Biophysical Journal, Volume 98

Supporting Material

A modeling approach to the self-assembly of the Golgi apparatus

Jens Kühnle, Julian Shillcock, Ole G. Mouritsen, and Matthias Weiss

A modeling approach to the self-assembly of the Golgi apparatus – Supplemental information

Jens Kühnle*,†, Julian Shillcock†, Ole G. Mouritsen† & Matthias Weiss*

* *Cellular Biophysics Group (BIOMS), German Cancer Research Center, c/o BIOQUANT, Im Neuenheimer Feld 267, D-69120 Heidelberg, Germany and*

† *MEMPHYS-Center for Biomembrane Physics, Dept. of Physics and Chemistry, University of Southern Denmark, DK-5230 Odense, Denmark*

Size and number of cisternae when protein exchange is limiting

To analytically estimate the number and size of cisternae, and the stack turnover time, a few approximations have to be applied to the model of the main text. We disregard spatial arrangements and assume, that all newly incoming VTCs fuse to the same aggregate as long as this aggregate has enough proteins of species A. Otherwise, the new VTCs will form a new cisterna.

Let $\tau_{\mathrm{f}}^{(1)}$ denote the time during which a single VTC is able to fuse with other VTCs. Each of these VTCs is assumed to carry an amount of proteins of species A that supports m vesicle budding events before being totally emptied. This situation can be captured by a binomial distribution: given the probability p of a single vesicle fusion/budding event, the probability of having the mth event in the $m+s$ th step reads

$$f\left(m, s; p\right) = \binom{m+s-1}{s} p^m \left(1-p\right)^s \tag{1}$$

This provides the mean number of total steps for a complete depletion of all proteins of species A hence leading to

$$\tau_{\mathrm{f}}^{(1)} = \Delta t\, m \frac{1-p_A}{p_A} \approx m \frac{1}{r_A} \qquad (p_A \ll 1) \tag{2}$$

where Δt denotes the time step and p_A, r_A are the loss probability and rate for species A, respectively.

If new VTCs enter the simulation box with a mean rate J_{in}, the mean time λ until the next one enters is given by $\lambda = 1/J_{in}$. If $\tau_{\mathrm{f}}^{(1)} \geq \lambda$, the new VTC is able to fuse with an old one, thus forming a dimer with a prolonged life time $\tau_{\mathrm{f}}^{(2)}$, during which fusion is possible. Generalizing this scenario for arbitrary cluster sizes yields:

$$\begin{aligned}
\tau_{\mathrm{f}}^{(2)} &= \frac{1}{2}\left(\tau_{\mathrm{f}}^{(1)} + \tau_{\mathrm{f}}^{(1)} + \lambda\right) && (3)\\
\tau_{\mathrm{f}}^{(n)} &= \frac{1}{n}\left(n\tau_{\mathrm{f}}^{(1)} + \sum_{j=1}^{n-1} j\lambda\right) && (4)\\
\tau_{\mathrm{f}}^{(n)} &= \frac{1}{n}\left(n\tau_{\mathrm{f}}^{(1)} + \frac{n}{2}\left(n-1\right)\lambda\right) && (5)\\
\tau_{\mathrm{f}}^{(n)} &= \tau_{\mathrm{f}}^{(1)} + \frac{n-1}{2}\lambda && (6)
\end{aligned}$$

The expected cisterna size (measured in number of VTCs), n_s, is determined by equating the lifetime $\tau_{\mathrm{f}}^{(n)}$ with the time needed to add n VTCs to the system. Therefore,

$$\tau_{\mathrm{f}}^{(1)} + \frac{n_s - 1}{2}\lambda = \left(n_s - 1\right)\,\lambda \tag{7}$$

$$n_s = \frac{2\tau_{\mathrm{f}}^{(1)}}{\lambda} + 1 = 2\tau_{\mathrm{f}}^{(1)}\, J_{\mathrm{in}} + 1 \tag{8}$$

$$n_s \approx 2\, m \frac{J_{\mathrm{in}}}{r_A} + 1 \qquad (p_A \ll 1) \tag{9}$$

Similarly, we can calculate the turnover time of the stack (12) and the expected number of cisternae (14). The time after which either a single VTC $\tau_{\mathrm{rm}}^{(1)}$ or a cluster of size n $\tau_{\mathrm{rm}}^{(n)}$ is removed from the system, can be determined exactly the same way as for species A:

$$\tau_{\mathrm{rm}}^{(1)} = \Delta t\, m \frac{1-p_B}{p_B} \approx m \frac{1}{r_B} \qquad (p_B \ll 1) \tag{10}$$

$$\tau_{\mathrm{rm}}^{(n)} = \frac{1}{n}\left(n\, \tau_{\mathrm{rm}}^{(1)} + \sum_{j=1}^{n-1} j\, \lambda\right) \tag{11}$$

$$\tau_{\mathrm{rm}}^{(n_s)} = \tau_{\mathrm{rm}}^{(1)} + \tau_{\mathrm{f}}^{(1)} \approx m\left(\frac{1}{r_A} + \frac{1}{r_B}\right) \tag{12}$$

where p_B and r_B denote the probability for an import event of species B and the corresponding rate, respectively. The lifetime of a cluster with expected size n_s is thus given by 12. The lifetime $\tau_{\mathrm{rm}}^{(n_s)}$ is equal to the turnover time of the stack since within the lifetime of the cluster the whole material of stack is replaced once. Now, we can calculate the expected amount of cisternae n_c.

$$\langle n_{\mathrm{tot}} \rangle = \tau_{\mathrm{rm}}^{(n_s)}\, J_{\mathrm{in}} \tag{13}$$

$$\langle n_{\mathrm{c}} \rangle = \frac{\langle n_{\mathrm{tot}} \rangle}{n_s} = \frac{\tau_{\mathrm{rm}}^{(1)} + \tau_{\mathrm{f}}^{(1)}}{2\tau_{\mathrm{f}}^{(1)} + \lambda} \approx \frac{\tau_{\mathrm{rm}}^{(1)} + \tau_{\mathrm{f}}^{(1)}}{2\tau_{\mathrm{f}}^{(1)}} = \frac{r_A}{2r_B} + \frac{1}{2} \qquad \left(\lambda \ll \tau_{\mathrm{f}}^{(1)}\right) \tag{14}$$

In equilibrium, the average total amount of VTCs in the simulation box $\langle n_{\mathrm{tot}} \rangle$ is constant and can be calculated by considering how many VTCs are produced within the turnover time (13).

Since the total amount of VTCs in the system and the average cisternal size are known, obtaining the total amount of cisternae (14) is straightforward.