

Biophysical Journal, Volume 98

**Supporting Material**

**Kinetic Hysteresis in Collagen Folding**

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## Appendix

### Model 1

abbreviation	meaning
IndVars	independent variables
DepVars	dependent variables
Params	parameters
T	time
TEMP	temperature (K)
F	fraction folded
CD	CD value (ellipticity or mean molar ellipticity)
K	equilibrium constant
K2	rate constant, $k_p$ ( $s^{-1}$ )
KMIN2	rate constant, $k_d$ ( $s^{-1}$ )
R	gas constant ( $8.31 \text{ J}\cdot\text{K}^{-1}\text{mol}^{-1}$ )
RATE	rate of heating or cooling ( $\text{K}\cdot\text{s}^{-1}$ )
TSTART	temperature at $t=0$
H	standard enthalpy ( $\text{J}\cdot\text{mol}^{-1}$ )
S	standard entropy ( $\text{J}\cdot\text{K}^{-1}\text{mol}^{-1}$ )
EA	activation energy ( $\text{J}\cdot\text{mol}^{-1}$ )
K02	rate constant $k_p$ at $7^\circ\text{C}$ ( $s^{-1}$ )
CDU	CD value of unfolded state
REFU	reference CD value of unfolded state at $0^\circ\text{C}$
DEU	slope of linear CD dependence of unfolded state
REFN	reference CD value of folded state at $0^\circ\text{C}$
DEN	slope of linear CD dependence of folded state

Symbols RATER, TEMPR, TSTARTR, FR, CDR, KR, K2R, KMIN2R have the same meaning as symbols without R as the last letter but refer to the reverse cooling pathway

Symbols in Model 2 are identical to those in Model 1 and in addition,

KA	rate constant, $k_{a, app}$ ( $\text{M}^{-2}\text{s}^{-1}$ )
KU	rate constant, $k_u$ ( $s^{-1}$ )
K0A	rate constant $k_{a, app}$ at reference temperature ( $7^\circ\text{C}$ )
K	equilibrium constant ( $\text{M}^{-2}$ )
C0	total peptide concentration $c_0$ (M)

Symbols in Model 3 are identical to those in Models 1 and 2 and in addition,

FS	fraction of $\text{H}^*$
Q	pre-equilibrium constant defined by Eq. 14 ( $\text{M}^{-2}$ )
Q7	pre-equilibrium constant at $7^\circ\text{C}$ ( $\text{M}^{-2}$ )
HQ	standard enthalpy of the pre-equilibrium reaction ( $\text{J}\cdot\text{mol}^{-1}$ )
SQ	standard entropy of the pre-equilibrium ( $\text{J}\cdot\text{mol}^{-1}\text{K}^{-1}$ )
FSR	fraction of $\text{H}^*$ in the reverse cooling pathway

### ***Model 1***

// Model 1

IndVars: T

DepVars: TEMP, F, CD, K, K2, KMIN2, TEMPR, FR, CDR, KR, K2R, KMIN2R

Params: H, S, EA, K02, DEU, REFU, DEN, REFN,

$$F' = 3 * K2 * (1 - F) - KMIN2 * F$$

$$FR' = 3 * K2R * (1 - FR) - KMIN2R * FR$$

$$R = 8.31$$

$$RATE = 0.008333333$$

$$TSTART = 302.96$$

$$RATER = -RATE$$

$$TSTARTR = 343.03$$

$$TEMP = TSTART + RATE * T$$

$$TEMPR = TSTARTR + RATER * T$$

$$TEMP0 = 280.15$$

$$K2 = K02 * \exp\left(\frac{EA}{R} * \left(\frac{1}{TEMP0} - \frac{1}{TEMP}\right)\right)$$

$$K2R = K02 * \exp\left(\frac{EA}{R} * \left(\frac{1}{TEMP0} - \frac{1}{TEMPR}\right)\right)$$

$$KMIN2 = K2 / K$$

$$K = \exp\left(-\frac{H - TEMP * S}{R * TEMP}\right)$$

$$KMIN2R = K2R / KR$$

$$KR = \exp\left(-\frac{H - TEMPR * S}{R * TEMPR}\right)$$

$$CDU = REFU + DEU * (TEMP - 273.15)$$

$$CDUR = REFU + DEU * (TEMPR - 273.15)$$

$$CDN = REFN + DEN * (TEMP - 273.15)$$

$$CDNR = REFN + DEN * (TEMPR - 273.15)$$

$$CD = F * (CDN - CDU) + CDU$$

$$CDR = FR * (CDNR - CDUR) + CDUR$$

//INITIAL CONDITIONS

$$F = 1$$

$$FR = 0$$

$$T = 0$$

\*\*\*\*\*

### ***Model 2***

//Model 2

IndVars: T

DepVars: TEMP, CD, CDR, F, TEMPR, FR, K, KA, KU, KAR, KUR, CDN

Params: H, S, EA, K0A, DEU, REFU, DEN, REFN

C0=0.00025

R=8.31

RATE=0.00138889

$F' = 3 * KA * C0^2 * (1 - F)^3 - KU * F$

$FR' = 3 * KAR * C0^2 * (1 - FR)^3 - KUR * FR$

TSTART=278.17

RATER=-RATE

TSTARTR=338.14

TEMP=TSTART+RATE\*T

TEMPR=TSTARTR+RATER\*T

TEMP0=280.15

$KA = K0A * \exp((EA/R) * (1/TEMP0 - 1/TEMP))$

$KAR = K0A * \exp((EA/R) * (1/TEMP0 - 1/TEMPR))$

$KU = KA / K$

$K = \exp(-(H - TEMP * S) / (R * TEMP))$

$KUR = KAR / KR$

$KR = \exp(-(H - TEMPR * S) / (R * TEMPR))$

$CDU = REFU + DEU * (TEMP - 273.15)$

$CDUR = REFU + DEU * (TEMPR - 273.15)$

$CDN = REFN + DEN * (TEMP - 273.15)$

$CDNR = REFN + DEN * (TEMPR - 273.15)$

$CD = F * (CDN - CDU) + CDU$

$CDR = FR * (CDNR - CDUR) + CDUR$

//INITIAL CONDITIONS

F=0.999999

FR=0.000001

T=0

\*\*\*\*\*

**Model 3**

// Model 3

IndVars: T

DepVars: TEMP, F, FS, CD, K, K2, KMIN2, TEMPR, FR, FSR, CDR, KR, K2R,  
KMIN2R, Q Params: H, S, HQ, SQ, EA, K02, DEU, REFU, DEN, REFN,

$$F' = K2 * FS - KMIN2 * F$$
$$FR' = K2R * FSR - KMIN2R * FR$$

$$C0 = 0.000082$$
$$R = 8.31$$
$$RATE = 0.002778$$
$$TSTART = 277.09$$
$$RATER = -RATE$$
$$TSTARTR = 352.44$$

$$TEMP = TSTART + RATE * T$$
$$TEMPR = TSTARTR + RATER * T$$

$$Q = \exp(-(HQ - TEMP * SQ) / (R * TEMP))$$
$$QR = \exp(-(HQ - TEMPR * SQ) / (R * TEMPR))$$
$$P = 1 / (3 * Q * (C0^2))$$
$$B = P * (1 - F)$$
$$W = (((B^2) / 4) + ((P^3) / 27))^{1/2}$$
$$U = ((-B / 2) + W)^{1/3}$$
$$V = -((-B / 2) + W)^{1/3}$$
$$FS = U + V + 1 - F$$

$$PR = 1 / (3 * QR * (C0^2))$$
$$BR = PR * (1 - FR)$$
$$WR = (((BR^2) / 4) + ((PR^3) / 27))^{1/2}$$
$$UR = ((-BR / 2) + WR)^{1/3}$$
$$VR = -((-BR / 2) + WR)^{1/3}$$
$$FSR = UR + VR + 1 - FR$$

$$TEMP0 = 280.15$$

$$K2 = K02 * \exp((EA / R) * (1 / TEMP0 - 1 / TEMP))$$
$$K2R = K02 * \exp((EA / R) * (1 / TEMP0 - 1 / TEMPR))$$

$$KMIN2 = K2 * Q / K$$
$$K = \exp(-(H - TEMP * S) / (R * TEMP))$$

$$KMIN2R = K2R * QR / KR$$
$$KR = \exp(-(H - TEMPR * S) / (R * TEMPR))$$

$$CDU = REFU + DEU * (TEMP - 273.15)$$
$$CDUR = REFU + DEU * (TEMPR - 273.15)$$

CDN=REFN+DEN\*(TEMP-273.15)  
CDNR=REFN+DEN\*(TEMPR-273.15)

CD=F\*(CDN-CDU)+CDU  
CDR=FR\*(CDNR-CDUR)+CDUR

//INITIAL CONDITIONS

F=0.999999  
FR=0.000001  
T=0

For global fitting, the second and the third data set were added for all of variables and parameters shown below as labeled TEMPB, TEMPC, FB, FC etc.

DepVars: TEMP, F, FS, CD, K, K2, KMIN2, TEMPR, FR, FSR, CDR, KR, K2R,  
KMIN2R, Q

and Params: DEU, REFU, DEN, REFN