Supporting Information

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SI Text

Outline of Supporting Information. The *SI Text* contains material of two sorts. Figs. S1 and S2 give further evidence for the classificatory ability of our measures. Tables S1, S2, and S3 provide information as follows: collect the basic information about all networks analyzed in the paper (Table S1), give numerical values and error estimates for all quantities measured in the directed assortativity analysis (Table S2), and elaborate the discussion of the cascade and niche models by showing the standard deviation for ensembles of model networks of each type (Table S3). We provide below an outline of the *SI Text*, briefly describing each component in order with a shortened version of the figure and table legends.

- Fig. S1 shows the similarities in Assortativity Significance Profile (ASP) between several real-world networks, as measured by the dot product between their ASPs, $R_{ij} = \sum_{\alpha,\beta} \mathrm{ASP}_i(\alpha,\beta) \times \mathrm{ASP}_j(\alpha,\beta)$.
- Fig. S2 is constructed as in Fig. S1, but omits the $\mathrm{ASP}(out, in)$ from the dot product. The classes are more clearly visible here.
- Table S1 shows network properties and sources. We show the class of network, the number of nodes N, the number of edges E, the average out degree $\langle k_{\mathrm{out}} \rangle$, whether or not the network has self-edges, the Pearson correlation between the in- and out-degrees of nodes in the network r_{auto}, and the source of the network. A list of sources follows the table for ease of reference.
- Table S2 shows directed assortativity results. For each network and each of the four possible pairs (α, β) we show the Pearson correlation $r(\alpha, \beta)$, the error σ_r^{rw} in this quantity as estimated by jackknife (see ref. 1), the average Pearson correlation of the random ensemble $\langle r_{\mathrm{rand}} \rangle$, the error of this average σ_r^{rand} (*Materials and Methods*), $Z(\alpha, \beta)$, and $\mathrm{ASP}(\alpha, \beta)$.
- Table S3 shows standard deviations in food-web models. We show the standard deviations in $r(\alpha, \beta)$ for 500 instances per real-world network of the cascade and niche model. Instances are constructed according to the procedure described in *Materials and Methods*, following ref. 2; note the large standard deviations of the niche model.

1. Newman MEJ (2003) Mixing patterns in networks. *Phys Rev E* 67:026126.

2. Built using Williams RJ, Martinez ND (2000) Simple rules yield complex food webs. *Nature* 404:180–183.

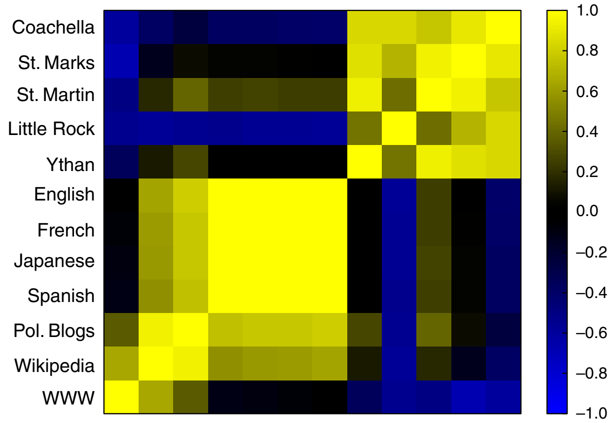

Fig. S1. Similarities between several real-world networks in the ASP measure. Each pair of real-world networks (i, j) is assigned a correlation by the dot product between their ASPs, $R_{ij} = \sum_{\alpha,\beta} \mathrm{ASP}_i(\alpha,\beta) \times \mathrm{ASP}_j(\alpha,\beta)$. As before, $\alpha, \beta\{in, out\}$ index the degree types. Because the ASPs are normalized, R_{ij} ranges from −1 to 1, with 1 indicating highly correlated ASPs. Note that all three classes of network are clearly visible in the heat map, with some overlap between the online networks and the word-adjacency networks. In Fig. S2 we identify the source of this overlap.

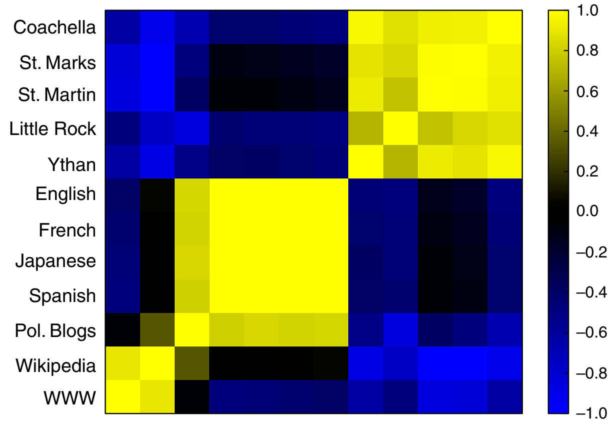

Fig. S2. This is constructed as in Fig. S1, but it omits the ASP(*out*, *in*) from the dot product. The classes are much more clearly visible, which suggests that the additional measures discussed in this paper are of greater discriminatory power than the typical assortativity measure of ref. 1. Note, however, that the political blog network is not grouped with the other online networks; this is consistent with its lacking the (*in*, *out*) *Z* assortativity of the World Wide Web (WWW) and Wikipedia.

Table S1. Network properties and sources

Network	Class	N	E	$\langle k_{out} \rangle$	Self-edges	r_{auto}	Source
Leadership	Social	32	96	3.000	No	0.053	(3)
Prison	Social	67	182	2.716	No	0.201	(3)
WWW	Online	325,729	1,497,135	4.596	Yes	0.211	(3)
Wikipedia	Online	1,598,583	19,753,078	12.357	Yes	0.203	(4)
Politcal blogs	Online	1,224	19,090	15.597	Yes	0.377	(5)
WWW model 1	Online	325,729	1,446,887	4.442	Yes	0.526	(6)
WWW model 2	Online	325,729	1,448,691	4.448	Yes	0.565	(6)
WWW model 3	Online	325,729	1,428,052	4.384	Yes	0.391	(6)
Coachella	Food web	29	262	9.034	Yes	−0.361	(7)
Little Rock	Food web	95	1,080	11.368	Yes	−0.242	(8)
St. Marks	Food web	48	221	4.604	Yes	−0.227	(9)
St. Martin	Food web	42	205	4.881	No	−0.368	(10)
Ythan	Food web	82	395	4.817	Yes	−0.055	(11)
Coachella niche	Food web	29	259	8.931	Yes	−0.408	(2)
Little Rock niche	Food web	95	1,056	11.116	Yes	−0.284	(2)
St. Marks niche	Food web	48	216	4.500	Yes	−0.258	(2)
St. Martin niche	Food web	41	208	5.073	No	−0.398	(2)
Ythan niche	Food web	82	386	4.707	Yes	−0.389	(2)
Coachella Cascade	Food web	29	267	9.207	No	−0.907	(2)
Little Rock Cascade	Food web	95	1,098	11.558	No	−0.859	(2)
St. Marks Cascade	Food web	48	223	4.646	No	−0.793	(2)
St. Martin Cascade	Food web	42	205	4.881	No	−0.662	(2)
Ythan Cascade	Food web	82	384	4.683	No	−0.702	(2)
Spanish	Word adjacency	11,586	45,129	3.895	No	0.913	(3)
Japanese	Word adjacency	2,704	8,300	3.070	No	0.927	(3)
French	Word adjacency	8,325	24,295	2.918	No	0.905	(3)
English	Word adjacency	8,525	74,921	8.788	Yes	0.876	(12)
Scrambled	Word adjacency	8,525	11,8161	13.861	Yes	0.999	(12)
Bipartite	Word adjacency	746	1,290	1.729	No	0.968	(3)

We show the class of network, the number of nodes N, the number of edges E, the average out degree $\langle k_{out} \rangle$, whether or not the network has self-edges, the Pearson correlation between the in- and out-degrees of nodes in the network r_{auto}, and the source (see the list below). Note that after reconstructing the adjacency matrix by hand from refs. 7–11, we performed a trophic aggregation on all food webs, meaning that if two species had identical interactions, we combined them into one node. Further, all parasites were removed from the Ythan food web.

3. Milo R, et al. (2004) Superfamilies of evolved and designed networks. *Science* 303:1538–1542. Available at http://www.weizmann.ac.il/mcb/UriAlon/groupNetworksData.html.
4. Gleich D (2009) Available at http://www.cise.ufl.edu/research/sparse/matrices/Gleich/index.html.
5. Adamic LA, Glance N (2005) The political blogosphere and the 2004 US Election, *Proceedings of the 3rd International Workshop on Link Discovery* (ACM, Chicago), pp 36–44.
6. Built using Krapivsky PL, Rodgers GJ, Redner S (2001) Degree distributions of growing networks. *Phys Rev Lett* 86:5401.
7. Polis GA (1991) Complex trophic interactions in deserts: An empirical critique of food-web theory. *Am Nat* 183:123–155.
8. Martinez ND (1991) Artifacts or attributes? Effects of resolution on the Little Rock Lake food web. *Ecol Monogr* 61:367–392.
9. Christian RR, Luczkovich JJ (1999) Organizing and understanding a winter's seagrass foodweb network through effective trophic levels. *Ecol Model* 117:99–124.
10. Goldwasser L, Roughgarden J (1993) Construction and analysis of a large Caribbean food web. *Ecology* 74:1216–1233.
11. Huxham M, Beaney S, Raffaeli D (1996) Do parasites reduce the chances of triangulation in a real food web? *Oikos* 76:284–300.
12. Darwin C (1859) *On the Origin of Species*, 6th Ed. Available at http://www.gutenberg.org/etext/2009.

Table S2. Directed assortativity results

Network	(α, β)	$r(\alpha, \beta)$	σ_r^{rw}	$\langle r_{rand} \rangle$	σ_r^{rand}	$Z(\alpha, \beta)$	ASP(α, β)
Leadership	(*out*, *in*)	−0.157	0.123	−0.030	0.0015	−1.419	−0.391
	(*in*, *out*)	0.214	0.107	−0.015	0.0014	2.344	0.646
	(*out*, *out*)	−0.199	0.010	−0.036	0.0013	−1.844	−0.508
	(*in*, *in*)	−0.083	0.089	−0.045	0.0013	1.504	0.415
Prison	(*out*, *in*)	0.129	0.072	−0.023	0.0010	2.152	0.492
	(*in*, *out*)	0.134	0.067	−0.012	0.0016	2.013	0.460
	(*out*, *out*)	0.206	0.073	−0.021	0.0016	3.214	0.734
	(*in*, *in*)	−0.053	0.070	−0.027	0.0016	−0.390	−0.089
WWW	(*out*, *in*)	−0.062	0.0001	−0.039	3.0×10^{-6}	−144.927	−0.388
	(*in*, *out*)	0.257	0.0002	0.000	1.8×10^{-5}	343.609	0.921
	(*out*, *out*)	−0.014	0.0001	−0.007	1.7×10^{-5}	−10.861	−0.029
	(*in*, *in*)	−0.023	0.0001	−0.021	1.5×10^{-5}	−3.258	−0.009
Wikipedia	(*out*, *in*)	−0.070	0.0002	−0.037	3.8×10^{-6}	−392.737	−0.941
	(*in*, *out*)	0.017	0.0028	−0.005	2.8×10^{-5}	125.057	0.299
	(*out*, *out*)	−0.032	0.0006	−0.024	3.0×10^{-5}	−48.970	−0.117
	(*in*, *in*)	−0.014	0.0008	−0.009	6.0×10^{-6}	−45.744	−0.110
Political blogs	(*out*, *in*)	−0.230	0.005	−0.133	4.5×10^{-5}	−25.689	−0.965
	(*in*, *out*)	−0.023	0.006	−0.020	5.8×10^{-5}	−0.609	−0.023
	(*out*, *out*)	−0.0515	0.006	−0.041	6.5×10^{-5}	−2.285	−0.086
	(*in*, *in*)	−0.094	0.006	−0.064	7.6×10^{-5}	−6.522	−0.245
WWW model 1	(*out*, *in*)	−0.040	0.0001	−0.043	4.5×10^{-7}	77.186	0.711
	(*in*, *out*)	−0.026	0.0003	−0.029	5.0×10^{-6}	27.230	0.251
	(*out*, *out*)	−0.033	0.0002	−0.037	8.0×10^{-6}	61.734	0.570
	(*in*, *in*)	−0.031	0.0002	−0.033	7.5×10^{-7}	35.574	0.328
WWW model 2	(*out*, *in*)	−0.050	0.0002	−0.054	6.5×10^{-7}	77.496	0.687
	(*in*, *out*)	−0.032	0.0003	−0.036	4.5×10^{-6}	29.586	0.262
	(*out*, *out*)	−0.051	0.0003	−0.060	1.8×10^{-5}	64.594	0.573
	(*in*, *in*)	−0.030	0.0002	−0.031	6.7×10^{-7}	40.795	0.362
WWW model 3	(*out*, *in*)	−0.036	0.0001	−0.037	1.9×10^{-7}	73.870	0.736
	(*in*, *out*)	−0.020	0.0003	−0.021	1.5×10^{-6}	19.573	0.195
	(*out*, *out*)	−0.031	0.0002	−0.033	4.5×10^{-6}	52.737	0.525
	(*in*, *in*)	−0.023	0.0001	−0.024	1.4×10^{-7}	38.111	0.380
Coachella	(*out*, *in*)	−0.143	0.068	−0.229	5.3×10^{-4}	2.642	0.357
	(*in*, *out*)	−0.170	0.059	−0.037	4.7×10^{-4}	−3.134	−0.424
	(*out*, *out*)	0.148	0.063	0.096	4.2×10^{-4}	1.459	0.197
	(*in*, *in*)	0.280	0.058	0.055	6.2×10^{-4}	5.971	0.808
Little Rock	(*out*, *in*)	−0.301	0.030	−0.197	2.3×10^{-4}	−5.902	−0.420
	(*in*, *out*)	−0.221	0.025	−0.029	2.6×10^{-4}	−7.464	−0.531
	(*out*, *out*)	0.317	0.029	0.098	2.6×10^{-4}	9.476	0.672
	(*in*, *in*)	0.142	0.029	0.049	4.3×10^{-4}	4.181	0.297
St. Marks	(*out*, *in*)	−0.027	0.065	−0.069	5.7×10^{-4}	0.735	0.081
	(*in*, *out*)	−0.344	0.054	−0.011	6.6×10^{-4}	−5.390	−0.595
	(*out*, *out*)	0.302	0.061	−0.010	6.7×10^{-4}	5.280	0.583
	(*in*, *in*)	0.298	0.061	0.004	0.0011	4.964	0.548
St. Martin	(*out*, *in*)	−0.204	0.068	−0.127	7.2×10^{-4}	−1.476	−0.204
	(*in*, *out*)	−0.392	0.042	−0.020	9.2×10^{-4}	−5.790	−0.800
	(*out*, *out*)	0.168	0.069	0.017	9.2×10^{-4}	2.492	0.344
	(*in*, *in*)	0.178	0.081	0.014	8.5×10^{-4}	3.244	0.448
Ythan	(*out*, *in*)	−0.179	0.047	−0.238	3.0×10^{-4}	−2.308	−0.493
	(*in*, *out*)	−0.338	0.033	−0.014	6.1×10^{-4}	−3.424	−0.732
	(*out*, *out*)	0.348	0.052	−0.062	6.1×10^{-4}	1.759	0.376
	(*in*, *in*)	0.288	0.056	−0.017	2.9×10^{-4}	1.321	0.282
Coachella Niche	(*out*, *in*)	−0.143	0.063	−0.195	7.4×10^{-4}	0.505	0.045
	(*in*, *out*)	−0.170	0.043	−0.020	5.6×10^{-4}	−6.383	−0.573
	(*out*, *out*)	0.148	0.049	0.085	5.4×10^{-4}	5.866	0.527
	(*in*, *in*)	0.280	0.061	0.031	6.6×10^{-4}	6.969	0.626
Little Rock Niche	(*out*, *in*)	−0.206	0.030	−0.073	4.2×10^{-4}	−5.197	−0.288
	(*in*, *out*)	−0.263	0.027	−0.006	3.4×10^{-4}	−9.467	−0.524
	(*out*, *out*)	0.337	0.027	0.013	3.3×10^{-4}	12.131	0.671
	(*in*, *in*)	0.198	0.030	0.001	3.3×10^{-4}	7.914	0.438
St. Marks Niche	(*out*, *in*)	−0.221	0.059	−0.113	0.00124	−1.964	−0.323
	(*in*, *out*)	−0.206	0.055	−0.013	0.00105	−3.099	−0.509
	(*out*, *out*)	0.282	0.061	0.046	8.6×10^{-4}	4.014	0.660
	(*in*, *in*)	0.163	0.066	0.004	8.5×10^{-4}	2.730	0.449
St. Martin Niche	(*out*, *in*)	−0.230	0.066	−0.181	4.4×10^{-4}	−1.230	−0.225
	(*in*, *out*)	−0.221	0.043	−0.038	5.6×10^{-4}	−2.926	−0.536
	(*out*, *out*)	0.312	0.062	0.083	5.3×10^{-4}	3.911	0.716
	(*in*, *in*)	0.182	0.081	0.067	9.1×10^{-4}	2.106	0.386
Ythan Niche	(*out*, *in*)	−0.193	0.058	−0.074	5.7×10^{-4}	−2.443	−0.324
	(*in*, *out*)	−0.243	0.037	−0.018	5.2×10^{-4}	−4.728	−0.616
	(*out*, *out*)	0.252	0.046	0.043	5.2×10^{-4}	4.414	0.585

Network	(α, β)	$r(\alpha, \beta)$	σ_r^{rw}	$\langle r_{rand} \rangle$	σ_r^{rand}	$Z(\alpha, \beta)$	ASP(α, β)
	(*in, in*)	0.158	0.060	0.020	5.7×10^{-4}	3.034	0.402
Coachella Cascade	(*out, in*)	−0.415	0.050	−0.229	4.7×10^{-4}	−5.713	−0.453
	(*in, out*)	−0.458	0.038	−0.037	2.1×10^{-4}	−6.891	−0.547
	(*out, out*)	0.436	0.048	0.096	3.2×10^{-4}	6.383	0.506
	(*in, in*)	0.433	0.043	0.055	3.8×10^{-4}	6.173	0.490
Little Rock Cascade	(*out, in*)	−0.363	0.027	−0.051	4.1×10^{-4}	−11.977	−0.465
	(*in, out*)	−0.417	0.020	−0.034	2.1×10^{-4}	−13.735	−0.533
	(*out, out*)	0.389	0.025	0.041	2.0×10^{-4}	12.756	0.495
	(*in, in*)	0.391	0.024	0.039	3.8×10^{-4}	13.033	0.506
St. Marks Cascade	(*out, in*)	−0.264	0.062	−0.040	9.2×10^{-4}	−3.627	−0.413
	(*in, out*)	−0.353	0.043	−0.020	6.7×10^{-4}	−5.146	−0.586
	(*out, out*)	0.294	0.055	0.025	6.7×10^{-4}	4.260	0.485
	(*in, in*)	0.305	0.053	0.024	7.5×10^{-4}	4.398	0.501
St. Martin Cascade	(*out, in*)	−0.289	0.066	−0.056	9.2×10^{-4}	−3.821	−0.424
	(*in, out*)	−0.371	0.056	−0.021	7.7×10^{-4}	−5.293	−0.587
	(*out, out*)	0.310	0.055	0.022	7.7×10^{-4}	4.536	0.503
	(*in, in*)	0.297	0.065	0.026	0.00145	4.265	0.473
Ythan Cascade	(*out, in*)	−0.257	0.046	−0.023	8.7×10^{-4}	−4.873	−0.431
	(*in, out*)	−0.346	0.041	−0.011	6.5×10^{-4}	−6.703	−0.592
	(*out, out*)	0.275	0.044	0.012	6.5×10^{-4}	5.401	0.477
	(*in, in*)	0.283	0.045	0.010	9.3×10^{-4}	5.495	0.486
Spanish	(*out, in*)	−0.280	0.002	−0.269	3.8×10^{-6}	−75.777	−0.599
	(*in, out*)	−0.256	0.002	−0.246	4.7×10^{-6}	−49.451	−0.391
	(*out, out*)	−0.282	0.002	−0.269	2.4×10^{-5}	−65.006	−0.514
	(*in, in*)	−0.254	0.002	−0.246	3.8×10^{-6}	−59.801	−0.473
Japanese	(*out, in*)	−0.266	0.004	−0.230	1.9×10^{-5}	−29.772	−0.634
	(*in, out*)	−0.231	0.004	−0.208	2.8×10^{-5}	−17.468	−0.372
	(*out, out*)	−0.240	0.004	−0.213	2.9×10^{-5}	−22.025	−0.469
	(*in, in*)	−0.255	0.004	−0.224	3.0×10^{-5}	−23.062	−0.491
French	(*out, in*)	−0.240	0.002	−0.210	6.2×10^{-6}	−75.777	−0.599
	(*in, out*)	−0.204	0.002	−0.183	1.3×10^{-5}	−49.451	−0.391
	(*out, out*)	−0.253	0.002	−0.220	2.8×10^{-5}	−65.006	−0.514
	(*in, in*)	−0.194	0.002	−0.174	4.8×10^{-6}	−59.801	−0.473
English	(*out, in*)	−0.226	0.001	−0.214	3.3×10^{-6}	−69.192	−0.671
	(*in, out*)	−0.203	0.001	−0.195	5.7×10^{-6}	−32.554	−0.316
	(*out, out*)	−0.193	0.001	−0.185	9.7×10^{-6}	−47.468	−0.460
	(*in, in*)	−0.238	0.001	−0.227	3.9×10^{-6}	−50.332	−0.488
Scrambled	(*out, in*)	−0.227	0.001	−0.235	4.3×10^{-6}	43.805	0.496
	(*in, out*)	−0.227	0.001	−0.235	5.3×10^{-6}	44.498	0.504
	(*out, out*)	−0.228	0.001	−0.235	5.4×10^{-6}	44.105	0.499
	(*in, in*)	−0.227	0.001	−0.234	4.6×10^{-6}	44.207	0.501
Bipartite	(*out, in*)	−0.974	0.001	−0.715	4.7×10^{-5}	−59.537	−0.511
	(*in, out*)	−0.973	0.001	−0.705	9.6×10^{-5}	−56.944	−0.488
	(*out, out*)	−0.974	0.001	−0.711	9.6×10^{-5}	−58.222	−0.499
	(*in, in*)	−0.973	0.001	−0.710	5.3×10^{-6}	−58.514	−0.502

For each network and each of the four possible pairs (α, β), we show the Pearson correlation $r(\alpha, \beta)$, the error σ_r^{rw} in this quantity as estimated by jackknife (see ref. 1), the average Pearson correlation of the random ensemble $\langle r_{rand} \rangle$, the error of this average σ_r^{rand} (*Materials and Methods*), $Z(\alpha, \beta)$, and ASP(α, β).

Table S3. Standard deviations in food-web models

Network	(α, β)	$\sigma_r^{\text{cascade}}$	σ_r^{niche}
Coachella	(*out*, *in*)	0.0268	0.1501
	(*in*, *out*)	0.0235	0.0826
	(*out*, *out*)	0.0289	0.1033
	(*in*, *in*)	0.0262	0.0739
Little Rock	(*out*, *in*)	0.0178	0.1314
	(*in*, *out*)	0.0127	0.0354
	(*out*, *out*)	0.0173	0.0777
	(*in*, *in*)	0.0166	0.0642
St. Marks	(*out*, *in*)	0.0583	0.1849
	(*in*, *out*)	0.0455	0.0729
	(*out*, *out*)	0.0592	0.1341
	(*in*, *in*)	0.0592	0.1046
St. Martin	(*out*, *in*)	0.0575	0.1841
	(*in*, *out*)	0.0436	0.0759
	(*out*, *out*)	0.0603	0.1276
	(*in*, *in*)	0.0582	0.1038
Ythan	(*out*, *in*)	0.0486	0.1636
	(*in*, *out*)	0.0342	0.0566
	(*out*, *out*)	0.0463	0.1116
	(*in*, *in*)	0.0467	0.0954

We show the standard deviations in $r(\alpha, \beta)$ for 500 instances per real-world network of the cascade and niche model. Instances are constructed according to the procedure described in the *Materials and Methods*, following ref. 2; note the large standard deviations of the niche model.