Web-based Supplementary Materials for ”A Bayesian approach to modeling associations between pulsatile hormones,” by Nichole E. Carlson, Timothy D. Johnson and Morton B. Brown.

1 Appendix A: The birth-death equations

In the birth-death stage, a finite length birth-death process is created where a birth of a pulse pair occurs with rate $\beta(\theta)$ and a death of pulse pair, j, occurs with rate $\delta_j(\theta)$. The total death rate is then $\delta(\theta) = \sum_j \delta_j(\theta)$. The time between a birth or a death event is exponentially distributed with mean, $\{\beta(\theta) + \delta(\theta)\}^{-1}$. An event is a birth with probability $\beta(\theta)/\{\beta(\theta) + \delta(\theta)\}$ and a death with probability $\delta(\theta)/\{\beta(\theta) + \delta(\theta)\}$. When the event is a birth, the parameters defining the new pulse pair are drawn from distribution $b(\theta)$. In our case, a pulse pair is added and the parameters defining the pulse pair (driver location and lag time and the pulse masses and widths) are drawn from specified distributions. The pulse masses and lags are drawn from their prior distributions, and the driver pulse locations are drawn from a $U[a, b]$; *i.e.*,

$$
\begin{aligned}
b(\theta) = \frac{1}{(b-a)} \frac{\beta^\alpha}{\Gamma(\alpha)} \tau^{\alpha-1} \times \\
\exp\{-\beta\tau\} \frac{1}{2\pi} |\kappa_{1i}^{-1}\Sigma_m|^{-1/2} \exp\{(\boldsymbol{A} - \boldsymbol{\mu}_m)' k_{i1} \Sigma_m^{-1} (\boldsymbol{A} - \boldsymbol{\mu}_m)\} I_{R^{2+}}(A) \\
\times \frac{1}{2\pi} |\kappa_{2i}^{-1}\Sigma_w|^{-1/2} \exp\{(\boldsymbol{\sigma_w^2} - \boldsymbol{\mu}_w)' k_{i2} \Sigma_w^{-1} (\boldsymbol{\sigma_w^2} - \boldsymbol{\mu}_w)\} I_{R^{2+}}(\sigma_w^2)
\end{aligned}
$$

In $b(\theta)$, $\boldsymbol{A} = (A_x, A_y)'$, $\boldsymbol{\sigma_w^2} = (\sigma_{xw}^2, \sigma_{yw}^2)'$. If we assume $\beta(\theta)$ is a constant, λ_b, as suggested by Stephens (2000a), then the death rate, $\lambda_j(w)$, for each pulse pair is:

$$
\begin{aligned}
\delta_j &= \lambda_b b(\theta) \frac{L_{-j}}{L} \frac{p(k-1|\lambda) p(t_{x1} \ldots t_{x,k-1}|\text{a,b}) \prod_{i=1}^{k-1} p(\boldsymbol{A_i}|\mu_i, \Sigma_m) p(\tau_k|\alpha, \beta)}{p(k|\lambda) p(t_{x1} \ldots t_{x,k}|\text{a,b}) \prod_{i=1}^{k} p(\boldsymbol{A_i}|\mu_i, \Sigma_m) p(\tau_k|\alpha, \beta) k} \\
&= \frac{\lambda_b}{\lambda} \frac{L_{-j}}{L} \frac{(t_{x,j+1} - t_{xj})(b-a)}{(t_{xj} - t_{x,j-1})(t_{x,j+1} - t_{xj})(4k+2)} \qquad (1)
\end{aligned}
$$

where L_{-j} is the likelihood with the jth pulse removed. The probability of a particular pulse pair dying is δ_j/δ and randomly chosen according to a multinomial.

2 Appendix B: Post simulation processing algorithm

When combining two secretion events into one event, we applied the following algorithm. We based this algorithm on the combine move of the split and combine moves in reversible jump MCMC algorithm for mixture distributions (Richardson and Green, 1997).

$$\begin{aligned} A_{\bullet,i'} &= A_{\bullet,i} + A_{\bullet,i+1} \\ A_{\bullet,i'} t_{\bullet,i'} &= (A_{\bullet,i} t_{\bullet,i} + A_{\bullet,i+1} t_{\bullet,i+1}) \\ \tau_{i'} &= t_{y,i'} - t_{x,i'} \end{aligned}$$

Table 1: Posterior means for the pulses masses in the bivariate and univariate BDMCMC estimates for the FSH series in the LH-FSH dataset in the full text.

	FSH			
	Bivariate		Univariate	
	Location	Mass	Location	Mass
Pulse 1	68.5	0.58	–	–
Pulse 2	156.18	0.61	162.54	0.43
Pulse 3	280.17	1.17	309.98	0.78
Pulse 4	338.08	0.26	–	–
Pulse 5	391.36	1.24	414.50	0.94
Pulse 6	467.68	0.75	–	–
Pulse 7	516.06	0.42	514.56	0.54
Pulse 8	611.16	0.38	–	–
Pulse 9	670.96	0.33	–	–
Pulse 10	714.18	0.48	721.94	0.47
Pulse 11	778.87	0.39	–	–
Pulse 12	824.67	0.32	–	–
Pulse 13	860.51	0.66	869.05	0.61
Pulse 14	914.02	0.68	–	–
Pulse 15	952.88	0.64	937.79	0.66
Pulse 16	997.70	0.50	1002.33	0.56
Pulse 17	1037.52	0.29	–	–
Pulse 18	1099.69	0.37	–	–
Pulse 19	1146.12	0.49	1158.96	0.48
Pulse 20	1210.80	0.78	–	–
Pulse 21	1278.65	0.63	1270.69	0.54
Pulse 22	1357.77	0.62	1368.89	0.55
Pulse 23	1387.19	0.39	–	–

Table 2: Summary statistics for the common parameters for the example simulated series from the bivariate and univariate fits: Mean=mean of the posterior and CI=credible interval

	LH		FSH	
	Bivariate	Univariate	Bivariate	Univariate
Parameter	Mean (95% CI)	Mean (95% CI)	Mean (95% CI)	Mean (95% CI)
No. Pulses	15.09 (15,16)	15.09 (14,16)	–	10.44 (6,17)
Baseline[a]	0.70 (0.08,1.30)	0.64 (0.04,1.28)	2.27 (0.42,3.25)	2.51 (0.58,3.38)
Half-Life[b]	63.75 (50.19,79.52)	66.10 (51.10,84.17)	309.01 (189.34,558.94)	601.86 (339.03,895.94)
Pulse Width[c]	5.32 (0.66,15.48)	5.59 (0.53,16.46)	29.28 (0.22,170.59)	48.83 (0.24,308.10)
Total pulse sec.[a]	62.09 (53.74,70.78)	61.01 (50.94,70.61)	10.22 (7.75,13.33)	5.56 (3.96,88.68)
Mean of Lag Dist'n[b]	16.13 (11.41,20.90)	–	–	–
Mean Log(Pulse) Mass	1.38 (1.17,1.57)	1.36 (1.11,1.57)	-0.47 (-0.88,-0.12)	-0.75 (-1.70,-0.20)
Log(Pulse) Mass Var	0.06 (0.02,0.15)	0.10 (0.03,0.24)	0.16 (0.04,0.45)	0.51 (0.02,2.50)
Mass Correlation	0.76 (0.35,0.95)	–	–	–

[a]units=ng/mL
[b]units=minutes
[c]units=minutes^2