Supporting Information

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1. Multiple Players with Two Strategies

1.1. Infinite Populations. We first address the replicator dynamics of multiplayer games with two strategies. If an A player interacts with k other A players, it obtains the payoff a_k. If a B player interacts with k A players, it obtains the payoff b_k. In an infinitely large population in which the fraction of A players is x, the probability that an A player interacts with k other A players is

$$\binom{d-1}{k} x^k (1-x)^{d-1-k}. \qquad \textbf{[S1]}$$

Here, $\binom{d-1}{k}$ is the number of possibilities of arranging the players. Thus, the average payoffs of A and B are given by

$$\begin{aligned} \pi_A &= \sum_{k=0}^{d-1} \binom{d-1}{k} x^k (1-x)^{d-1-k} a_k \\ \pi_B &= \sum_{k=0}^{d-1} \binom{d-1}{k} x^k (1-x)^{d-1-k} b_k. \end{aligned} \qquad \textbf{[S2]}$$

These average payoffs are subject to the condition that the order of the players does not matter. For example, in a $d = 3$ game, let the player in the first position play A. Then, the remaining two players can play a combination of A and B. The possible combinations are AAB and ABA. By writing the payoffs in the above-mentioned manner, we assume that such combinations have the same payoffs.

If the order of players does matter, then the payoff values are given by $\beta_{i_0,i_1,i_2,i_3,\ldots,i_{d-1}}$. Here, i_0 is the strategy of the focal player. The i_p are the strategies of the type in position p. For random matching of players, we can map the $\beta_{i_0,i_1,i_2,i_3,\ldots,i_{d-1}}$ to modified payoffs $\tilde{a}_k$ and $\tilde{b}_k$ without changing the average payoffs of the strategies. As an example, for $d = 4$, we have the modified payoffs $\tilde{a}_k$ and $\tilde{b}_k$ as

$$\begin{aligned} \tilde{a}_0 &= \beta_{A,B,B,B} & \tilde{b}_0 &= \beta_{B,B,B,B} \\ \tilde{a}_1 &= \frac{\beta_{A,A,B,B}+\beta_{A,B,A,B}+\beta_{A,B,B,A}}{3} & \tilde{b}_1 &= \frac{\beta_{B,A,B,B}+\beta_{B,B,A,B}+\beta_{B,B,B,A}}{3} \\ \tilde{a}_2 &= \frac{\beta_{A,A,A,B}+\beta_{A,A,B,A}+\beta_{A,B,A,A}}{3} & \tilde{b}_2 &= \frac{\beta_{B,A,A,B}+\beta_{B,A,B,A}+\beta_{B,B,A,A}}{3} \\ \tilde{a}_3 &= \beta_{A,A,A,A} & \tilde{b}_3 &= \beta_{B,A,A,A}. \end{aligned} \qquad \textbf{[S3]}$$

We just need to substitute the above payoffs in place of a_k and b_k in Eq. **S2** to take into account the effect of the arrangement of players. For any number of players such a generalization can be easily obtained. Thus, the evolutionary dynamics under random-interaction group formation remains unaffected by the fact that the order of players does matter. When interaction groups are not formed at random, this argument will, of course, fail in most cases.

The following analysis deals with π_A and π_B as in Eq. **S2**, but it also holds when the order of players matters but interaction groups are formed at random. The replicator equation is thus given by (1, 2)

$$\dot{x} = x(1-x)(\pi_A - \pi_B). \qquad \textbf{[S4]}$$

Both π_A and π_B are polynomials of degree $d-1$. This implies that the replicator equation can have up to $d-1$ interior fixed points (3).

Maximum number of interior fixed points. For a d-player game to have $d-1$ interior fixed points, the quantities $a_k - b_k$ and $a_{k+1} - b_{k+1}$ must have different signs for all k. For example, in a three-player game with $a_0 = +1$, $a_1 = -\lambda$, $a_2 = +1$ and $b_0 = -1$, $b_1 = +\lambda$, $b_2 = -1$, we have two internal equilibria at $\frac{1}{2}(1 \pm \sqrt{\frac{\lambda-1}{\lambda+1}})$ for $\lambda > 1$. However, this condition is necessary (because the direction of selection can only change $d-1$ times if the payoff difference $a_k - b_k$ changes sign $d-1$ times), but not sufficient. For example, in the above three-player game, there are no internal equilibria for $\lambda < 1$.

Single interior fixed point. A d-player game has a single internal equilibrium if $a_k - b_k$ has a different sign from $a_{k+1} - b_{k+1}$ for a single value of k: In this case, A individuals are disadvantageous at low frequency and advantageous at high frequency (or vice versa). If $a_k - b_k$ changes sign only once, then the direction of selection can obviously at most change once. Thus, this condition is sufficient.

1.2. Finite Populations. Let us now turn to the evolutionary dynamics in finite populations. In a population of size N with j individuals of type A, the probability of choosing a group that consists of k A players and $d-1-k$ B players is given by a hypergeometric distribution. The probability that an A player interacts with k other A players is given by

$$H(k,d;j,N) = \frac{\binom{j-1}{k}\binom{N-j}{d-1-k}}{\binom{N-1}{d-1}}. \qquad \textbf{[S5]}$$

This leads to the average payoffs

$$\begin{aligned} \pi_A &= \sum_{k=0}^{d-1} \frac{\binom{j-1}{k}\binom{N-j}{d-1-k}}{\binom{N-1}{d-1}} a_k \\ \pi_B &= \sum_{k=0}^{d-1} \frac{\binom{j}{k}\binom{N-j-1}{d-1-k}}{\binom{N-1}{d-1}} b_k. \end{aligned} \qquad \textbf{[S6]}$$

We assume that strategies spread by a frequency-dependent Moran process (4–6). The fitness is given by $f_A = \exp(+w\pi_A)$ for A players and $f_B = \exp(+w\pi_B)$ for B players, where w measures the intensity of selection (7). For $w \ll 1$, selection is weak. For $w \gg 1$, selection is strong and only the fitter type reproduces. In the Moran process, an individual is selected for reproduction at random but proportional to its fitness. The individual produces identical offspring. Another individual is chosen at random for death. Consider j individuals of type A in a population of size N. The number of A individuals increases with probability T_j^+ from j to $j+1$ if an A individual is selected for reproduction and a B individual dies. We have

$$T_j^+ = \frac{jf_A}{jf_A + (N-j)f_B}\frac{N-j}{N} \qquad \textbf{[S7]}$$

$$T_j^- = \frac{(N-j)f_B}{jf_A + (N-j)f_B}\frac{j}{N}. \qquad \textbf{[S8]}$$

The fixation probability of a single A individual in a population of N is given by (8)

$$\rho_A = \frac{1}{1 + \sum_{m=1}^{N-1} \prod_{j=1}^{m} \frac{T_j^-}{T_j^+}}. \quad \textbf{[S9]}$$

For the ratio of transition probabilities, we have

$$\frac{T_j^-}{T_j^+} = \frac{f_B}{f_A} = e^{-w(\pi_A - \pi_B)} \approx 1 - w(\pi_A - \pi_B). \quad \textbf{[S10]}$$

The approximation is valid for weak selection, $w \ll 1$. Note that this is the only approximation we make, such that our result is valid for any birth-death process with

$$\frac{T_j^-}{T_j^+} \approx 1 - w(\pi_A - \pi_B). \quad \textbf{[S11]}$$

For weak selection, the product in the fixation probabilities can be approximated by a sum, which leads to

$$\rho_A \approx \frac{1}{N} + \frac{w}{N} \underbrace{\sum_{m=1}^{N-1} \sum_{j=1}^{m} (\pi_A - \pi_B)}_{\Gamma}. \quad \textbf{[S12]}$$

In *Appendix A*, we show that

$$\Gamma = \frac{1}{d(d+1)} \left[N^2 \left(\sum_{k=0}^{d-1} (d-k)(a_k - b_k) \right) - N \left(\sum_{k=0}^{d-1} (k+1) a_k + \sum_{k=1}^{d-1} (d-k) b_k - d^2 b_0 \right) \right]. \quad \textbf{[S13]}$$

As seen from Eq. **S12**, a strategy is favored by selection; that is, it has a fixation probability larger than $1/N$ if $\Gamma > 0$. For any N, $\Gamma > 0$ can be represented by

$$\sum_{k=0}^{d-1} \left[N(d-k) - k - 1 \right] a_k > \sum_{k=0}^{d-1} \left[\left(N+1 \right)(d-k) b_k - (d+1) b_0 \right]. \quad \textbf{[S14]}$$

For $d = 2$, this condition reduces to the condition $(2N-1)a_0 + (N-2)a_1 > (2N-4)b_0 + (N+1)b_1$, exactly as developed by Nowak et al. (9). For a large population size, the condition can be simplified to

$$\sum_{k=0}^{d-1} (d-k) a_k > \sum_{k=0}^{d-1} (d-k) b_k. \quad \textbf{[S15]}$$

In large populations, we have $\rho_A > 1/N$ if the condition Eq. **S15** is fulfilled. In the usual case of $d = 2$, the fixation probability of strategy A is larger than $1/N$ if $2a_0 + a_1 > 2b_0 + b_1$. This can be rearranged to

$$x^* = \frac{b_0 - a_0}{a_1 - a_0 - b_1 + b_0} < \frac{1}{3}. \quad \textbf{[S16]}$$

This is the 1/3-law first derived in ref. 9: A mutant takes over the population with probability larger than neutral if the mutant is advantageous when it has reached a fraction of 1/3. Condition Eq. **S15** represents a generalization of the 1/3 law for general d-player games.

We can also compare the fixation probability ρ_A of a single A player to the fixation probability ρ_B of a single B player. It has been shown (7, 8) that

$$\frac{\rho_B}{\rho_A} = \prod_{j=1}^{N-1} \frac{T_j^-}{T_j^+} = \exp\left[\underbrace{-w \sum_{j=1}^{N-1} (\pi_A - \pi_B)}_{\Phi} \right]. \quad \textbf{[S17]}$$

Note that if our previous approximation Eq. **S11** holds, then we obtain $\frac{\rho_B}{\rho_A} \approx 1 - w\Phi$. Because we do not make any further approximations, our calculation remains valid for any birth-death process fulfilling Eq. **S11** under weak selection. As shown in *Appendix B*,

$$\Phi = \frac{N}{d} \sum_{k=0}^{d-1} (a_k - b_k) + b_0 - a_{d-1}. \quad \textbf{[S18]}$$

From Eq. **S17**, it is clear that $\rho_A > \rho_B$ if $\Phi > 0$. This is equivalent to the condition

$$\sum_{k=0}^{d-1} (N a_k - a_{d-1}) > \sum_{k=0}^{d-1} (N b_k - b_0). \quad \textbf{[S19]}$$

Note that this condition is valid for any intensity of selection for the process we use. For weak selection, it is valid for all processes with $\frac{T_j^-}{T_j^+} \approx 1 - w(\pi_A - \pi_B)$. For $d = 2$, expression Eq. **S19** reduces to $(N - 2)\ (a_1 - b_0) > N(b_1 - a_0)$, which is the risk dominance condition developed in ref. 10 for finite population size (see also ref. 11 for the generality of this finding). For a large population, the condition can be further simplified:

$$\sum_{k=0}^{d-1} a_k > \sum_{k=0}^{d-1} b_k. \quad \textbf{[S20]}$$

For two-player games, this reduces to risk dominance, $a_0 + a_1 > b_0 + b_1$.

We can also incorporate mutations, which will complicate the transition probabilities. For symmetric mutation rates, $\mu_{A \to B} = \mu_{B \to A}$, the condition $\rho_A > \rho_B$ is equivalent to a higher average abundance of A compared to B given that $\mu_{A \to B}$ and $\mu_{B \to A}$ are small. For $d = 2$, it has recently been shown that the abundance condition does in fact depend neither on the mutation rate nor on the intensity of selection (11). For $d > 2$, this statement no longer holds, which can be seen from the high mutation limit: If the mutation rates are very high, then the system will be driven toward the point where the two abundances are identical. The dynamics at this point, however, does not depend on the parameters in the same way as $\rho_A > \rho_B$ when it comes to d-player games.

2. Multiplayer Games with Multiple Strategies

2.1. Infinite Populations. In the full multiverse, we have multiple players playing multiple strategies. We are interested in the maximum number of internal equilibria of a system, which will help us understand the general features of the dynamics. Consider a system with d players with n possible strategies. Here we resort to the payoff values as given by $\beta_{i_0,i_1,i_2,i_3,\ldots,i_{d-1}}$, because for random group formation a system where the order of players does matter can always be reduced to a system where the order does not matter. Here, i_0 is the strategy of the focal player. The i_p are the strategies of the type in position p. Then the average payoff of the focal player is given by

$$\pi_{i_0} = \sum_{i_1=1}^{n} \sum_{i_2=1}^{n} \cdots \sum_{i_{d-1}=1}^{n} \left(\prod_{k=i_1}^{i_{d-1}} x_k \right) \beta_{i_0,i_1,i_2,i_3,\ldots,i_{d-1}}. \quad \textbf{[S21]}$$

From this it is clear that each variable x_k is at most of degree $d - 1$. Also, as there are n strategies, we have $i_0 = (1, 2, \ldots, n)$, that is, n such multivariate polynomials. Each multivariate polynomial is in $n - 1$ variables (because of the normalization $\sum_{l=1}^{n} x_l = 1$). At the fixed points, all these polynomials will be equal. Hence, if we subtract one of the polynomials (say π_n) from all, we have a system of $n - 1$ multivariate polynomials, $\Delta\pi_{i_0}$, equal to zero (where i_0 goes from 1 to $n - 1$). In each variable x_k, the multivariate polynomial $\Delta\pi_{i_0}$ is at most of degree $d - 1$. Hence, there are at most $d-1$ roots of $\Delta\pi_{i_0}$ in x_k. Because this is valid for all $n-1$ functions of $\Delta\pi_{i_0}$, there can be up to $(d-1)^{n-1}$ simultaneous roots of all $\Delta\pi_{i_0}$. These are the interior fixed points of the replicator dynamics. Thus, there can be at most

$$(d-1)^{n-1} \qquad \textbf{[S22]}$$

fixed points in the interior of the system. This holds for the full system but also for any subspace in which fewer strategies are available. For example, a game with $d = 3$ players and $n = 4$ strategies has up to 8 fixed points in the interior of the simplex S_4. On the faces of the simplex S_4, represented by the simplex S_3, there can be up to 4 fixed points.

We now have an analytical method to deduce the maximum number of internal equilibria. The question that now arises is: With what probability do we see this maximum number of equilibria? We address the problem by generating 10^8 payoff matrices where the payoff values $a_k, b_k, \ldots$, are drawn from a uniform distribution for different configurations of d and n. As discussed in the main text, the probability of obtaining the maximum number of internal equilibria in a game with random payoff entries reduces as the complexity increases in d as well as n.

An example for d = 4 and n = 3. In this section, we describe the parameters of Fig. 2 in the main text. The number of players $d = 4$ and the number of strategies $n = 3$. The total number of payoff values is therefore n^d, which is 81. Thus, for each strategy there are 27 payoff values. This is the number of values we have to consider when the order of player matters. If the payoffs are the same for different arrangements then we reduce the payoff values, but we have to weight them by the number of their occurrence. Consider the three strategies to be A, B, and C. Solving the replicator equation using the average payoffs calculated from the payoffs from Table S1, we numerically obtain 9 fixed points in the interior of the simplex. At these points, the frequencies of all of the strategies are nonzero and the average payoff to each strategy is equal.

2.2. Finite Populations. For finite populations and more than two strategies, few analytical tools are available. The average abundance under weak selection can be addressed using tools from coalescence theory (12, 13).

For small mutation rates, the dynamics reduces to an embedded Markov chain on the pure states of the system [see Fudenberg and Imhof (14) for a proof]. Essentially, this means that the dynamics is governed by dynamics on the edges of the simplex S_n where only two strategies are present. This result can be applied in a variety of contexts (15–17).

Both approaches can be adapted to d-player games.

Appendix A

Condition for the Comparison of One Strategy with Neutrality. We first repeat the condition to prove

$$\sum_{m=1}^{N-1}\sum_{j=1}^{m}(\pi_A-\pi_B) = \frac{1}{d(d+1)}\left[N^2\left(\sum_{k=0}^{d-1}(d-k)(a_k-b_k)\right) - N\left(\sum_{k=0}^{d-1}(k+1)a_k+\sum_{k=1}^{d-1}(d-k)b_k-d^2b_0\right)\right], \qquad \textbf{[S23]}$$

where the payoffs are defined in Eq. **S6**. Because all of the a_ks come from π_A and all of the b_ks from π_B, we can solve each separately. For π_A we have to show that

$$\sum_{m=1}^{N-1}\sum_{j=1}^{m}\sum_{k=0}^{d-1}\frac{\binom{j-1}{k}\binom{N-j}{d-k-1}}{\binom{N-1}{d-1}}a_k = \sum_{k=0}^{d-1}\frac{N^2(d-k)-N(k+1)}{d(d+1)}a_k. \qquad \textbf{[S24]}$$

Because this should hold for any choice of a_ks, we must show that

$$\sum_{m=1}^{N-1}\sum_{j=1}^{m}\frac{\binom{j-1}{k}\binom{N-j}{d-k-1}}{\binom{N-1}{d-1}} = \frac{N^2(d-k)-N(k+1)}{d(d+1)}. \qquad \textbf{[S25]}$$

We take out the factor $\binom{N-1}{d-1}^{-1}$ on the left-hand side and get back to the full expression only at the end. We consider the quantity

$$\sum_{m=1}^{N-1}\sum_{j=1}^{m}\binom{j-1}{k}\binom{N-j}{d-k-1}. \qquad \textbf{[S26]}$$

Using the identity $\sum_{m=1}^{N-1}\sum_{j=1}^{m} = \sum_{j=1}^{N-1}\sum_{m=j}^{N-1}$, we obtain

$$\begin{aligned}\sum_{m=1}^{N-1}\sum_{j=1}^{m}&\binom{j-1}{k}\binom{N-j}{d-k-1}\\ &=\sum_{j=1}^{N-1}\sum_{m=j}^{N-1}\binom{j-1}{k}\binom{N-j}{d-k-1}\\ &=\sum_{j=1}^{N-1}\binom{j-1}{k}\binom{N-j}{d-k-1}(N-j),\end{aligned} \qquad \textbf{[S27]}$$

where we performed the sum over m. Let us use the factor $N-j$ to split this expression into two sums. The first sum with the factor N is given by

$$\Sigma_1 = N\sum_{j=1}^{N-1}\binom{j-1}{k}\binom{N-j}{d-k-1}. \qquad \textbf{[S28]}$$

We change the summation index by one, $i = j-1$, and then extend the sum up to $N-1$,

$$\begin{aligned}\Sigma_1 &= N\sum_{i=0}^{N-2}\binom{i}{k}\binom{N-i-1}{d-k-1}\\ &= N\left[\sum_{i=0}^{N-1}\binom{i}{k}\binom{N-i-1}{d-k-1}-\binom{N-1}{k}\binom{0}{d-k-1}\right].\end{aligned} \qquad \textbf{[S29]}$$

The last term is zero as long as $d-k-1>0$, that is, $k<d-1$. We can now apply a variant of Vandermonde's convolution, $\sum_{i=0}^{l}\binom{l-i}{m}\binom{q+i}{n}=\binom{l+q+1}{m+n+1}$ (18), on the first term and obtain for $k<d-1$ the result $\Sigma_1 = N\binom{N}{d}$. For the special case of $k = d-1$, we start from Eq. **S28**,

$$\Sigma_1 = N\sum_{j=1}^{N-1}\binom{j-1}{d-1}\binom{N-j}{0} = N\sum_{j=1}^{N-1}\binom{j-1}{d-1}. \qquad \textbf{[S30]}$$

Using the identity $\sum_{j=1}^{N-1}\binom{j-1}{d-1}=\binom{N-1}{d}$, we obtain $\Sigma_1 = N\binom{N-1}{d} = (N-d)\binom{N}{d}$. To summarize, we have for Σ_1

$$\Sigma_1 = \begin{cases} N\binom{N}{d} & \text{for } 0 \le k < d-1 \\ N\binom{N-1}{d} = (N-d)\binom{N}{d} & \text{for } k = d-1 \end{cases}. \quad \textbf{[S31]}$$

The second sum in Eq. **S27** involving the additional factor j can be rewritten as

$$\Sigma_2 = \sum_{j=1}^{N-1} j\binom{j-1}{k}\binom{N-j}{d-k-1} = (k+1)\sum_{j=1}^{N-1}\binom{j}{k+1}\binom{N-j}{d-k-1}, \quad \textbf{[S32]}$$

where we have used $j\binom{j-1}{k} = (k+1)\binom{j-1}{k+1}$. We again shift the summation index by one, $i = j-1$, and extend the sum up to $N-1$,

$$\Sigma_2 = (k+1)\sum_{i=0}^{N-2}\left[\binom{i+1}{k+1}\binom{N-i-1}{d-k-1}\right] = (k+1)\sum_{i=0}^{N-1}\left[\binom{i+1}{k+1}\binom{N-i-1}{d-k-1}\right] - (k+1)\left[\binom{N}{k+1}\binom{0}{d-k-1}\right]. \quad \textbf{[S33]}$$

The last term is zero for $k < d-1$. For the first term, we can apply the same variant of Vandermonde's convolution as above, $\sum_{i=0}^{l}\binom{l-i}{m}\binom{q+i}{n} = \binom{l+q+1}{m+n+1}$, and obtain

$$\Sigma_2 = (k+1)\binom{N+1}{d+1}. \quad \textbf{[S34]}$$

For $k = d-1$, we again start from Eq. **S32**, which yields

$$\Sigma_2 = d\sum_{j=1}^{N-1}\binom{j}{d}\binom{N-j}{0} = d\sum_{j=1}^{N-1}\binom{j}{d} = d\binom{N}{d+1}. \quad \textbf{[S35]}$$

We slightly rearrange these two results to a common binomial,

$$\Sigma_2 = \begin{cases} (k+1)\dfrac{N+1}{d+1}\dbinom{N}{d} & \text{for } 0 \le k < d-1 \\ \dfrac{d}{d+1}(N-d)\dbinom{N}{d} & \text{for } k = d-1 \end{cases}. \quad \textbf{[S36]}$$

Combining these results with Eq. **S31**, we obtain

$$\Sigma_1 - \Sigma_2 = \binom{N}{d}\frac{1}{d+1}\times\begin{cases} N(d-k)-k-1 & \text{for } 0 \le k < d-1 \\ N-d & \text{for } k = d-1 \end{cases}. \quad \textbf{[S37]}$$

Note that these two expressions have the same form, such that we obtain a single expression for $\Sigma_1 - \Sigma_2$ or, equivalently, for Eq. **S27**,

$$\sum_{m=1}^{N-1}\sum_{j=1}^{m}\binom{j-1}{k}\binom{N-j}{d-k-1} = \Sigma_1 - \Sigma_2 = \binom{N}{d}\frac{N(d-k)-k-1}{d+1}. \quad \textbf{[S38]}$$

Together with the common factor $\binom{N-1}{d-1}^{-1}$, we obtain

$$\sum_{m=1}^{N-1}\sum_{j=1}^{m}\frac{\binom{j-1}{k}\binom{N-j}{d-k-1}}{\binom{N-1}{d-1}} = \frac{N^2(d-k)-N(k+1)}{d(d+1)}, \quad \textbf{[S39]}$$

which is Eq. **S25**.

The sums over π_B can be solved in a similar way. In that case, the special case is $k = 0$ rather than $k = d-1$, which also indicates the symmetry of the result. For the sums over π_B, we obtain

$$\sum_{m=1}^{N-1}\sum_{j=1}^{m}\frac{\binom{j}{k}\binom{N-j-1}{d-k-1}}{\binom{N-1}{d-1}} = \begin{cases} \dfrac{N(N-d)}{d+1} & \text{for } k = 0 \\ \dfrac{N(N+1)(d-k)}{d(d+1)} & \text{for } 1 \le k \le d-1 \end{cases}. \quad \textbf{[S40]}$$

Appendix B

Condition for the Comparison of Two Strategies. The statement to prove is

$$\sum_{j=1}^{N-1}(\pi_A - \pi_B) = \frac{N}{d}\sum_{k=0}^{d-1}(a_k - b_k) + b_0 - a_{d-1}. \quad \textbf{[S41]}$$

As the a_ks are contributed only by π_A and the b_ks only by π_B, we first need to show that

$$\sum_{j=1}^{N-1}\pi_A = \frac{N}{d}\sum_{k=0}^{d-1}a_k - a_{d-1}, \quad \textbf{[S42]}$$

with the payoffs from Eq. **S26**. This holds for any choice of a_ks. Thus, we only have to show that

$$\frac{1}{\binom{N-1}{d-1}}\sum_{j=1}^{N-1}\binom{j-1}{k}\binom{N-j}{d-k-1} = \begin{cases} \dfrac{N}{d} & \text{for } 0 \le k < d-1 \\ \dfrac{N}{d}-1 & \text{for } k = d-1 \end{cases}. \quad \textbf{[S43]}$$

The sum has been solved above, cf Eq. **S28**, where we have shown that $\sum_{j=1}^{N-1}\binom{j-1}{k}\binom{N-j}{d-k-1} = \binom{N}{d}$ for $0 \le k < d-1$ and $\sum_{j=1}^{N-1}\binom{j-1}{k}\binom{N-j}{d-k-1} = \frac{N-d}{N}\binom{N}{d}$ for $k = d-1$. Using the identity $\binom{N}{d} = \frac{N}{d}\binom{N-1}{d-1}$, we directly obtain Eq. **S43**.

The equivalent condition for π_B can be derived based on a similar argument. As above, we have $k = 0$ as the special case instead of $k = d-1$ in the equivalent of Eq. **S43**,

$$\frac{1}{\binom{N-1}{d-1}}\sum_{j=1}^{N-1}\binom{j}{k}\binom{N-j-1}{d-k-1} = \begin{cases} \dfrac{N}{d}-1 & \text{for } k = 0 \\ \dfrac{N}{d} & \text{for } 0 < k \le d-1 \end{cases}. \quad \textbf{[S44]}$$

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Table S1. The reduced payoff table for the d = 4 and n = 3 game in Fig. 2 in the main text

Weight (Total 27)	1	3	3	3	6	3	1	3	3	1
Configuration	AAA	AAB	AAC	ABB	ABC	ACC	BBB	BBC	BCC	CCC
A	−9.30	3.83	3.86	−1.03	−1.00	−0.96	0.10	0.33	0.16	0.20
B	0.10	−1.03	0.13	3.83	−1.00	0.16	−9.30	4.06	−0.96	0.20
C	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20	0.00	0.00

In total, there would be $n^d = 3^4 = 81$ payoff entries. For each strategy, we would have had 27 entries. But when we consider that the ordering does not matter, we just weight each configuration by the different ways of ordering; for example, there are three orderings for *AAB*, that is, *AAB*, *ABA*, and *BAA*. In this way, we reduce the number of payoff entries from 81 to 30.