

Supporting Information for:

**Rates and equilibria for probe capture by an antibody with infinite
affinity**

Tolulope A. Aweda,[†] Heather Beck,[†] Anna Wu,[‡] Liu Wei,[‡] Wolfgang Weber,[§] and
Claude F. Meares^{†*}

[†]Chemistry Department, University of California, Davis, California

[‡]Department of Molecular and Medical Pharmacology, David Geffen School of
Medicine, University of California, Los Angeles, California

[§]Department of Nuclear Medicine, University of Freiburg, Freiburg, Germany

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Key Mathematica instructions for solving Equations 1–4 to prepare for Figure 5.

Units for numerical calculation

First-order rate const: min⁻¹; second-order rate const: conc⁻¹ min⁻¹, where all conc expressed in molecules/ μ^3

a+b->c a+b->cc c->d c->a+b cc->a+b

a=site concn b=DOTA concn c=reactive complex cc=non-reactive complex

d=product

ka1 = .35; kd1 = .42; kirr = i; b0 = 600; a0 = 6000; fxn = j;

```
Do[Do[solution = NDSolve[{
  a'[t] == -ka1 a[t] b[t] + kd1 c[t] + kd1 cc[t],
  b'[t] == -ka1 a[t] b[t] + kd1 c[t] + kd1 cc[t],
  cc'[t] == (1. - j)*ka1 a[t] b[t] - kd1 cc[t],
  c'[t] == j*ka1 a[t] b[t] - i c[t] - kd1 c[t],
  d'[t] == i c[t],
  a[0] == a0, b[0] == b0, c[0] == 0, cc[0] == 0, d[0] == 0}, {a, b, c, cc,
  d}, {t, 0, 150}];
```

```
modelf2d = Function[{t}, Evaluate[First[d[t] /. solution]]];
data2 = {{1, 23}, {2, 30}, {4, 34}, {8, 45}, {16, 66}, {32, 85}, {64,
  95}, {96, 95}, {128, 98}};
v1 = (modelf2d[1]*95/573 - 23)^2;
v2 = (modelf2d[2]*95/573 - 30)^2;
v4 = (modelf2d[4]*95/573 - 34)^2;
v8 = (modelf2d[8]*95/573 - 45)^2;
v16 = (modelf2d[16]*95/573 - 66)^2;
v32 = (modelf2d[32]*95/573 - 85)^2;
v64 = (modelf2d[64]*95/573 - 95)^2;
v96 = (modelf2d[96]*95/573 - 95)^2;
v128 = (modelf2d[128]*95/573 - 98)^2;
sumv = v1 + v2 + v4 + v8 + v16 + v32 + v64 + v96 + v128;
Print[i, " ", j, " ", sumv], {i, 0, 10, .1}], {j, 0, 1, .1}]
```

Key Mathematica instructions for solving Equations 5–9 to prepare for Figure 6.

u is AABD(Y) concn, c is free receptor concn, v is ligand-receptor reactive complex concn, vv is ligand-receptor non-reactive complex concn, w is covalent product concn.
radius of capillary=10 μ , max radius of cord=110 μ , time minutes, conc molecules/ μ^3

```
difco = 20000;
k1 = .35;
k2 = .42;
kir = 1.5; fxn = .2; preexp = i;
Do[solution =
  NDSolve[{D[u[r, t], t] ==
    difco*(D[u[r, t], r, r] + D[u[r, t], r]/r) - k1*c[r, t]*u[r, t] +
    k2*v[r, t] + k2*vv[r, t],
    D[c[r, t], t] == -k1*c[r, t]*u[r, t] + k2*v[r, t] + k2*vv[r, t],
    D[vv[r, t], t] == (1 - fxn)*k1*c[r, t]*u[r, t] - k2*vv[r, t],
    D[v[r, t], t] == fxn*k1*c[r, t]*u[r, t] - (k2 + kir)*v[r, t],
    D[w[r, t], t] == kir*v[r, t], u[r, 0] == i*Exp[-(r - 10)^2],
    u[10, t] == i*Exp[-0.12*t], (D[u[r, t], r] /. r -> 110) == 0,
    c[r, 0] == 28.2, vv[r, 0] == 0, vv[10, t] == 0, v[r, 0] == 0,
    v[10, t] == 0, w[r, 0] == 0, w[10, t] == 0}, {u, c, vv, v, w}, {r, 10,
    110}, {t, 0, 60},
  Method -> {"MethodOfLines",
    "SpatialDiscretization" -> {"TensorProductGrid",
    "DifferenceOrder" -> "Pseudospectral"} }];
```

```
toall = Table[
  1.67*10^-4*
  NIntegrate[
    Evaluate[
      First[u[r, ti] /. solution]*r + First[vv[r, ti] /. solution]*r +
      First[v[r, ti] /. solution]*r + First[w[r, ti] /. solution]*r,
      {r, 10, 110}, WorkingPrecision -> 24, PrecisionGoal -> 10,
      MaxRecursion -> 40], {ti, 60}];
sumvar = (toall[[1]]/.103256 - 3.8)^2 + (toall[[5]]/.103256 -
  5.2)^2 + (toall[[10]]/.103256 - 6.4)^2 + (toall[[20]]/.103256 -
  6.6)^2 + (toall[[30]]/.103256 - 6.7)^2 + (toall[[40]]/.103256 -
  7.5)^2 + (toall[[50]]/.103256 - 7.1)^2 + (toall[[60]]/.103256 - 7.7)^2;
Print[i, " ", sumvar, " ", toall[[1]]/.103256, toall[[5]]/.103256,
  toall[[10]]/.103256, toall[[20]]/.103256, toall[[30]]/.103256,
  toall[[40]]/.103256, toall[[50]]/.103256, toall[[60]]/.103256, {i, 0.05,
  0.7, .05}]
```