(\beta,t)}{r_{ik}(\beta,t)}-\frac{s_k^{(1)}(\beta_0,t)}{s_k^{(0)}(\beta_0,t)}\right] Y_{ik}(t)(\hat{r}_{ik}(\beta_0,t)-r_{ik}(\beta_0,t))\lambda_{0k}(t)dt+o_p(1) \\
= & -n^{-\frac{1}{2}}\sum_{k=1}^{K}\sum_{i=1}^{\overline{n}_v}\int_0^\tau \left[\begin{pmatrix} \frac{\phi_{ik}^{(1)}(\beta_{10},t)}{\phi_{ik}(\beta_{10},t)}-\frac{s_k^{(11)}(\beta_0,t)}{s_k^{(0)}(\beta_0,t)} \\ Z_{ik}(t)-\frac{s_k^{(12)}(\beta_0,t)}{s_k^{(0)}(\beta_0,t)}\end{pmatrix}\right] Y_{ik}(t)e^{\beta_{20}'Z_{ik}(t)}(\hat{\phi}_{ik}(\beta_{10},t)-\phi_{ik}(\beta_{10},t))\lambda_{0k}(t)dt \\
& +o_p(1), \qquad (4)
\end{aligned}
$$

here $s_k^{(11)}(\beta_{10},t)$ is the first m elements of the vector $s_k^{(1)}(\beta_{10},t)$, and $s_k^{(12)}(\beta_{10},t)$ contains the remaining p elements. Let

$$
\Psi_k = -n^{-\frac{1}{2}}\sum_{i=1}^{\overline{n}_v}\int_0^\tau \left(\frac{\phi_{ik}^{(1)}(\beta_{10},t)}{\phi_{ik}(\beta_{10},t)}-\frac{s_k^{(11)}(\beta_0,t)}{s_k^{(0)}(\beta_0,t)}\right)Y_{ik}(t)e^{\beta_{20}'Z_{ik}(t)}(\hat{\phi}_{ik}(\beta_{10},t)-\phi_{ik}(\beta_{10},t))\lambda_{0k}(t)dt
$$

be the first m elements of (4), then by assumptions (i)-(vi), we have

$$
\begin{aligned}
\Psi_k = & -\int_0^\tau \sum_{m=1}^{L}\left[\frac{\phi_{a_mk}^{(1)}(\beta_{10},t)}{\phi_{a_mk}(\beta_{10},t)}-\frac{s_k^{(11)}(\beta_{10},t)}{s_k^{(0)}(\beta_{10},t)}\right]\frac{1}{Pr(Y_{ik}(t)=1,A_{ik}^*(t)=a_m)} \\
& \times\sqrt{n}\left(\frac{1}{n_v}\sum_{l=1}^{n_v}I(A_{lk}^*(t)=a_m)Y_{lk}(t)(e^{\beta_{10}'E_{lk}^*(t)}-\phi_{a_mk}(\beta_{10},t))\right) \\
& \times\frac{1}{n}\sum_{i=1}^{\overline{n}_v}Y_{ik}(t)e^{\beta_{20}'Z_{ik}(t)}I(A_{ik}^*(t)=a_m)\lambda_{0k}(t)dt+o_p(1) \\
= & -n^{-1/2}\frac{\overline{n}_v}{n_v}\int_0^\tau\sum_{l=1}^{n_v}Y_{lk}(t)[e^{\beta_{10}'E_{lk}^*(t)}-\phi_{lk}(\beta_{10},t)]E(e^{\beta_{20}'Z_{lk}(t)}|Y_{lk}(t)=1,A_{lk}^*(t)) \\
& \times\left[\frac{\phi_{lk}^{(1)}(\beta_{10},t)}{\phi_{lk}(\beta_{10},t)}-\frac{s_k^{(11)}(\beta_0,t)}{s_k^{(0)}(\beta_0,t)}+o_p(1)\right]\lambda_{0k}(t)dt.
\end{aligned}
$$

Therefore, the first m elements of the vector in (4) is asymptotically equivalent to $-n^{-\frac{1}{2}}\frac{\bar{n}_v}{n_v}\sum_{k=1}^{K}\sum_{l=1}^{n_v}Q_{lk}(\beta_0)$, where $Q_{lk}(\beta_0)$ is defined as in Theorem 2. Using similar arguments we can show that the remaining elements of the vector in (4) is asymptotically equivalent to $-n^{-\frac{1}{2}}\frac{\bar{n}_v}{n_v}\sum_{k=1}^{K}\sum_{l=1}^{n_v}H_{lk}(\beta_0)$, where $H_{lk}(\beta_0,t)$ is defined as in Theorem 2.

Hence $n^{-\frac{1}{2}}\hat{U}(\beta_0)$ is asymptotically equivalent to

$$n^{-\frac{1}{2}}\sum_{k=1}^{K}\sum_{i=1}^{\overline{n}_v}\int_0^{\tau}\left(\frac{r_{ik}^{(1)}(\beta_0,t)}{r_{ik}(\beta_0,t)}-\frac{s_k^{(1)}(\beta_0,t)}{s_k^{(0)}(\beta_0,t)}\right)dM_{ik}(t)$$
$$+n^{-\frac{1}{2}}\sum_{k=1}^{K}\sum_{j=1}^{n_v}\left[\int_0^{\tau}\left(\frac{r_{jk}^{(1)}(\beta_0,t)}{r_{jk}(\beta_0,t)}-\frac{s_k^{(1)}(\beta_0,t)}{s_k^{(0)}(\beta_0,t)}\right)dM_{jk}(t)-\frac{\overline{n}_v}{n_v}\begin{pmatrix} Q_{jk}(\beta_0) \\ H_{jk}(\beta_0)\end{pmatrix}\right]. \tag{5}$$

The first and second summation of (5) are independent. Let

$$G_k(\beta_0)=\sum_{i=1}^{\overline{n}_v}\int_0^{\tau}\left(\frac{r_{ik}^{(1)}(\beta_0,t)}{r_{ik}(\beta_0,t)}-\frac{s_k^{(1)}(\beta_0,t)}{s_k^{(0)}(\beta_0,t)}\right)dM_{ik}(t),$$

then the first term of (5) equals $n^{-\frac{1}{2}}\sum_{k=1}^{K}G_k(\beta_0)$. By the multivariate central limit theorem, we have $\overline{n}_v{}^{-\frac{1}{2}}\begin{pmatrix} G_1(\beta_0) \\ G_2(\beta_0) \\ \cdots \\ G_K(\beta_0)\end{pmatrix}$ converges in distribution to a zero-mean normal random vector with variance-covariance matrix $E(G_k(\beta_0)G_j'(\beta_0))$. Therefore, the first term converges in distribution to a zero-mean normal random vector with asymptotic covariance matrix $\Sigma_1(\beta_0)$ as defined in Theorem 2.

The second term of (5) is a summation of i.i.d. terms from subjects in the validation sample. By central limit theorem, it converges to a normal distribution with mean

$$E\left[\int_0^{\tau}(\frac{r_{ik}^{(1)}(\beta_0,t)}{r_{ik}(\beta_0,t)}-\frac{s_k^{(1)}(\beta_0,t)}{s_k^{(0)}(\beta_0,t)})dM_{ik}(t)-\frac{(1-q)}{q}\begin{pmatrix} Q_{ik}(\beta_0) \\ H_{ik}(\beta_0)\end{pmatrix}\right].$$

Observe that these terms have mean zero since $M_{ik}(t)$ is a martingale, and $E(Q_{ik}(\beta_0))=0$ and $E(H_{ik}(\beta_0))=0$. Therefore the second term of (5) is asymptotically normally distributed with mean zero and variance $\Sigma_2(\beta_0,t)$. Since these two terms are independent, it follows that $n^{-\frac{1}{2}}\hat{U}(\beta_0)$ converges to a mean zero normal distribution variable with variance $\Sigma_1(\beta_0)+\Sigma_2(\beta_0)$. These results together with the Taylor expansion in (3) give the desired normality result for $\hat{\beta}_E$ in Theorem 2.

2 Web Table 1

Web Table 1 presents some additional simulation results under the model: $\lambda_{ik}(t;E_{ik}(t),Z_{ik}(t))=\lambda_{0k}(t)\exp\{\beta_{k1}E_{ik}(t)+\beta_2 Z_{ik}(t)\}(k=1,2)$, where $\beta_{11}=\log(2)=0.693,\beta_{21}=\log(1.3)=0.262$,

$\beta_2 = -0.2$. Censoring rate is 20%. These results illustrate performance of the proposed methodology in the case when the sample size of the validation set is fixed at 60 and 120, while the total sample size are 200, 1000, and 3000. The simulation studies are conducted with 500 replications. Values for other parameters are: $\sigma = 0.1$, $\theta = 0.25$, $r = 0.8$.

Web Table 1: Additonal Simulation Results

		validation sample size=60			validation sample size=120		
True Value		$\hat{\beta}$	SD	SE	$\hat{\beta}$	SD	SE
				n=200			
$\beta_{11} = 0.693$	$\hat{\beta}_V$	0.7214	0.1892	0.1709	0.6980	0.1230	0.1179
	$\hat{\beta}_E$	0.6968	0.0985	0.1225	0.6957	0.0968	0.1024
$\beta_{21} = 0.262$	$\hat{\beta}_V$	0.2842	0.1736	0.1510	0.2716	0.1051	0.1060
	$\hat{\beta}_E$	0.2650	0.0894	0.1657	0.2669	0.0868	0.1075
$\beta_2 = -0.2$	$\hat{\beta}_V$	-0.1998	0.1132	0.1065	-0.1980	0.0782	0.0742
	$\hat{\beta}_E$	-0.1926	0.0598	0.0587	-0.1952	0.0608	0.0614
				n=1000			
$\beta_{11} = 0.693$	$\hat{\beta}_V$	0.7357	0.1852	0.1719	0.7142	0.1224	0.1189
	$\hat{\beta}_E$	0.6870	0.0478	0.0962	0.6899	0.0439	0.0761
$\beta_{21} = 0.262$	$\hat{\beta}_V$	0.2797	0.1807	0.1528	0.2634	0.1175	0.1055
	$\hat{\beta}_E$	0.2545	0.0433	0.0909	0.2565	0.0397	0.0780
$\beta_2 = -0.2$	$\hat{\beta}_V$	-0.2070	0.1155	0.1081	-0.2003	0.0784	0.0751
	$\hat{\beta}_E$	-0.1913	0.0257	0.0594	-0.1920	0.0256	0.0420
				n=3000			
$\beta_{11} = 0.693$	$\hat{\beta}_V$	0.7149	0.1921	0.1712	0.7063	0.1244	0.1186
	$\hat{\beta}_E$	0.6698	0.0231	0.0630	0.6705	0.0212	0.0431
$\beta_{21} = 0.262$	$\hat{\beta}_V$	0.2717	0.1643	0.1508	0.2657	0.1076	0.1049
	$\hat{\beta}_E$	0.2518	0.0221	0.0603	0.2521	0.0205	0.0404
$\beta_2 = -0.2$	$\hat{\beta}_V$	-0.2052	0.1190	0.1081	-0.2030	0.0767	0.0749
	$\hat{\beta}_E$	-0.1915	0.0146	0.0418	-0.1921	0.0145	0.0284

References

[1] Andersen, P. K. and Gill, R. D. (1982). Cox's regression model for counting processed: A large sample study. *Ann.Statist.* **10**, 1100-20.

[2] Foutz, R. V. (1977). On the Unique consistent solution to the likelihood equations. *Journal of the American Statistical Association* , **72**, 147-148.