Web-based Supplementary Materials for “Association Models for Clustered Data with Binary and Continuous Responses”
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Web Appendix

We derive (10)-(11) for $X_{ij} = 0$ in the following. When $X_{ij} = 0$, we have $E[Y_{1ij}|B_i, W_i] = \exp(\beta_0 + B_i)/(1 + \exp(\beta_0 + B_i))$, and $E[Y_{2ij}|B_i, W_i, Y_{1ij}] = \alpha_0 + \gamma(Y_{1ij} - \pi_{1ij}) + W_i$. Covariance between the binary responses of different subjects within the same cluster is

$$\begin{aligned}\text{Cov}[Y_{1ij}, Y_{1ij'}] &= E[(Y_{1ij} - \pi_{1ij})(Y_{1ij'} - \pi_{1ij})] = EE[(Y_{1ij} - \pi_{1ij})(Y_{1ij'} - \pi_{1ij})|B_i] \\ &= E[\{E(Y_{1ij} - \pi_{1ij}|B_i)\}^2] = E\left[\left\{\frac{\exp(\beta_0 + B_i)}{1 + \exp(\beta_0 + B_i)} - \pi_{1ij}\right\}^2\right].\end{aligned}$$

The covariance between the binary response and the cluster random effect for the corresponding continuous response from same subject is

$$\begin{aligned}\text{Cov}[Y_{1ij}, W_i] &= E[(Y_{1ij} - \pi_{1ij})W_i] = EE[Y_{1ij}W_i|B_i] - \pi_{1ij}E[W_i] = E[E[Y_{1ij}|B_i]E[W_i|B_i]] \\ &= E\left[\frac{\exp(\beta_0 + B_i)}{1 + \exp(\beta_0 + B_i)} E[W_i|B_i]\right] \\ &= E\left[\frac{\exp(\beta_0 + B_i)}{1 + \exp(\beta_0 + B_i)} W_i\right].\end{aligned}$$