Differentiation of Cognitive Abilities across the Lifespan

Online Supplement

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This online supplement reports the results of an alternative set of analyses performed on a single sample of children and adults. That is, rather than fitting models with nonlinear and age-modified factor loadings separately to child and adult subsamples, a single group analysis was conducted by combining the data from all three subsamples. Note that with the data combined, some sample restrictions are no longer needed (e.g. it is no longer desirable to exclude older adults attending college). A slightly larger sample size (N=6,641 compared to N=6,273) was therefore available.

Analyses

Removing cross-sectional age trends from the data

As discussed in the main text of the paper, it is important to control for the main effects of any variables that are included in polynomial or interaction terms. Therefore, as a first step, each of the 7 composite scores representative of the seven broad abilities was residualized for their cross-sectional age trends by way of locally smoothed regression. These locally smoothed age trends are presented in Figure S1.

Development of a fully age-heterogeneous single group model

As with the models reported in the main text of the paper, one can begin by constructing a liner factor model,

$$G[x]_n = \upsilon[x] + \lambda[x]\cdot g_n + u[x]_n, \qquad \text{Eq. S1}$$

and adding a quadratic term to examine the ability differentiation hypothesis,

$$G[x]_n = \upsilon[x] + \lambda_1[x]\cdot g_n + \lambda_2[x]\cdot g_n^2 + u[x]_n. \qquad \text{Eq. S2}$$

To examine age differentiation-dedifferentiation in a single child and adult sample, a more complex model is required. The main issue is that childhood age differentiation and adult age dedifferentiation are hypotheses that each need to be tested independent of one-another (e.g. it is possible that factor loadings decrease in childhood but remain stable in adulthood). No simple parametric function lends itself to such a goal. A piecewise “linear-linear” function was therefore employed. To remain consistent with developmental theory, the transition point is specified to occur at 21 years of age (the approximate transition point between childhood development and adult aging). Further included are terms to test a “connected” segments model against a “disconnected” segments model. It is important to allow for discontinuities, because, if present but not modeled, discontinuities could produce results suggestive of continuous trends, even when no such continuous trends exist. In the current dataset, the child and adult transition point is confounded with the approximate transition point between sampling from schools/universities and sampling from communities. One should therefore be cautious in substantively interpreting any such discontinuities that may exist. The trends that exist *within* childhood and *within* adulthood are not, alternatively, threatened by this potential selection effect.

The single group model is developed hierarchically as follows:

$$G[x]_n = \upsilon[x] + \lambda[x]_n \cdot g_n + u[x]_n, \qquad \text{Eq. S3a}$$

$$\lambda[x]_n = \lambda_{1,C}[x] + \lambda_{3,C}[x] \cdot (age_{C,n} - 21) + \lambda_{1,A}[x] \cdot (Adult_n) + \lambda_{3,A}[x] \cdot (age_{A,n} - 21), \qquad \text{Eq. S3b}$$

where

$age_{C,n}$ = participant age IF participant age ≤ 21,

$age_{C,n}$ = 0 IF participant age > 21,

$age_{A,n}$ = 0 IF participant age ≤ 21,

$age_{A,n}$ = participant age IF participant age > 21,

$Adult_n$ = 0 IF participant age ≤ 21,

and

$Adult_n$ = 1 IF participant age > 21.

Therefore, $\lambda_{3,C}$ corresponds to the linear age trend in the factor loading during childhood, and $\lambda_{3,A}$ corresponds to the linear age trend in the factor loading during adulthood. $\lambda_{1,C}$ corresponds to the fixed component of the factor loading during childhood (centered at 21 years of age), and $\lambda_{1,A}$ corresponds to any discontinuity in the child and adult segments occurring at 21 years of age. These $\lambda_{1,A}$ terms can be dropped from the model to test a connected segments model against a disconnected segments model.

Equations S3a and S3b can be combined to form

$$G[x]_n = \upsilon[x] + \lambda_{1,C}[x] \cdot g_n + \lambda_{3,C}[x] \cdot (age_{C,n} - 21) \cdot g_n + \lambda_{1,A}[x] \cdot (Adult_n) \cdot g_n + \lambda_{3,A}[x] \cdot (age_{A,n} - 21) \cdot g_n + u[x]_n. \qquad \text{Eq. S4}$$

A quadratic loading term can be added to Equation S4 to simultaneously examine the age and ability based hypotheses,

$$G[x]_n = \upsilon[x] + \lambda_{1,C}[x] \cdot g_n + \lambda_2[x] \cdot g_n^2 + \lambda_{3,C}[x] \cdot (age_{C,n} - 21) \cdot g_n + \lambda_{1,A}[x] \cdot (Adult_n) \cdot g_n + \lambda_{3,A}[x] \cdot (age_{A,n} - 21) \cdot g_n + u[x]_n. \qquad \text{Eq. S5}$$

Results

Models were fit sequentially and compared using χ^2 difference testing. Note that because the locally-smoothed cross-sectional age trends were removed from the data, no age trends were expected to remain in the data. Nevertheless, to ensure that no remaining age effects influenced results, the main effects of $age_{C,n}$, $age_{A,n}$, and Adult_n were included as covariates in all fitted models (none of these effects were statistically significant).

Results of the nested comparisons are provided in Table S1. It can be seen that all comparisons were significant, supporting the inclusion of both nonlinear factor loadings and age modification of factor loadings (taking on a "disconnected segments" form).

Parameter estimates from four key models (the linear model, the nonlinear model, the disconnected segments age-modification model, and the full model that included both nonlinearity and age-modification) are provided in Table S2. Results from the nonlinear model and the full model strongly support the ability differentiation hypothesis, with all λ_2 coefficients negative in direction, and five out of seven of these coefficients significant at the $p<.01$ level. Results from the age modification model suggest that, to some extent, abilities become more related with increasing childhood age, and less related with increased adult age, as the $\lambda_{3,C}$ coefficients tend to be positive (three out of seven are significant) and the $\lambda_{3,A}$ coefficients tend to be negative (two out of seven are significant). This pattern is, however, less apparent in the full model. These results contradict the traditional differentiation-dedifferentiation hypothesis, and are instead more consistent with views that the positive manifold emerges with child development (e.g. Dickens, unpublished), and views that cognitive aging is characterized by (partially) independent domain-specific losses (e.g. Buckner, 2004). Also note that there is evidence for age-discontinuities in the relations among abilities, as the $\lambda_{1,A}$ parameters tend to be positive and significant. This can likely be attributed to the adult sample being more heterogeneous as the result of being selected from communities, rather than schools. It is finally of note that all of these results are consistent with those reported in the main text of the paper. Key results for the full model are plotted in Figure S2.

Table S1. Fit Indices and Comparisons of Stepwise Models.

Model	Loglikelihood	Free Parameters	Difference Test Relative to	χ^2 of Difference	Degrees of Freedom of Difference	AIC	BIC
Linear Model	-144159.537	42				288403.1	288688.7
Nonlinear Model	-143596.925	49	Linear Model	789.631	7	287291.8	287625.1
Connected Segments Age Modification Model	-143617.881	56	Linear Model	573.181	14	287347.8	287728.6
Disconnected Segments Age Modification Model	-143476.475	63	Connected Segments Age Modification Model	209.490	7	287079.0	287507.4
Full Model	-143278.904	70	Nonlinear Model	379.425	21	286697.8	287173.9

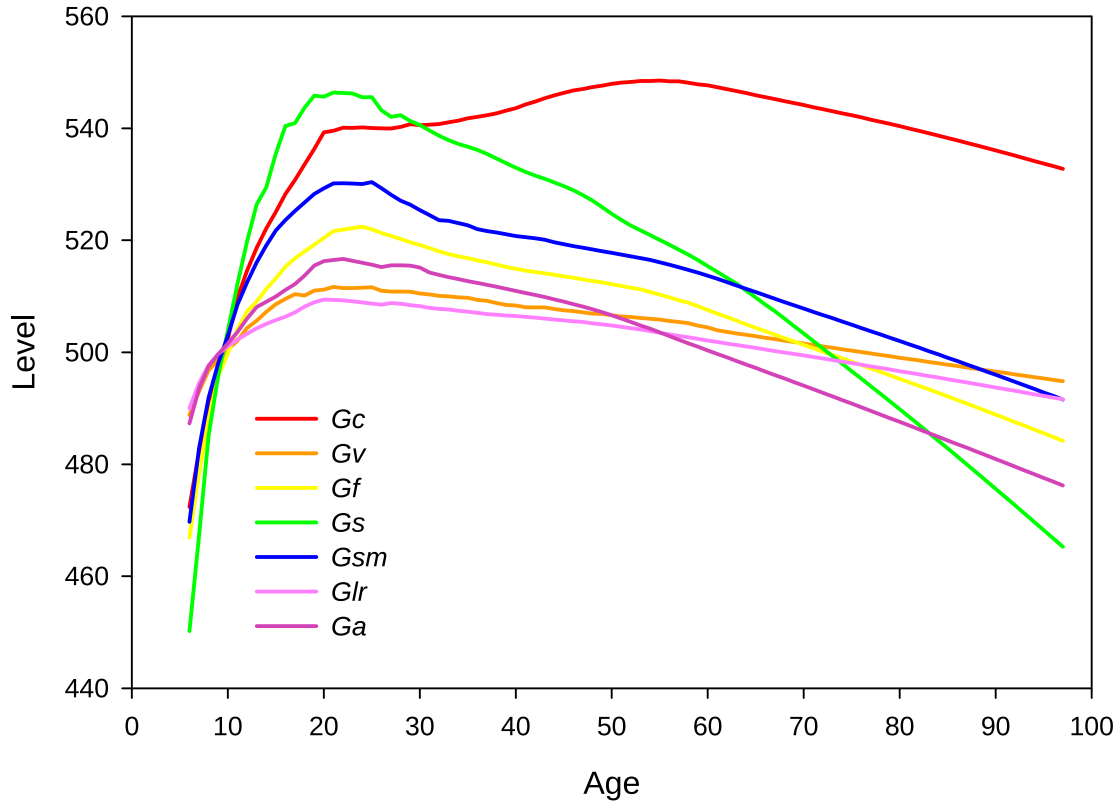
Note: The Nonlinear Model and the Age Modification Model were alternative models considered separately for the second step. Each χ^2 of difference was computed using model-specific scaling factors. All model comparisons were significant at $p<.01$.

Table S2. Parameter Estimates (and 99% Confidence Intervals) from Key Models.

Parameter	$\lambda_{1,C}$	λ_2	$\lambda_{3,C}$	$\lambda_{1,A}$	$\lambda_{3,A}$	σ^2_u
Coefficient on	g_n	g_n^2	$(age_{C,n}-21)\cdot g_n$	$(Adult_n)\cdot g_n$	$(age_{A,n}-21)\cdot g_n$	
Full Model						
Gc	**15.021 (13.051, 16.99)**	**-2.611 (-3.229, -1.994)**	**0.365 (0.192, 0.539)**	**8.817 (3.089, 14.546)**	-0.068 (-0.204, 0.068)	**92.118 (83.588, 100.647)**
Gv	**5.349 (4.247, 6.45)**	-0.24 (-0.599, 0.119)	0.074 (-0.035, 0.183)	1.204 (-0.473, 2.88)	-0.036 (-0.085, 0.014)	**57.641 (53.7, 61.582)**
Gf	**12.217 (10.66, 13.775)**	**-1.154 (-1.587, -0.721)**	0.078 (-0.081, 0.236)	3.313 (-0.07, 6.695)	0.003 (-0.085, 0.091)	**96.228 (88.376, 104.08)**
Gs	**10.242 (8.343, 12.141)**	**-1.918 (-2.645, -1.191)**	0.053 (-0.125, 0.231)	**5.192 (0.129, 10.256)**	0.011 (-0.124, 0.147)	**236.575 (221.52, 251.63)**
Gsm	**14.209 (12.227, 16.19)**	**-0.942 (-1.573, -0.311)**	0.150 (-0.039, 0.339)	0.365 (-3.228, 3.957)	0.074 (-0.018, 0.167)	**220.932 (206.906, 234.957)**
Glr	**4.317 (3.662, 4.972)**	**-0.29 (-0.486, -0.095)**	0.011 (-0.054, 0.077)	**3.238 (1.738, 4.738)**	**-0.063 (-0.096, -0.029)**	**13.55 (12.299, 14.802)**
Ga	**6.641 (5.551, 7.731)**	-0.446 (-0.903, 0.012)	0.065 (-0.041, 0.172)	**3.688 (0.776, 6.6)**	0.009 (-0.068, 0.087)	**60.147 (55.379, 64.914)**
"Disconnected Segments" Age Modification Model						
Gc	**15.426 (13.516, 17.337)**		**0.441 (0.285, 0.596)**	**15.255 (8.882, 21.628)**	**-0.183 (-0.346, -0.02)**	**94.587 (85.72, 103.455)**
Gv	**5.371 (4.303, 6.439)**		0.091 (-0.013, 0.195)	**2.133 (0.31, 3.957)**	-0.048 (-0.103, 0.006)	**57.732 (53.846, 61.619)**
Gf	**12.694 (11.172, 14.217)**		**0.162 (0.018, 0.306)**	**6.208 (2.425, 9.991)**	-0.045 (-0.147, 0.057)	**95.322 (87.522, 103.121)**
Gs	**10.689 (8.815, 12.564)**		0.114 (-0.053, 0.281)	**8.905 (2.751, 15.059)**	-0.041 (-0.210, 0.129)	**237.065 (221.569, 252.56)**
Gsm	**14.503 (12.568, 16.437)**		**0.211 (0.036, 0.386)**	2.670 (-1.297, 6.637)	0.047 (-0.059, 0.154)	**220.216 (206.238, 234.193)**
Glr	**4.263 (3.65, 4.877)**		0.020 (-0.040, 0.081)	**4.893 (3.091, 6.696)**	**-0.090 (-0.130, -0.050)**	**13.632 (12.439, 14.826)**
Ga	**6.654 (5.651, 7.657)**		0.086 (-0.010, 0.182)	**5.503 (1.879, 9.127)**	-0.018 (-0.114, 0.077)	**60.377 (55.962, 64.792)**
Nonlinear Model						
Gc	**15.108 (13.879, 16.336)**	**-3.923 (-4.741, -3.104)**				**92.320 (83.9, 100.739)**
Gv	**5.054 (4.613, 5.494)**	-0.306 (-0.654, 0.041)				**57.716 (53.742, 61.689)**
Gf	**12.837 (12.073, 13.6)**	**-1.466 (-1.915, -1.017)**				**96.642 (88.97, 104.315)**
Gs	**11.715 (10.727, 12.703)**	**-2.338 (-3.206, -1.471)**				**236.663 (221.613, 251.712)**
Gsm	**14.010 (13.143, 14.877)**	**-0.963 (-1.633, -0.293)**				**223.077 (208.655, 237.499)**
Glr	**4.791 (4.484, 5.099)**	**-0.351 (-0.566, -0.136)**				**13.724 (12.423, 15.025)**
Gaw	**7.549 (7.01, 8.088)**	**-0.855 (-1.449, -0.261)**				**61.030 (56.117, 65.944)**
Linear Model						
Gc	**16.324 (14.68, 17.968)**					**109.244 (97.897, 120.592)**

Gv	**5.069** **(4.576, 5.562)**	**57.815** **(53.929, 61.700)**
Gf	**13.226** **(12.375, 14.077)**	**95.017** **(87.367, 102.668)**
Gs	**12.684** **(11.402, 13.965)**	**236.056** **(220.172, 251.941)**
Gsm	**14.199** **(13.248, 15.151)**	**222.445** **(207.304, 237.587)**
Glr	**4.897** **(4.544, 5.250)**	**13.854** **(12.640, 15.068)**
Ga	**7.776** **(7.070, 8.483)**	**60.625** **(56.117, 65.133)**

Note: Values in bold indicate $p<.01$. The Full model is given by Equation S5.



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Age
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Figure S1

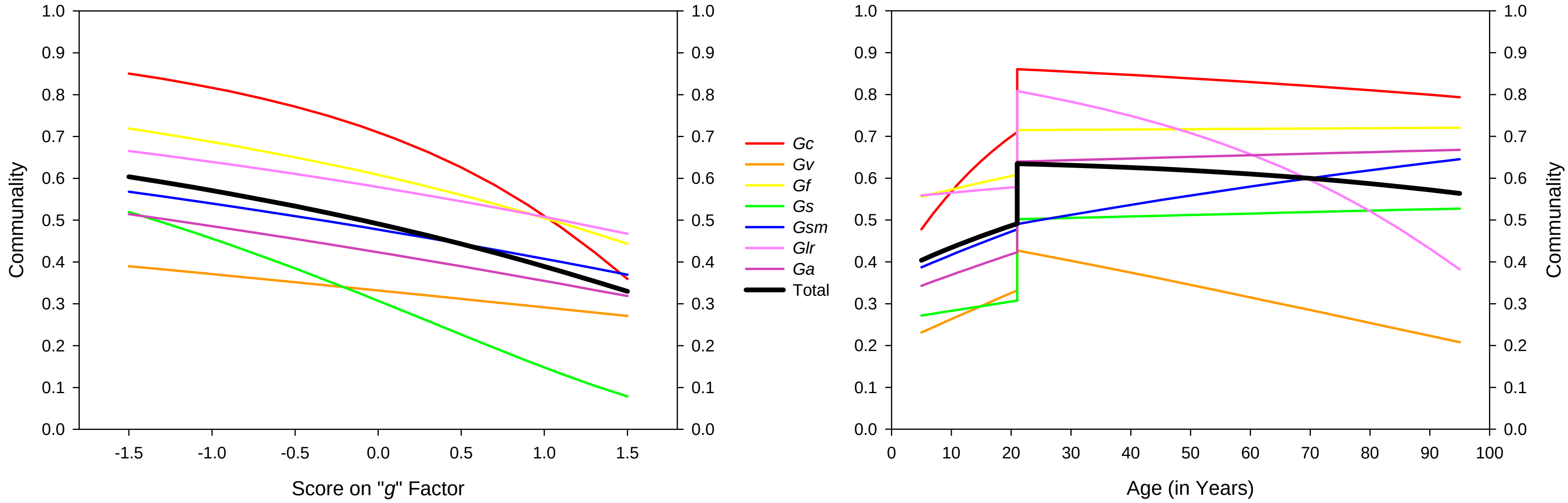
Communality
Score on "g" Factor
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Figure S2