

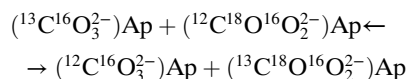
# Supporting Information

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## SI Text

**Modeling  $^{13}\text{C}$ - $^{18}\text{O}$  Clumping in Apatite—Methods and Results.** An initial theoretical thermodynamic model of the  $^{13}\text{C}$ - $^{18}\text{O}$  clumping equilibrium in carbonate-substituted apatite was created to provide an independent temperature calibration. This model is based on a B-type carbonate substitution into fluorapatite—where a  $\text{CO}_3^{2-}$  group replaces one  $\text{PO}_4^{3-}$  group in each unit cell. To maintain charge balance, one  $\text{Na}^+$  ion is substituted for a  $\text{Ca}^{2+}$  in each unit cell as well. B-type substitution is thought to be the dominant mechanism for incorporating carbonate in biogenic apatites (1), although it is less clear whether  $\text{Na}^+$  for  $\text{Ca}^{2+}$  is the main charge-balancing substitution. Fluorapatite is chosen because it has a simpler unit cell than hydroxylapatite (e.g., 2), facilitating density-functional theory calculations. One carbonate group per unit cell (out of six phosphate sites) corresponds to  $\sim 4.5\%$  wt.  $\text{CO}_2$  in the structure, similar to biogenic apatites (1, 3).

Much like conventional stable isotope fractionations,  $^{13}\text{C}$ - $^{18}\text{O}$  clumping equilibria are controlled mainly by vibrational energy differences between isotopically substituted species (isotopologues) in an isotope exchange reaction; i.e.,



Where  $(^x\text{C}^y\text{O}^z\text{O}_2^{2-})\text{Ap}$  is the relevant isotopologous substitution in apatite. These equilibria can be successfully estimated if a suitable model of the vibrational (phonon) frequencies of each isotopologue can be generated (4, 5). In the case of apatite, where carbonate is a minor nonstoichiometric substituent, a structural model for the carbonate substitution is needed before vibrational frequencies can be calculated.

**Modeling Method.** The positions of substituted  $\text{CO}_3^{2-}$  and  $\text{Na}^+$  groups in the model fluorapatite structure are determined by an initial empirical-potential optimization followed by electronic structure calculations using density-functional theory (DFT). Initial optimization was performed in the General Utility Lattice Program (GULP; 6), using a previously generated empirical-potential model (7). We attempted to survey all symmetrically distinct Na-compensated B-type substitution configurations. Initial guess carbonate  $\leftarrow \rightarrow$  phosphate replacement structures were generated by substituting one carbon atom for phosphorus and deleting one oxygen atom at a time in the same  $\text{PO}_4^{3-}$  group. Each guess configuration was allowed to relax under constant-volume then constant-pressure constraints ( $10^5$  Pa). Among the relaxed structures, the lowest potential energy was observed when  $\text{Na}^+$  substituted into the Ca(2) site—which shares bonds with fluoride in pure fluorapatite and hydroxyl groups in hydroxylapatite—and the oxygen vacancy was located on a phosphatic O(2) or O(3) site nearby. Three of the thirty tested configurations shared indistinguishable calculated lattice energies ( $-446.6303$  eV/unit cell), the lowest in the survey. These three structures were crystallographically equivalent after empirical-potential relaxation.

This minimum-energy substituted structure was then used as an initial guess structure in a DFT-based structural optimization using the Quantum Espresso package (8; [www.quantum-espresso.org](http://www.quantum-espresso.org)). Electronic structure calculations used the PBE gradient-corrected functional (9) and the plane-wave pseudopotentials C.pbe-rrkjus.UPF, O.pbe-rrkjus.UPF, F.pbe-n-van.UPF, Na.pbe-sp-van\_ak.UPF, P.pbe-van\_ak.UPF, and Ca.pbe-nsp-van.UPF,

available from the [www.quantum-espresso.org](http://www.quantum-espresso.org) distribution. The plane-wave energy cutoff was set at 40 Rydberg, sufficient to obtain convergence of structural parameters within  $\sim 0.1\%$ —test calculations at a cutoff energy of 50 Rydberg showed insignificant changes in the structure—as was also found for calcite and pure fluorapatite. One electronic wave vector is sampled, (0, 0, 1/2). Atomic positions and unit cell dimensions were allowed to relax until forces acting on each atom were less than  $10^{-4}$  Rydberg/bohr ( $4 \times 10^{-12}$  N) and stresses on the unit cell were less than 0.5 kbar. Fig. S5 and Table S5 show the resulting structure, compared to stoichiometric apatite. The carbonate group is coincident and coplanar with a mirror plane in the parent fluorapatite structure, consistent with the structural arrangement inferred from neutron powder diffraction measurements (10).

Phonon frequencies at zero phonon wave vector were then calculated for the resulting DFT-optimized structural model, assuming average atomic masses for each atom. The matrix of force constants generated in this calculation was then used to determine frequencies for each isotopologue in the exchange reaction, following the procedure of Schauble et al. 2006 (4). All three possible  $^{18}\text{O}$ -substitution sites in the carbonate molecule were substituted in turn. Phonon frequencies are all positive real numbers, indicating that the optimized structure represents a local energy minimum at that unit cell size. Subsequent calculations at the (0, 0, 1/2) phonon wave vector suggest that this structure is dynamically unstable with respect to a supercell (doubled along the z-axis), consistent with positional disorder observed in actual apatites (10). Since isotopic ordering and fractionation properties are dominantly controlled by short-range valence bonding interactions (4) it is expected that the long-range ordering forces responsible for this instability are not very important in determining the clumping equilibrium. Indeed, the equilibrium constant calculated using all positive frequencies at the (0, 0, 1/2) phonon wave vector is within  $3 \times 10^{-7}$  (0.0003‰) of the (0, 0, 0) wave-vector estimate at  $250 \pm ^\circ\text{K}$ .

Phonon frequencies calculated with the PBE functional tend to be lower than measured frequencies (4, 11). In order to correct for this, we calculated optimized crystal structures (Table S6) and zero wave-vector frequencies for calcite and pure fluorapatite, and compared them to infrared and Raman spectroscopic measurements. For calcite, the estimated frequency scale factor is  $1.032 \pm 0.004$ , very close to what was used in an earlier calculation of clumping equilibria. The best-fit scale factor for fluorapatite is somewhat larger,  $1.055 \pm 0.002$ . It seems most likely that this reflects slightly different systematic errors in force constants associated with internal vibrations of  $\text{CO}_3^{2-}$  vs.  $\text{PO}_4^{3-}$ , because the associated vibrational modes dominate the high-frequency part of each mineral's spectrum. Since  $^{13}\text{C}$ - $^{18}\text{O}$  clumping is dominantly controlled by internal vibrations of the carbonate molecule, the calcite scale factor is used. Substituting the average of the apatite and calcite scale factors would raise the estimated clumping  $K_{\text{eq}}$  by approximately  $1 \times 10^{-5}$  (0.01‰) at  $25^\circ\text{C}$ .

Calculated phonon frequencies for all model structures are shown in Table S7.

**$K_{\text{eq}}$  (38)** Calculated  $^{13}\text{C}$ - $^{18}\text{O}$  clumping equilibria ( $K_{\text{eq}}$ ) are shown in Fig. 2. A polynomial fit is also given to facilitate interpolation between the reported temperature points.

Carbonate fluorapatite:

$$K_{\text{eq}}(38) = 1 - 8.5580 \times 10^{-3}/T + 2.3632/T^2 + 2.1480 \times 10^4/T^3 - 3.1506 \times 10^6/T^4$$

Calcite:

$$K_{\text{eq}}(38) = 1 - 6.6699 \times 10^{-3}/T - 1.1483/T^2 + 2.2466 \times 10^4/T^3 - 3.2146 \times 10^6/T^4$$

Where  $T$  is the absolute temperature. These polynomials reproduce calculated equilibrium constants within  $3 \times 10^{-6}$  at temperatures above 250 °K.

The positionally averaged  $K_{\text{eq}}$  of B-type carbonate-substituted fluorapatite is remarkably similar to the analogous calcite model, as well as to a previously described calcite model created with different software and pseudopotentials (4). As noted in that study, empirical calibrations suggest somewhat greater temperature sensitivity than first-principles models. All three models yield temperature estimates within 1–2 °C for a given  $\Delta_{47} \text{CO}_2$ . The clumping vs. temperature relationship in carbonate-substituted apatite thus seems likely to be indistinguishable from calcite. As a theoretical matter this conclusion is tentative—it will be necessary to model alternate defect structures in both fluorapatite and hydroxyl apatite to examine possible heterogeneity—but it does support the empirically observed match between calcite and apatite calibrations.

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