Annex 1

1 HIV and HSV-2 statuses of females

For females, the probability of being infected was estimated based on data from a published survey [1]. Indeed, the statuses of the males' partners are unknown. These statuses e_k (k=0, 1, 2, 3) are expressed as four configurations (states) detailed in Table 1. For each female, the covariates are age, HIV and HSV-2 statuses and reported number of sexual partners in the past 12 months. The probability of being in state e_k was estimated by a multi-class logistic regression. This probability depends on age, q and the reported number of sexual partners in the past 12 months, y. Thus, we obtain the following formula:

$$\pi_k(q,y) = \frac{e^{\alpha_k q + \beta_k y}}{\sum_{j=0}^{3} e^{\alpha_j q + \beta_j y}}.$$

The reported number of sexual partners in the past 12 months was modeled by a Poisson distribution Y, which mean λ was estimated from the survey data using the maximum likelihood method. Hence:

$$\pi_k(q) = \sum_{y=0}^{+\infty} P_r(Y = y) \times \pi_k(q,y).$$

	HIV	HSV-2
e_0	-	-
e_1	+	-
e_2	-	+
e_3	+	+

Table 1: Statuses of men and women.

2 Notations and models

2.1 Settings

The HIV and HSV-2 statuses of males are determined at recruitment $\upsilon = 0$ and at the end of each of the 3 follow-up visits $\upsilon = 1,2,3$.

Consider an individual i. For $\upsilon = 1,2,3$, consider the period between visits $\upsilon - 1$ and υ; then, for that period, the observed data consist of:

$s_i^{(\upsilon)}$, the number of partners of i, $r_i^{(\upsilon,l)}$, his number of sexual contacts with his l^{th} partner, $q_i^{(\upsilon,l)}$, the age of his l^{th} partner, $\gamma_i^{(\upsilon,l)} = 1$ denotes the use of condom with a partner (0 otherwise), and $c_i^{(\upsilon)} = 1$ denotes a circumcision (0 otherwise).

2.2 Per-sex-act transmission model

2.2.1 Notations

We define the transmission probability per sex act of a given virus as the probability that a person becomes infected after a sexual contact with a person infected with the virus.

Let:

$P(hiv,\overline{hsv2})$ be the transmission probability per sex act of HIV from a woman not infected by HSV-2 to a man not infected by HSV-2, not circumcised and not condom user.

$P(hsv2,\overline{hiv})$ be the transmission probability per sex act of HSV-2 from a woman not infected by HIV to a man not infected by HIV, not circumcised and not condom user.

$RR(1)$ be the relative risk of HIV transmission to males associated with the presence of HSV-2

$RR(2)$ be the relative risk of HSV-2 transmission to males associated with the presence of HIV

$RR(0,1)$ be the relative risk of HIV transmission to males associated with condom use

$RR(1,0)$ be the relative risk of HIV transmission to males associated with male circumcision

$RR(0,2)$ be the relative risk of HSV-2 transmission to males associated with condom use

$RR(2,0)$ be the relative risk of HSV-2 transmission to males associated with male circumcision.

2.2.2 Probabilities of transmission between two visits

We derive the expression of the transmission probability per sex act, $\tilde{P}$, for a male i with his l^{th} partner between the visits $\upsilon-1$ and υ ($\upsilon=1,2,3$):

$\tilde{P}(hiv,\overline{hsv2},\gamma_i^{(\upsilon,l)},c_i^{(\upsilon)}) = RR(0,1)^{\gamma_i^{(\upsilon,l)}} \times RR(1,0)^{c_i^{(\upsilon)}} \times P(hiv,\overline{hsv2})$ is the transmission probability of HIV when neither the man nor his partner is infected by HSV-2

$\tilde{P}(hiv,hsv2+,\gamma_i^{(\upsilon,l)},c_i^{(\upsilon)}) = RR(0,1)^{\gamma_i^{(\upsilon,l)}} \times RR(1,0)^{c_i^{(\upsilon)}} \times RR(1) \times P(hiv,\overline{hsv2})$ is the transmission probability of HIV when either the male or his partner is infected by HSV-2

$\tilde{P}(hiv,hsv2++,\gamma_i^{(\upsilon,l)},c_i^{(\upsilon)}) = RR(0,1)^{\gamma_i^{(\upsilon,l)}} \times RR(1,0)^{c_i^{(\upsilon)}} \times RR(1)^2 \times P(hiv,\overline{hsv2})$ is the transmission probability of HIV when both the male and his partner are infected by HSV-2

$\tilde{P}(hsv2,\overline{hiv},\gamma_i^{(\upsilon,l)},c_i^{(\upsilon)}) = RR(0,2)^{\gamma_i^{(\upsilon,l)}} \times RR(2,0)^{c_i^{(\upsilon)}} \times P(hsv2,\overline{hiv})$ is the transmission probability of HSV-2 when neither the man nor his partner is infected by HIV

$\tilde{P}(hsv,hiv+,\gamma_i^{(\upsilon,l)},c_i^{(\upsilon)}) = RR(0,2)^{\gamma_i^{(\upsilon,l)}} \times RR(2,0)^{c_i^{(\upsilon)}} \times RR(2) \times P(hiv,hsv2)$ is the transmission probability of HSV-2 when either the male or his partner is infected by HIV

$\tilde{P}(hsv2,hiv++,\gamma_i^{(\upsilon,l)},c_i^{(\upsilon)}) = RR(0,2)^{\gamma_i^{(\upsilon,l)}} \times RR(2,0)^{c_i^{(\upsilon)}} \times RR(2)^2 \times P(hsv2,hiv)$ is the transmission probability of HSV-2 when both the male and his partner are infected by HIV.

2.2.3 Likelihood

Let

$P = (P(hiv,\overline{hsv2}), P(hsv2,\overline{hiv}))$ and $RR = (RR(1), RR(0,1), RR(1,0), RR(2), RR(0,2), RR(2,0))$.

Let i be an individual and $\varphi(P,RR,\gamma_i^{(\upsilon,l)},c_i^{(\upsilon)})$, the vector which coordinates are:

$1-\tilde{P}(hiv,\overline{hsv2},\gamma_i^{(\upsilon,l)},c_i^{(\upsilon)})$, $1-\tilde{P}(hsv2,\overline{hiv},\gamma_i^{(\upsilon,l)},c_i^{(\upsilon)})$, $1-\tilde{P}(hiv,hsv2+,\gamma_i^{(\upsilon,l)},c_i^{(\upsilon)})$,

$1-\tilde{P}(hsv2,hiv+,\gamma,c)$, $1-\tilde{P}(hiv,hsv2++,\gamma_i^{(\upsilon,l)},c_i^{(\upsilon)})$,

and $1-\tilde{P}(hsv2,hiv++,\gamma,c)$, respectively.

For the periods 1, 2, 3 (between visits $\upsilon-1$ and υ, υ=1, 2, 3), the contribution to the likelihood of each individual i of the sample ($L_{\upsilon,i}(e_k,e_j)$, where e_k is the state at visit $\upsilon-1$, and e_j is the state at visit υ, $k,j=0,1,2,3$) is obtained as follow:

$$L_{\upsilon,i}(e_0,e_0)=\prod_{j=1}^{s_i^{(\upsilon)}} pm(r_i^{(\upsilon,l)},q_i^{(\upsilon,l)},\varphi_i^{(\upsilon,l)});$$

$$L_{\upsilon,i}(e_0,e_1)=\sum_{t=1}^{s_i^{(\upsilon)}}\left(\prod_{j=1}^{t-1} pm(r_i^{(\upsilon,l)},q_i^{(\upsilon,l)},\varphi_i^{(\upsilon,l)})\right)\times um(r_i^{(\upsilon,l)},q_i^{(\upsilon,l)},\varphi_i^{(\upsilon,l)})\times\left(\prod_{j=t+1}^{s_i^{(\upsilon)}} qm(r_i^{(\upsilon,l)},q_i^{(\upsilon,l)},\varphi_i^{(\upsilon,l)})\right)$$

$$L_{\upsilon,i}(e_0,e_2)=\sum_{t=1}^{s_i^{(\upsilon)}}\left(\prod_{j=1}^{t-1} pm(r_i^{(\upsilon,l)},q_i^{(\upsilon,l)},\varphi_i^{(\upsilon,l)})\right)\times um(r_i^{(\upsilon,l)},q_i^{(\upsilon,l)},\varphi_i^{(\upsilon,l)})\times\left(\prod_{j=t+1}^{s_i^{(\upsilon)}} qm(r_i^{(\upsilon,l)},q_i^{(\upsilon,l)},\varphi_i^{(\upsilon,l)})\right);$$

$$L_{\upsilon,i}(e_0,e_3)=1-L_{\upsilon,i}(e_0,e_0)-L_{\upsilon,i}(e_0,e_1)-L_{\upsilon,i}(e_0,e_2);$$

$$L_{\upsilon,i}(e_1,e_1)=\prod_{j=1}^{s_i^{(\upsilon)}} qm(r_i^{(\upsilon,l)},q_i^{(\upsilon,l)},\varphi_i^{(\upsilon,l)});$$

$$L_{\upsilon,i}(e_1,e_3)=1-\prod_{j=1}^{s_i^{(\upsilon)}} qm(r_i^{(\upsilon,l)},q_i^{(\upsilon,l)},\varphi_i^{(\upsilon,l)});$$

$$L_{\upsilon,i}(e_2,e_2)=\prod_{j=1}^{s_i^{(\upsilon)}} rm(r_i^{(\upsilon,l)},q_i^{(\upsilon,l)},\varphi_i^{(\upsilon,l)});$$

$$L_{\upsilon,i}(e_2,e_3)=1-\prod_{j=1}^{s_i^{(\upsilon)}} rm(r_i^{(\upsilon,l)},q_i^{(\upsilon,l)},\varphi_i^{(\upsilon,l)});$$

With

$\varphi_i^{(\upsilon,j)}=\varphi(P,RR,\gamma_i^{(\upsilon,j)},c_i^{(\upsilon)})$,

and for any

$\xi=(\xi_1,\xi_2,\xi_3,\xi_4,\xi_5,\xi_6), 0<\xi_1,\xi_2,\xi_3,\xi_4,\xi_5,\xi_6<1$,

r,q integers:

$$pm(r,q,\xi)=\pi_0(q)+\pi_1(q)\xi_1^{\,r}+\pi_2(q)\xi_2^{\,r}+\pi_3(q)\xi_3^{\,r}\xi_4^{\,r};$$

$$qm(r,q,\xi)=\pi_0(q)+\pi_1(q)+\pi_2(q)\xi_3^{\,r}\xi_4^{\,r}+\pi_3(q)\cdot\xi_6^{\,r}+\pi_2(q)(1-\xi_3)\xi_4\sum_{j=1}^{r}(\xi_3\xi_4)^{r-j}\xi_6^{\,j-1};$$

$$rm(r,q,\xi)=\pi_0(q)+\pi_2(q)+\pi_1(q)\xi_3^{\,r}\xi_4^{\,r}+\pi_3(q)\cdot\xi_5^{\,r}+\pi_1(q)(1-\xi_4)\xi_3\sum_{j=1}^{r}(\xi_3\xi_4)^{r-j}\xi_5^{\,j-1};$$

$$um(r,q,\xi)=\pi_1(q)\left(1-\xi_1^{\,r}\right)+\pi_3(q)\xi_4(1-\xi_3)\sum_{j=1}^{r}\xi_6^{\,j-1}(\xi_3\xi_4)^{r-j};$$

$$vm(r,q,\xi)=\pi_2(q)\left(1-\xi_2^{\,r}\right)+\pi_3(q)\xi_3(1-\xi_4)\sum_{j=1}^{r}\xi_5^{\,j-1}(\xi_3\xi_4)^{r-j}.$$

2.3 Per-partnership transmission model

2.3.1 Notation

In this setting the transmission probability refers to the transmission probability per partnership from a woman to a man not infected. Thus, the previous model for transmission probability can be used with the following convention:

$P(hiv,\overline{hsv2})$ denotes the transmission probability per partnership of HIV from a woman not infected by HSV-2 to a man not infected by HSV-2.

$P(hsv2,\overline{hiv})$ denotes the transmission probability per partnership of HSV-2 from a woman not infected by HIV to a man not infected by HIV.

2.3.2 Probabilities of transmission between two visits

When a male i is with his l^{th} partner between the visits $\upsilon-1$ and υ ($\upsilon=1,2,3$), the transmission probabilities per partnership are given by the same formulas as those of section 2.2.3.

2.3.3 Likelihood

The likelihood is obtained by setting $r_i^{(\upsilon,l)}=1$ for all $i,\upsilon,l,$ and using the formulas of section 2.2.3.