

1. Equations A - 11 to A - 14, as defined by boundary conditions (A) in the Appendix.

$$D0 / Rp^2 == C1 * Rp + D1 / Rp^2$$

$$C1 * Rm + D1 / Rm^2 == C2 * Rm + D2 / Rm^2$$

$$C2 * rp + D2 / rp^2 == C3 * rp + D3 / rp^2$$

$$C3 * rm + D3 / rm^2 == C4 * rm$$

2. Equations A - 15 to A - 18, as defined by boundary conditions (B) in the Appendix.

$$S0 * (M + 2 * D0 / Rp^3) == S1 * (M + 2 * D1 / Rp^3 - C1)$$

$$S1 * (M + 2 * D1 / Rm^3 - C1) == S2 * (M + 2 * D2 / Rm^3 - C2)$$

$$S2 * (M + 2 * D2 / rp^3 - C2) == S3 * (M + 2 * D3 / rp^3 - C3)$$

$$S3 * (M + 2 * D3 / rm^3 - C3) == S4 * (M - C4)$$

3. Solve equations A - 11 to A - 18 to obtain the unknown coefficients D0 - D3 and C1 - C4.

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Solve[{D0 / Rp^2 == C1 * Rp + D1 / Rp^2,
C1 * Rm + D1 / Rm^2 == C2 * Rm + D2 / Rm^2,
C2 * rp + D2 / rp^2 == C3 * rp + D3 / rp^2,
C3 * rm + D3 / rm^2 == C4 * rm,
S0 * (M + 2 * D0 / Rp^3) == S1 * (M + 2 * D1 / Rp^3 - C1),
S1 * (M + 2 * D1 / Rm^3 - C1) == S2 * (M + 2 * D2 / Rm^3 - C2),
S2 * (M + 2 * D2 / rp^3 - C2) == S3 * (M + 2 * D3 / rp^3 - C3),
S3 * (M + 2 * D3 / rm^3 - C3) == S4 * (M - C4)},
{D0, C1, D1, C2, D2, C3, D3, C4}]
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D0

$$\left( M Rp^3 \left( -rm^3 \left( 2 Rm^6 (S0 + 2 S1) (S1 - S2) (S2 - S3) + 2 rp^3 Rp^3 (S0 - S1) (S1 - S2) (S2 + 2 S3) + Rm^3 \left( 2 Rp^3 (S0 - S1) (2 S1 + S2) (S2 - S3) + rp^3 (S0 + 2 S1) (S1 + 2 S2) (S2 + 2 S3) \right) \right) (S3 - S4) - rp^3 \left( 2 rp^3 Rp^3 (S0 - S1) (S1 - S2) (S2 - S3) + Rm^6 (S0 + 2 S1) (S1 - S2) (2 S2 + S3) + Rm^3 \left( rp^3 (S0 + 2 S1) (S1 + 2 S2) (S2 - S3) + Rp^3 (S0 - S1) (2 S1 + S2) (2 S2 + S3) \right) \right) (2 S3 + S4) \right) / \left( 2 rm^3 \left( 2 Rm^6 (S0 - S1) (S1 - S2) (S2 - S3) + rp^3 Rp^3 (2 S0 + S1) (S1 - S2) (S2 + 2 S3) + Rm^3 \left( Rp^3 (2 S0 + S1) (2 S1 + S2) (S2 - S3) + rp^3 (S0 - S1) (S1 + 2 S2) (S2 + 2 S3) \right) \right) (S3 - S4) + rp^3 \left( 2 rp^3 Rp^3 (2 S0 + S1) (S1 - S2) (S2 - S3) + 2 Rm^6 (S0 - S1) (S1 - S2) (2 S2 + S3) + Rm^3 \left( 2 rp^3 (S0 - S1) (S1 + 2 S2) (S2 - S3) + Rp^3 (2 S0 + S1) (2 S1 + S2) (2 S2 + S3) \right) \right) (2 S3 + S4) \right)$$

C1

$$\left( M (S0 - S1) \left( 2 rm^3 \left( 2 Rm^6 (S1 - S2) (S2 - S3) - rp^3 Rp^3 (S1 - S2) (S2 + 2 S3) + Rm^3 \left( -Rp^3 (2 S1 + S2) (S2 - S3) + rp^3 (S1 + 2 S2) (S2 + 2 S3) \right) \right) (S3 - S4) + rp^3 \left( -2 rp^3 Rp^3 (S1 - S2) (S2 - S3) + 2 Rm^6 (S1 - S2) (2 S2 + S3) + Rm^3 \left( 2 rp^3 (S1 + 2 S2) (S2 - S3) - Rp^3 (2 S1 + S2) (2 S2 + S3) \right) \right) (2 S3 + S4) \right) / \left( 2 rm^3 \left( 2 Rm^6 (S0 - S1) (S1 - S2) (S2 - S3) + rp^3 Rp^3 (2 S0 + S1) (S1 - S2) (S2 + 2 S3) + Rm^3 \left( Rp^3 (2 S0 + S1) (2 S1 + S2) (S2 - S3) + rp^3 (S0 - S1) (S1 + 2 S2) (S2 + 2 S3) \right) \right) (S3 - S4) + rp^3 \left( 2 rp^3 Rp^3 (2 S0 + S1) (S1 - S2) (S2 - S3) + 2 Rm^6 (S0 - S1) (S1 - S2) (2 S2 + S3) + Rm^3 \left( 2 rp^3 (S0 - S1) (S1 + 2 S2) (S2 - S3) + Rp^3 (2 S0 + S1) (2 S1 + S2) (2 S2 + S3) \right) \right) (2 S3 + S4) \right)$$



**C4**

$$\begin{aligned} & \left( M \left( 2 \text{rm}^3 \left( 2 \text{Rm}^6 (S_0 - S_1) (S_1 - S_2) (S_2 - S_3) + \text{rp}^3 \text{Rp}^3 (2 S_0 + S_1) (S_1 - S_2) (S_2 + 2 S_3) + \right. \right. \right. \\ & \quad \left. \left. \left. \text{Rm}^3 \left( \text{Rp}^3 (2 S_0 + S_1) (2 S_1 + S_2) (S_2 - S_3) + \text{rp}^3 (S_0 - S_1) (S_1 + 2 S_2) (S_2 + 2 S_3) \right) \right) (S_3 - S_4) + \right. \right. \\ & \quad \left. \left. \text{rp}^3 \left( 2 \text{rp}^3 \text{Rp}^3 (2 S_0 + S_1) (S_1 - S_2) (S_2 - S_3) (2 S_3 + S_4) + \right. \right. \right. \\ & \quad \left. \left. \left. 2 \text{Rm}^6 (S_0 - S_1) (S_1 - S_2) (2 S_2 + S_3) (2 S_3 + S_4) + \right. \right. \right. \\ & \quad \left. \left. \left. \text{Rm}^3 \left( 2 \text{rp}^3 (S_0 - S_1) (S_1 + 2 S_2) (S_2 - S_3) (2 S_3 + S_4) + \text{Rp}^3 \left( S_1 (2 S_1 + S_2) (2 S_2 + S_3) (2 S_3 + S_4) + \right. \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. S_0 (2 S_2 (2 S_2 + S_3) (2 S_3 + S_4) + S_1 (-65 S_2 S_3 + 8 S_3^2 + 8 S_2 S_4 + 4 S_3 S_4) \right) \right) \right) \right) \right) / \\ & \left( 2 \text{rm}^3 \left( 2 \text{Rm}^6 (S_0 - S_1) (S_1 - S_2) (S_2 - S_3) + \text{rp}^3 \text{Rp}^3 (2 S_0 + S_1) (S_1 - S_2) (S_2 + 2 S_3) + \right. \right. \\ & \quad \left. \left. \text{Rm}^3 \left( \text{Rp}^3 (2 S_0 + S_1) (2 S_1 + S_2) (S_2 - S_3) + \text{rp}^3 (S_0 - S_1) (S_1 + 2 S_2) (S_2 + 2 S_3) \right) \right) (S_3 - S_4) + \right. \\ & \quad \left. \left. \text{rp}^3 \left( 2 \text{rp}^3 \text{Rp}^3 (2 S_0 + S_1) (S_1 - S_2) (S_2 - S_3) + 2 \text{Rm}^6 (S_0 - S_1) (S_1 - S_2) (2 S_2 + S_3) + \right. \right. \right. \\ & \quad \left. \left. \left. \text{Rm}^3 \left( 2 \text{rp}^3 (S_0 - S_1) (S_1 + 2 S_2) (S_2 - S_3) + \text{Rp}^3 (2 S_0 + S_1) (2 S_1 + S_2) (2 S_2 + S_3) \right) \right) (2 S_3 + S_4) \right) \right) \end{aligned}$$

**4. Transmembrane potential on the cytoplasmic membrane**

$$V_1 = C_1 * \text{xx} + D_1 / \text{xx}^2$$

$$V_m = (C_1 * \text{Rm} + D_1 / \text{Rm}^2) - (C_1 * \text{Rp} + D_1 / \text{Rp}^2)$$

**Simplify[Vm]**

$$\begin{aligned} & \left( M \left( 3 \text{Rm}^3 \text{Rp} S_0 \left( \text{rm}^3 \left( 2 \text{Rm}^3 (S_1 - S_2) (S_2 - S_3) + \text{rp}^3 (S_1 + 2 S_2) (S_2 + 2 S_3) \right) (S_3 - S_4) + \right. \right. \right. \\ & \quad \left. \left. \left. \text{rp}^3 \left( \text{rp}^3 (S_1 + 2 S_2) (S_2 - S_3) + \text{Rm}^3 (S_1 - S_2) (2 S_2 + S_3) \right) (2 S_3 + S_4) \right) - \right. \right. \\ & \quad \left. \left. 3 \text{Rm} \text{Rp}^3 S_0 \left( \text{rm}^3 \left( 2 \text{Rm}^3 (S_1 - S_2) (S_2 - S_3) + \text{rp}^3 (S_1 + 2 S_2) (S_2 + 2 S_3) \right) (S_3 - S_4) + \right. \right. \right. \\ & \quad \left. \left. \left. \text{rp}^3 \left( \text{rp}^3 (S_1 + 2 S_2) (S_2 - S_3) + \text{Rm}^3 (S_1 - S_2) (2 S_2 + S_3) \right) (2 S_3 + S_4) \right) + \right. \right. \\ & \quad \left. \left. \text{Rm} (S_0 - S_1) \left( 2 \text{rm}^3 \left( 2 \text{Rm}^6 (S_1 - S_2) (S_2 - S_3) - \text{rp}^3 \text{Rp}^3 (S_1 - S_2) (S_2 + 2 S_3) + \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \text{Rm}^3 \left( -\text{Rp}^3 (2 S_1 + S_2) (S_2 - S_3) + \text{rp}^3 (S_1 + 2 S_2) (S_2 + 2 S_3) \right) \right) (S_3 - S_4) + \right. \right. \right. \\ & \quad \left. \left. \left. \text{rp}^3 \left( -2 \text{rp}^3 \text{Rp}^3 (S_1 - S_2) (S_2 - S_3) + 2 \text{Rm}^6 (S_1 - S_2) (2 S_2 + S_3) + \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \text{Rm}^3 \left( 2 \text{rp}^3 (S_1 + 2 S_2) (S_2 - S_3) - \text{Rp}^3 (2 S_1 + S_2) (2 S_2 + S_3) \right) \right) (2 S_3 + S_4) \right) - \right. \right. \\ & \quad \left. \left. \text{Rp} (S_0 - S_1) \left( 2 \text{rm}^3 \left( 2 \text{Rm}^6 (S_1 - S_2) (S_2 - S_3) - \text{rp}^3 \text{Rp}^3 (S_1 - S_2) (S_2 + 2 S_3) + \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \text{Rm}^3 \left( -\text{Rp}^3 (2 S_1 + S_2) (S_2 - S_3) + \text{rp}^3 (S_1 + 2 S_2) (S_2 + 2 S_3) \right) \right) (S_3 - S_4) + \right. \right. \right. \\ & \quad \left. \left. \left. \text{rp}^3 \left( -2 \text{rp}^3 \text{Rp}^3 (S_1 - S_2) (S_2 - S_3) + 2 \text{Rm}^6 (S_1 - S_2) (2 S_2 + S_3) + \right. \right. \right. \right. \\ & \quad \left. \left. \left. \left. \text{Rm}^3 \left( 2 \text{rp}^3 (S_1 + 2 S_2) (S_2 - S_3) - \text{Rp}^3 (2 S_1 + S_2) (2 S_2 + S_3) \right) \right) (2 S_3 + S_4) \right) \right) \right) / \\ & \left( 2 \text{rm}^3 \left( 2 \text{Rm}^6 (S_0 - S_1) (S_1 - S_2) (S_2 - S_3) + \text{rp}^3 \text{Rp}^3 (2 S_0 + S_1) (S_1 - S_2) (S_2 + 2 S_3) + \right. \right. \\ & \quad \left. \left. \text{Rm}^3 \left( \text{Rp}^3 (2 S_0 + S_1) (2 S_1 + S_2) (S_2 - S_3) + \text{rp}^3 (S_0 - S_1) (S_1 + 2 S_2) (S_2 + 2 S_3) \right) \right) (S_3 - S_4) + \right. \\ & \quad \left. \left. \text{rp}^3 \left( 2 \text{rp}^3 \text{Rp}^3 (2 S_0 + S_1) (S_1 - S_2) (S_2 - S_3) + 2 \text{Rm}^6 (S_0 - S_1) (S_1 - S_2) (2 S_2 + S_3) + \right. \right. \right. \\ & \quad \left. \left. \left. \text{Rm}^3 \left( 2 \text{rp}^3 (S_0 - S_1) (S_1 + 2 S_2) (S_2 - S_3) + \text{Rp}^3 (2 S_0 + S_1) (2 S_1 + S_2) (2 S_2 + S_3) \right) \right) (2 S_3 + S_4) \right) \right) \end{aligned}$$

**5. Transmembrane potential on the internal membrane**

$$V_{\text{internal}} = (C_3 * \text{rm} + D_3 / \text{rm}^2) - (C_3 * \text{rp} + D_3 / \text{rp}^2)$$

**Simplify[Vinternal]**

$$\begin{aligned}
 & \left( M \left( 27 \text{rm}^3 \text{Rm}^3 \text{rp} \text{Rp}^3 \text{S0 S1 S2} (S3 - S4) + 27 \text{rm} \text{Rm}^3 \text{rp}^3 \text{Rp}^3 \text{S0 S1 S2} (-S3 + S4) + \right. \right. \\
 & \quad \text{rm} \left( 2 \text{rm}^3 \left( 2 \text{Rm}^6 (S0 - S1) (S1 - S2) (S2 - S3) + \text{rp}^3 \text{Rp}^3 (2 \text{S0} + \text{S1}) (S1 - S2) (S2 + 2 \text{S3}) + \right. \right. \\
 & \quad \quad \left. \left. \text{Rm}^3 \left( \text{Rp}^3 (2 \text{S0} + \text{S1}) (2 \text{S1} + \text{S2}) (S2 - S3) + \text{rp}^3 (S0 - S1) (S1 + 2 \text{S2}) (S2 + 2 \text{S3}) \right) \right) (S3 - S4) + \right. \\
 & \quad \left. \text{rp}^3 \left( 2 \text{rp}^3 \text{Rp}^3 (2 \text{S0} + \text{S1}) (S1 - S2) (S2 - S3) + 2 \text{Rm}^6 (S0 - S1) (S1 - S2) (2 \text{S2} + \text{S3}) + \right. \right. \\
 & \quad \quad \left. \left. \text{Rm}^3 \left( 2 \text{rp}^3 (S0 - S1) (S1 + 2 \text{S2}) (S2 - S3) + \right. \right. \right. \\
 & \quad \quad \quad \left. \left. \left. \text{Rp}^3 (S1 (2 \text{S1} + \text{S2}) (2 \text{S2} + \text{S3}) + \text{S0} (2 \text{S2} (2 \text{S2} + \text{S3}) + \text{S1} (-19 \text{S2} + 4 \text{S3})) \right) \right) \right) (2 \text{S3} + \text{S4}) \right) - \\
 & \quad \left. \text{rp} \left( 2 \text{rm}^3 \left( 2 \text{Rm}^6 (S0 - S1) (S1 - S2) (S2 - S3) + \text{rp}^3 \text{Rp}^3 (2 \text{S0} + \text{S1}) (S1 - S2) (S2 + 2 \text{S3}) + \right. \right. \right. \\
 & \quad \quad \left. \left. \left. \text{Rm}^3 \left( \text{Rp}^3 (2 \text{S0} + \text{S1}) (2 \text{S1} + \text{S2}) (S2 - S3) + \text{rp}^3 (S0 - S1) (S1 + 2 \text{S2}) (S2 + 2 \text{S3}) \right) \right) \right) (S3 - S4) + \right. \\
 & \quad \left. \text{rp}^3 \left( 2 \text{rp}^3 \text{Rp}^3 (2 \text{S0} + \text{S1}) (S1 - S2) (S2 - S3) + 2 \text{Rm}^6 (S0 - S1) (S1 - S2) (2 \text{S2} + \text{S3}) + \right. \right. \\
 & \quad \quad \left. \left. \text{Rm}^3 \left( 2 \text{rp}^3 (S0 - S1) (S1 + 2 \text{S2}) (S2 - S3) + \text{Rp}^3 (S1 (2 \text{S1} + \text{S2}) (2 \text{S2} + \text{S3}) + \right. \right. \right. \\
 & \quad \quad \quad \left. \left. \left. \text{S0} (2 \text{S2} (2 \text{S2} + \text{S3}) + \text{S1} (-19 \text{S2} + 4 \text{S3})) \right) \right) \right) (2 \text{S3} + \text{S4}) \right) \left. \right) / \\
 & \left( 2 \text{rm}^3 \left( 2 \text{Rm}^6 (S0 - S1) (S1 - S2) (S2 - S3) + \text{rp}^3 \text{Rp}^3 (2 \text{S0} + \text{S1}) (S1 - S2) (S2 + 2 \text{S3}) + \right. \right. \\
 & \quad \left. \left. \text{Rm}^3 \left( \text{Rp}^3 (2 \text{S0} + \text{S1}) (2 \text{S1} + \text{S2}) (S2 - S3) + \text{rp}^3 (S0 - S1) (S1 + 2 \text{S2}) (S2 + 2 \text{S3}) \right) \right) (S3 - S4) + \right. \\
 & \quad \left. \text{rp}^3 \left( 2 \text{rp}^3 \text{Rp}^3 (2 \text{S0} + \text{S1}) (S1 - S2) (S2 - S3) + 2 \text{Rm}^6 (S0 - S1) (S1 - S2) (2 \text{S2} + \text{S3}) + \right. \right. \\
 & \quad \quad \left. \left. \text{Rm}^3 \left( 2 \text{rp}^3 (S0 - S1) (S1 + 2 \text{S2}) (S2 - S3) + \text{Rp}^3 (2 \text{S0} + \text{S1}) (2 \text{S1} + \text{S2}) (2 \text{S2} + \text{S3}) \right) \right) (2 \text{S3} + \text{S4}) \right)
 \end{aligned}$$