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Supporting Material

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Acuity of a cryptochrome and vision based magnetoreception system in birds (Supplementary Material)

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Theory

In this section the filter function formalism describing the effect of orientational disorder on cryptochrome magnetic acuity is introduced. First, the magnetic field filter function is defined. Second, it is demonstrated in principle how magnetic field mediated signalling of cryptochrome is affected by protein orientational fluctuations. The formalism introduced is then employed for the calculation of the magnetic field modulated visual pattern in the retina of a bird's eye.

Magnetic field mediated signalling yield of cryptochrome

For our analysis an X, Y, Z-coordinate frame is associated with the bird's eye as shown in Fig. 2. At each point of the retina is affixed a magnetic field-sensitive cryptochrome molecule. We assume that the orientations of these molecules are highly coordinated and, in fact, obtained from the X, Y, Z-coordinate frame through a general rotational transformation D specific for each point on the retina. D is a 3×3-matrix with elements (1, 2)

$$D = \begin{pmatrix} \cos\alpha\cos\beta\cos\gamma - \sin\alpha\sin\gamma & -\cos\alpha\cos\beta\sin\gamma - \sin\alpha\cos\gamma & \cos\alpha\sin\beta \\ \sin\alpha\cos\beta\cos\gamma + \cos\alpha\sin\gamma & -\sin\alpha\cos\beta\sin\gamma + \cos\alpha\cos\gamma & \sin\alpha\sin\beta \\ -\sin\beta\cos\gamma & \sin\beta\sin\gamma & \cos\beta \end{pmatrix}. \tag{S1}$$

The orientation of the cryptochrome molecules distributed over the retina are specified through an x, y, z-coordinate systems as shown in Fig. 2. This coordinate system is affixed to the FADH prosthetic group in cryptochrome, as illustrated in Fig. 2. The key part of the FADH group, as shown in Fig. 2, is a planar flavin moiety and we define the z-axis of the x, y, z-coordinate frame to be perpendicular to the flavin plane, while the x- and y-axes are oriented in the flavin plane as seen in Fig. 2.

The orientation of cryptochrome in the retina is chosen such that its z-axis is perpendicular to the retina surface, as shown in Figs. 3a-b. This assumption is rather arbitrary, but does not violate the generality of our rationale because after having the coordinate frames defined we study all possible spatial reorientations of cryptochrome. However, crucial is that all cryptochromes assume an orientation defined through a systematic transformation D that rotates the X, Y, Z coordinate system into the local x, y, z-coordinate system. We note that for any vector $\vec{v}$ with Cartesian coordinates $v_j,\ j = 1, 2, 3$ in the X, Y, Z-coordinate frame and Cartesian coordinates $\tilde{v}_j,\ j = 1, 2, 3$ in the x, y, z-coordinate frame, holds

$$\tilde{v}_j = \sum_n D_{nj} v_n \ . \tag{S2}$$

The rotation D is conveniently characterized through three Euler angles α, β, γ and then denoted $D(\alpha, \beta, \gamma)$. The angles are functions of each position $\vec{r}$ on the retina (see Fig. 3a-b). The definition of the Euler angles is connected with $\vec{r}$ as follows: One can express the position $\vec{r}$ on the retina through so-called spherical coordinates r, ϑ, φ, namely, $\vec{r} = r(\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$, i.e., each position on the retina is assigned, besides the retina radius r, a pair of angles (ϑ, φ) as shown in Fig. 3a. The Euler angles are then chosen as $(\alpha, \beta, \gamma) = (\varphi, \pi - \vartheta, 0)$ as illustrated in Fig. 3b. The choice of $\gamma = 0$ is due to the fact that for the definition of an ideally oriented coordinate frame only two angles are necessary.

Cryptochrome wiggling introduces orientational disorder of the ideally oriented proteins. As a result, the ideal x, y, z-coordinate frame is to be replaced by a randomly oriented x', y', z'-coordinate frame, as illustrated in Fig. 3c. This disorder can be captured mathematically through a second transformation, $D(\alpha', \beta', \gamma')$, that takes the x, y, z-frame for an ideally oriented cryptochrome to the x', y', z' frame of the randomly reoriented cryptochrome. Thus, the components of the vector $\tilde{v}_j$ in Eq. (S2) in the x', y', z'-coordinate frame can be written as

$$\tilde{v}'_j = \sum_m D_{mj}(\alpha', \beta', \gamma')\tilde{v}_m \ . \tag{S3}$$

Substituting Eq. (S2) into Eq. (S3) one relates the cartesian coordinates of a vector $\vec{v}$ in the X, Y, Z-coordinate frame with the cartesian coordinates $\tilde{v}'_j,\ j = 1, 2, 3$ in the x', y', z'-coordinate frame through two consecutive transformations

$$\tilde{v}'_j = \sum_{m,n} D_{mj}(\alpha', \beta', \gamma') D_{nm}(\alpha, \beta, \gamma) v_n \ . \tag{S4}$$

In the X, Y, Z-coordinate frame the Earth magnetic field vector $\vec{B}$ is characterized by the polar angle Θ, azimuthal angle Φ and the magnitude B_0 as illustrated in Fig. 3d

$$\vec{B} = (B_X,\ B_Y,\ B_Z) = B_0(\sin\Theta\cos\Phi,\ \sin\Theta\sin\Phi,\ \cos\Theta). \tag{S5}$$

Substituting Eq. (S5) into Eq. (S4) allows one to express then the magnetic field vector in the x', y', z'-coordinate frame, i.e., the field vector as seen relative to a specific cryptochrome molecule, as a function of α, β, γ, α', β', γ', Θ and Φ

$$B'_j(\alpha,\beta,\gamma,\alpha',\beta',\gamma',\Theta,\Phi) = \sum_{m,n} D_{mj}(\alpha',\beta',\gamma')D_{nm}(\alpha,\beta,\gamma)B_n(\Theta,\Phi) \ . \tag{S6}$$

Equivalently, the magnetic field vector in the x', y', z'-coordinate frame can be written in terms of angles θ' and ϕ' (see Fig. 3e) as

$$\vec{B}' = (B_{x'},\ B_{y'},\ B_{z'}) = B_0(\sin\theta'\cos\phi',\ \sin\theta'\sin\phi',\ \cos\theta'). \tag{S7}$$

The dependence of cryptochrome's activation yield on the cryptochrome intrinsic field orientation θ' and ϕ' was studied in (3, 4). In these earlier papers we considered a fixed coordinate frame associated with cryptochrome, and calculated the change in protein signalling due to the reorientation of the external magnetic field, i.e., due to a change in θ' and ϕ' as defined here. To understand how cryptochrome disorder in the retina affects the magnetic field modulated signal detected by a bird we assume $\vec{B}$ to be fixed and consider the change in protein orientation with respect to it.

According to the prior investigations cryptochrome has an intrinsic magnetic field sensing anisotropy axis (signalling axis), which is roughly perpendicular to the plane of the flavin radical, responsible for cryptochrome's functioning (3–6). For this reason, we have fixed in the present treatment the cryptochrome x, y, z-coordinate system to the FADH prosthetic group, choosing the z-axis perpendicular to the flavin plane as illustrated in Fig. 2. Cryptochrome signalling is dominated by the anisotropic hyperfine interactions in the two nitrogen atoms (6), highlighted in Fig. 2. If the magnetic field is applied along the signalling axis, cryptochrome signalling is significantly enhanced, while applying the magnetic field in perpendicular directions reduces cryptochrome signalling. Thus, the magnetic field effect in cryptochrome is mainly governed by the relative orientation of cryptochrome's signalling axis with respect to the magnetic field direction, measured by the angle θ' defined in Eq. (S7); cryptochrome signalling is insensitive to the angle ϕ' (3–6). Because of the retina spherical symmetry, the external magnetic field is expected to be detected efficiently if the signalling axes of cryptochromes are oriented symmetrically around the retina, for example perpendicular to its surface. This particular spatial organization of cryptochromes is one of the possibilities, in which for any orientation of the external magnetic field there are always at least several cryptochromes whose signalling axes are collinear with the magnetic field vector and therefore the cryptochromes represent efficiently the magnetic field orientation. For the sake of concreteness, we assume the signalling axis of ideally oriented cryptochromes to be perpendicular to the retina surface. We stress, however, that there are many other cryptochrome orientational arrangements that could lead to a visual magnetic field response that is representative of the bird's orientation in the Earth' magnetic field..

Figure S1 shows the cryptochrome magnetic field mediated signalling yield calculated for a field strength of 0.5 G, i.e., the strength characteristic for the geomagnetic field. Figure S1a shows the orientational dependence of the cryptochrome activation yield in the external magnetic field, as determined in (3). Figure S1b shows the orientational dependence on the relative duration of cryptochrome's dark reaction as determined in (4). The results in (3, 4) show that the magnetic field mediated signalling yield of cryptochrome is not significantly affected by the change of the angle ϕ'; this can be seen in Figure S1a; actually Fig. S1b does not show the ϕ'-dependence as it was argued in (4) that the dependence is weak.

The angle θ', defined in Eq. (S7), can be related to α, β, γ, α', β', γ', Θ and Φ. Consider a scalar product of a unit vector $\vec{z}'$ directed along the z' axis with the magnetic field vector $\vec{B}$ (see Fig. 3e)

$$\vec{B} \cdot \vec{z}' = |\vec{B}| \cdot |\vec{z}'| \cos\theta'. \tag{S8}$$

With $|\vec{z}'| = 1$ and $|\vec{B}| = B_0$ follows from Eq. (S8)

$$\cos\theta' = \frac{B_X z'_X + B_Y z'_Y + B_Z z'_Z}{B_0} = z'_X \sin\Theta\cos\Phi + z'_Y \sin\Theta\sin\Phi + z'_Z\cos\Theta, \tag{S9}$$

where B_X, B_Y and B_Z are defined in Eq. (S5) and z'_X, z'_Y, z'_Z are the cartesian components of the $\vec{z}'$ vector in the X, Y, Z-coordinate frame. In the x', y', z'-coordinate frame the cartesian components of the $\vec{z}'$ vector are equal to (0,0,1). Thus, substituting $\vec{z}' = (z'_X,\, z'_Y,\, z'_Z)$ into Eq. (S4) one obtains

$$\begin{aligned}
z'_X &= -\sin\alpha\sin\alpha'\sin\beta' + \cos\alpha(\cos\beta'\sin\beta + \cos\alpha'\cos\beta\sin\beta') && \text{(S10)}\\
z'_Y &= \cos\beta'\sin\alpha\sin\beta + \sin\beta'(\cos\alpha'\cos\beta\sin\alpha + \cos\alpha\sin\alpha') && \text{(S11)}\\
z'_Z &= \cos\beta\cos\beta' - \cos\alpha'\sin\beta\sin\beta'. && \text{(S12)}
\end{aligned}$$

Equations (S10)-(S12) are obtained assuming $\gamma = 0$, which does not violate the generality of the derivations. Figure 3b shows that one can always define the x, y, z-coordinate frame in such a way that $\gamma = 0$ holds at all points of the retina.

Substituting Eqs. (S10)-(S12) into Eq. (S9) one obtains $\cos\theta'$ as a function of α, β, α', β', Θ and Φ. The resulting expression reveals that θ' does not depend on the Euler angle γ', defined in Fig. 3c. Therefore, one concludes that magnetoreceptive properties of cryptochrome are γ'-independent, i.e., cryptochromes are free to reorient around the axis defining γ'.

As described in (3, 4), the accurate calculation of the magnetic field mediated signalling yield of cryptochrome, shown in Fig. S1, is a non-trivial task. However, for the sake of simplicity one may assume, without introducing a significant error, a Gaussian-shaped parametric dependence of the signaling yield. The deviation of the z'-axis from the external magnetic field vector is a measure for the magnetic field mediated signalling yield of cryptochrome

$$F = F_0 \exp\left[-\frac{(n_{x'}^2 + n_{y'}^2)}{\sigma^2}\right]. \tag{S13}$$

Here $n_{x'} = B_{x'}/B_0$ and $n_{y'} = B_{y'}/B_0$ are the normalized projections of the magnetic field vector on the $(x'y')$-plane of the cryptochrome's intrinsic coordinate frame, i.e., on the flavin plane. F_0 is the maximal value of the signalling yield, and σ defines the signalling acuity. Substituting $n_{x'}$ and $n_{y'}$ into Eq. (S13) one obtains

$$F(\theta') = F(\alpha, \beta, \alpha', \beta', \Theta, \Phi) = F_0 \exp\left[-\frac{\sin^2\theta'}{\sigma^2}\right]. \tag{S14}$$

$F(\theta')$ is cryptochrome's signaling efficiency. If the magnetic field is applied along the signalling axis of cryptochrome ($\theta' = 0$) the protein efficiency is maximal, namely, equal to $F(0) = F_0$, while if it is perpendicular to that axis ($\theta' = \pi/2$) the efficiency reaches its minimum value. An important parameter, defining magnetoreceptive properties of cryptochrome, is the relative change of the signalling yield, Δ

$$\Delta \equiv \frac{|F(0) - F(\pi/2)|}{F(0)} = 1 - \exp\left[-\frac{1}{\sigma^2}\right] \tag{S15}$$

which defines how efficiently cryptochrome responds to reorientation of the external magnetic field. Figure S1b shows that in a magnetic field of 0.5 G, Δ can be as large as 0.17. Equation (S15) allows one to express σ as a function of Δ

$$\sigma = \sqrt{\frac{1}{\ln\left[1/(1-\Delta)\right]}}. \tag{S16}$$

To average the magnetic field mediated signalling yield of cryptochrome over all possible orientations of the protein, we assume that the change in cryptochrome orientation, i.e., the change in the angles α', β' and γ', arising in the course of cryptochrome's wiggling, is linked to a potential of mean force

$$E_{pot} = E(\alpha', \beta', \gamma'). \tag{S17}$$

According to statistical mechanics (7, 8) the average value of the magnetic field mediated signalling yield of cryptochrome, $\langle F\rangle$ is

$$\begin{aligned}\langle F(\alpha, \beta, \Theta, \Phi)\rangle &= \frac{1}{Z}\int_0^{2\pi}\int_0^{\pi}\int_0^{2\pi} F(\alpha, \beta, \alpha', \beta', \Theta, \Phi) \times \\ &\quad \times \exp\left(-E_{pot}/k_{\mathrm{B}}T\right)\sin\beta' d\alpha' d\beta' d\gamma',\end{aligned} \tag{S18}$$

where $F(\alpha, \beta, \alpha', \beta', \Theta, \Phi)$ is defined in Eq. (S14) and Z is the partition function of the system defined as

$$Z = \int_0^{2\pi}\int_0^{\pi}\int_0^{2\pi} \exp\left(-E_{pot}/k_{\mathrm{B}}T\right)\sin\beta' d\alpha' d\beta' d\gamma'. \tag{S19}$$

T in Eqs. (S18)-(S19) is the temperature of the system and k_{B} is the Botzmann factor.

Equation (S18) shows that $\langle F(\alpha, \beta, \Theta, \Phi)\rangle$ depends on E_{pot} which in turn is α', β' and γ' dependent. It is impossible to write out explicitly the exact functional form of the potential energy term, because it involves many different biologically related factors which are presently unknown. However, since the magnetoreceptive properties of cryptochrome are γ'-independent (as follows from Eqs. (S10)-(S12)) and are influenced mainly by the angle θ', as illustrated in Fig. S1, the only angle that is expected to govern the orientational ordering of cryptochrome is β'. Thus, cryptochrome orientational wiggling is relevant mainly along β'. Taking this into account we assume

$$E_{pot}(\alpha', \beta', \gamma') = V(\beta'). \tag{S20}$$

We let $V(\beta')$ adopt a very simple functional form characterized through the depth of the potential well, defined by $\varepsilon_0 k_\mathrm{B} T$ where ε_0 is a dimensionless parameter with a minimum at $\beta' = 0, \pi$ and a barrier of $\varepsilon_0 k_\mathrm{B} T$ at $\beta' = \pi/2$. Such potential is given by

$$V(\beta') = \varepsilon_0 k_\mathrm{B} T \sin^2 \beta'. \tag{S21}$$

Substituting Eq. (S20)-(S21) and Eq. (S14) into Eq. (S18) one obtains

$$\begin{aligned} \langle F(\alpha, \beta, \Theta, \Phi) \rangle &= \frac{F_0}{\int_0^{2\pi} \int_0^{\pi} \int_0^{2\pi} \exp\left[-\varepsilon_0 \sin^2 \beta'\right] \sin \beta' d\alpha' d\beta' d\gamma'} \times \\ &\times \int_0^{2\pi} \int_0^{\pi} \int_0^{2\pi} \exp\left[-\frac{\sin^2 \theta'}{\sigma^2} - \varepsilon_0 \sin^2 \beta'\right] \sin \beta' d\alpha' d\beta' d\gamma' \end{aligned} \tag{S22}$$

Here $\sin^2 \theta'$ can be calculated from Eq. (S9). It depends on α, β, α', β', Θ and Φ. Since θ' is γ' independent, Eq. (S22) can be further simplified

$$\begin{aligned} \langle F(\alpha, \beta, \Theta, \Phi) \rangle &= \frac{F_0}{2\pi \int_0^{\pi} \exp\left[-\varepsilon_0 \sin^2 \beta'\right] \sin \beta' d\beta'} \times \\ &\times \int_0^{\pi} \int_0^{2\pi} \exp\left[-\frac{\sin^2 \theta'}{\sigma^2} - \varepsilon_0 \sin^2 \beta'\right] \sin \beta' d\alpha' d\beta'. \end{aligned} \tag{S23}$$

The integrals in Eq. (S23) can be evaluated numerically, although an analytical solution is possible for several limiting cases. Dividing $\langle F \rangle$ in Eq. (S23) by F_0 one obtains the relative magnetic field mediated signalling yield of cryptochrome (magnetic filter function), which is an important quantity because a sensory system of a bird likely detects relative changes in biochemical processes rather than the absolute values. The magnetic filter function is determined by the orientation of the external magnetic field defined by the angles Θ and Φ and varies at different sites of the retina, characterized by the angles α and β

$$\langle F(\alpha, \beta, \Theta, \Phi) \rangle = \frac{\int_0^{\pi} \int_0^{2\pi} \exp\left[-(1 - \cos^2 \theta')/\sigma^2 - \varepsilon_0 \sin^2 \beta'\right] \sin \beta' d\alpha' d\beta'}{2\pi \int_0^{\pi} \exp\left[-\varepsilon_0 \sin^2 \beta'\right] \sin \beta' d\beta'}. \tag{S24}$$

σ is defined in Eq. (S16), which is not substituted into Eq. (S24) for the sake of simplicity.

Repetitive cryptochrome activation increases acuity

Suppose that cryptochrome, like many other sensory proteins, affects a signalling cascade, which results in the activation of specific molecules, which in turn perform a further biological function such as opening or closing an ion-channel. Let N_0 be the number of activated molecules generated in this step in the absence of the external magnetic field. The external magnetic field, according to our assumption, reduces the number of activated molecules, described by the filter function $\langle F(\alpha, \beta, \Theta, \Phi)\rangle$, defined in Eq. (S24). After the cryptochrome signalling cycle is completed the number of activated molecules in the magnetic field mediated step of the transduction reaction is then

$$N_1 = N_0 \langle F(\alpha, \beta, \Theta, \Phi)\rangle. \tag{S25}$$

The termination of cryptochrome signalling does not necessarily coincide with the termination of the transduction reaction. Before the latter reaction is actually complete, cryptochrome may get activated a second time, further quenching the number of activated molecules in the magnetic field mediated step of the transduction reaction

$$N_2 = N_1 \langle F(\alpha, \beta, \Theta, \Phi)\rangle = N_0 \langle F(\alpha, \beta, \Theta, \Phi)\rangle^2. \tag{S26}$$

In general, cryptochrome activation may occur η times, thereby significantly changing the number of the activated molecules in the magnetic field mediated step of the transduction process, according to the expression

$$N_\eta = N_{\eta-1} \langle F(\alpha, \beta, \Theta, \Phi)\rangle = N_0 \langle F(\alpha, \beta, \Theta, \Phi)\rangle^\eta. \tag{S27}$$

The number N_η of activated molecules defines the efficiency, defined as

$$\frac{N_\eta}{N_0} = I(\alpha, \beta) = \langle F(\alpha, \beta, \Theta, \Phi)\rangle^\eta, \tag{S28}$$

of a cryptochrome-containing receptor cell. $I(\alpha, \beta)$ measures to which extent the cell characterized through its retinal position (α, β) contributes to the retinal image. $I(\alpha, \beta)$ is measured in arbitrary units varying between 0 and 1, reflecting the modulation level of the virtual visual image in a bird's eye by the magnetic field.

From Eq. (S24) and Eq. (S28) follows that at a fixed orientation of the magnetic field the varying (with α, β) efficiency of the cells in the retina (due to the varying relative orientation of the magnetic field relative to the local cryptochromes) leads to a formation of a disc-shaped virtual visual pattern in the bird's field of view. Size and intensity of this pattern are related to the acuity of the vision-based magnetic compass. Let I_{max} and I_{min} be the maximal and the minimal values of $I(\alpha, \beta)$, respectively. Then the maximal variation of the magnetic field mediated pattern, A, is

$$A = I_{max} - I_{min}. \tag{S29}$$

The modulation level of the visual signal through the magnetic field, defined in Eq. (S28), allows one to determine the total variation, S, of the magnetic field mediated pattern defined as

$$S = \int_0^{2\pi} \int_0^{\pi/2} (I(\alpha, \beta) - I_{min}) \sin \beta d\beta d\alpha. \tag{S30}$$

According to this definition holds $0 \leq S \leq 2\pi$. Another important characteristic of the magnetic filter function is the size of the magnetic field mediated disc-shaped pattern at half intensity, $\Delta\Omega$, which can be calculated numerically as the solution of the equation

$$\frac{A}{2} = I\left(\Omega_{max} - \Delta\Omega/2\right) - I_{min}, \tag{S31}$$

where $\Omega_{max} = (\alpha, \beta)_{max}$ denotes α and β at which $I(\alpha, \beta)$ reaches its maximal value and where A is the maximal intensity of the magnetic field mediated pattern defined in Eq. (S29). The quantities A, S, and $\Delta\Omega$ in Eqs. (S29-S31) define the acuity of the visual-based compass. A is the measure of the maximal intensity of the magnetic field mediated pattern (increasing A leads to an increase of the magnetic field mediated signal in the retina), S indicates to which extend the retina is influenced by the magnetic field and $\Delta\Omega$ defines the size of the magnetic field mediated pattern. A small value of $\Delta\Omega$ corresponds to a well localized magnetic field mediated spot on the retina, allowing a bird to resolve the magnetic field better than in case of a large spot, i.e., large $\Delta\Omega$.

Mapping of the magnetic field mediated pattern to the visual field

The position of the magnetic field mediated pattern in the retina is defined in Eq. (S9) by the angle θ' (see Fig. 3), which in turn is Φ and Θ dependent. The angles Θ and Φ change upon bird rotation in the horizontal plane, causing the displacement of the magnetic field mediated pattern mapped to the animal's visual field.

Let the angle ω characterize the turn of a bird heading with respect to magnetic North as shown in Fig. S2a. Thus, the Cartesian components of the vector $\vec{B}$ in the (X, Y, Z) coordinate frame change according to

$$\vec{B}' = R_y \vec{B}, \tag{S32}$$

where R_y is the rotation matrix, which describes the rotation of the (X, Y, Z) coordinate frame (see Fig. 3) around the Y-axis

$$R_y = \begin{pmatrix} \cos\omega & 0 & -\sin\omega \\ 0 & 1 & 0 \\ \sin\omega & 0 & \cos\omega \end{pmatrix}. \tag{S33}$$

Substituting Eq. (S5) and Eq. (S33) into Eq. (S32) one obtains

$$\begin{aligned} \vec{B}' \;=\; & B_0(-\cos\Theta\sin\omega + \cos\omega\cos\Phi\sin\Theta,\ \sin\Theta\sin\Phi, \\ & \cos\omega\cos\Theta + \cos\Phi\sin\omega\sin\Theta). \end{aligned} \tag{S34}$$

Let Θ_0 and Φ_0 be the reference angles of $\vec{B}$ in the (X, Y, Z) coordinate frame (see Fig. 3d), corresponding to a bird flying towards magnetic North. According to this definition Θ_0 is the inclination angle of the magnetic field vector and Φ_0 is given by $\Phi_0 = \pi/2$, allowing one to rewrite Eq. (S34) as

$$\vec{B}' = B_0(-\cos\Theta_0\sin\omega,\ \sin\Theta_0,\ \cos\Theta_0\cos\omega). \tag{S35}$$

Equation (S35) and Eq. (S5) give the Cartesian coordinates of $\vec{B}$ in the (X, Y, Z)-coordinate frame as a function of (Θ_0, ω) and (Θ, Φ), respectively. Equating the components of $\vec{B}$ in both equations results in

$$\begin{aligned}
\sin\Theta\cos\Phi &= -\cos\Theta_0\sin\omega & \text{(S36)}\\
\sin\Theta\sin\Phi &= \sin\Theta_0 & \text{(S37)}\\
\cos\Theta &= \cos\Theta_0\cos\omega. & \text{(S38)}
\end{aligned}$$

Substituting Eqs. (S36-S38) into Eq. (S9) one obtains

$$\cos\theta' = -z'_X\cos\Theta_0\sin\omega + z'_Y\sin\Theta_0 + z'_Z\cos\Theta_0\cos\omega, \qquad \text{(S39)}$$

where z'_X, z'_Y, z'_Z are defined in Eqs. (S10-S12).

A pattern on a sphere can be conveniently mapped to a plane by the so-called Miller cylindrical projection (9), in which a spot on the sphere with longitude λ and latitude ϕ is mapped to a spot with coordinates (x, y) defined through the relationships

$$\begin{aligned}
x &= \lambda & \text{(S40)}\\
y &= \frac{5}{4}\ln\left[\tan\left(\frac{\pi}{4}+\frac{2}{5}\phi\right)\right]. & \text{(S41)}
\end{aligned}$$

The Miller projection can be used to map the magnetic field mediated pattern in the retina to the visual field of a bird. Hence, λ and ϕ characterize different spots in the retina as illustrated in Fig. S2b. The coordinates (λ, ϕ) are related to the Euler angles (α, β), defined in Fig. 3b, as

$$\begin{aligned}
\cos\beta &= \cos\phi\sin\lambda & \text{(S42)}\\
\sin\beta &= \sqrt{1-(\cos\phi\sin\lambda)^2} & \text{(S43)}\\
\cos\alpha &= -\frac{\cos\phi\cos\lambda}{\sqrt{1-(\cos\phi\sin\lambda)^2}} & \text{(S44)}\\
\sin\alpha &= -\frac{\sin\phi}{\sqrt{1-(\cos\phi\sin\lambda)^2}}. & \text{(S45)}
\end{aligned}$$

Here $\lambda \in [0\cdots\pi]$ and $\phi \in [-\pi/2\cdots\pi/2]$ describe different spots on the retina (see Fig. S2b). According to Eqs. (S40-S41), the coordinates of different points of the visual field are measured in arbitrary units, such that $x \in [0\cdots\pi]$ and $y \in [-2.303\cdots2.303]$.

Substituting Eqs. (S42-S45) into Eqs. (S10-S12) and the resulting equations into Eq. (S39) one obtains $\cos\theta'$ as a function of ϕ, λ, Θ_0, ω, α' and β'. Finally, substituting $\cos\theta'$ into Eq. (S24) one obtains $\langle F(\Theta_0, \omega, \lambda, \phi)\rangle$, which in turn allows one to rewrite $I(\alpha, \beta)$ in Eq. (S28) as $I(\Theta_0, \omega, \lambda, \phi)$.

$I(\Theta_0, \omega, \lambda, \phi)$ describes the modulation level of cells in the retina by a magnetic field characterized through the coordinates (λ, ϕ), which can be expressed as a function of the coordinates (x, y) in the visual field through

$$\lambda = x \tag{S46}$$

$$\phi = \frac{5}{2}\left(\arctan\left[\exp\left(4y/5\right)\right] - \frac{\pi}{4}\right). \tag{S47}$$

Substituting Eqs. (S46-S47) into $I(\Theta_0, \omega, \lambda, \phi)$ allows one to calculate the modulation level of the visual field through the magnetic field mediated pattern in the retina.

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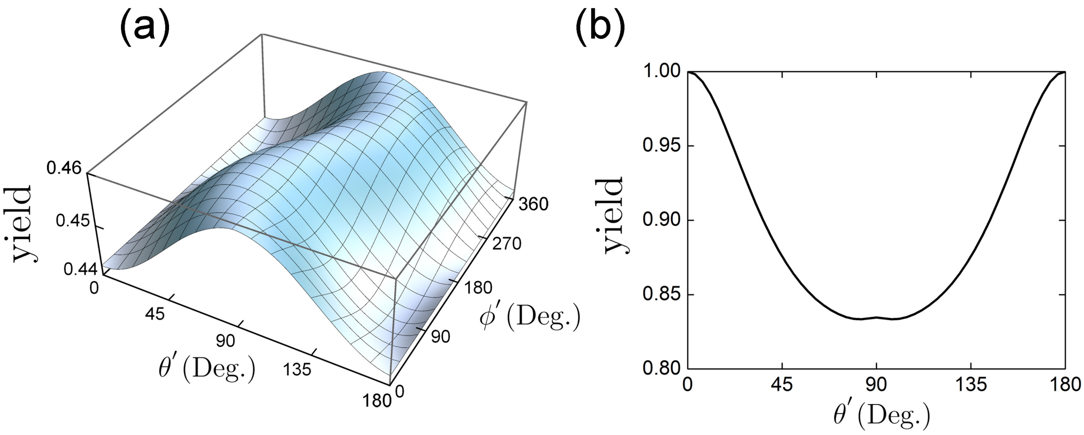

Figure S1: Cryptochrome signalling yield due to a magnetic field strength of 0.5 G. (a) shows the orientational dependence of cryptochrome activation yield in the external magnetic field, as determined in (3). (b) shows the relative duration of cryptochrome dark reaction as established in (4).

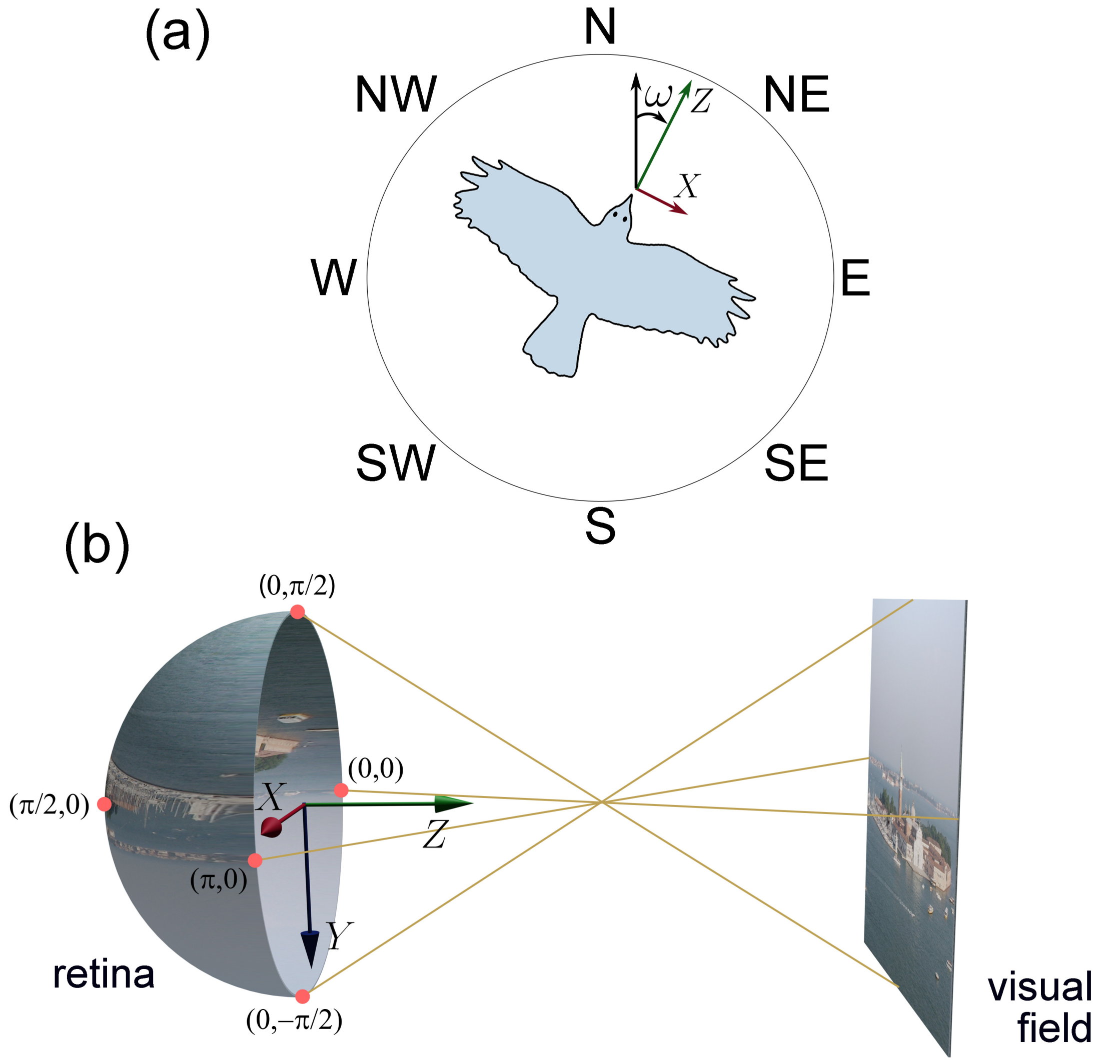

Figure S2: The angle ω characterizes the turn of a bird with respect to magnetic North (a). $\omega = 0°$ corresponds to the the bird flying directly towards magnetic North. (b) shows schematically the inverted projection of a visual field onto the retina of a bird's eye. The longitude and latitude coordinates (λ, ϕ) are indicated for several points on the retina. The (X, Y, Z) coordinate frame, associated with the retina, is also shown.

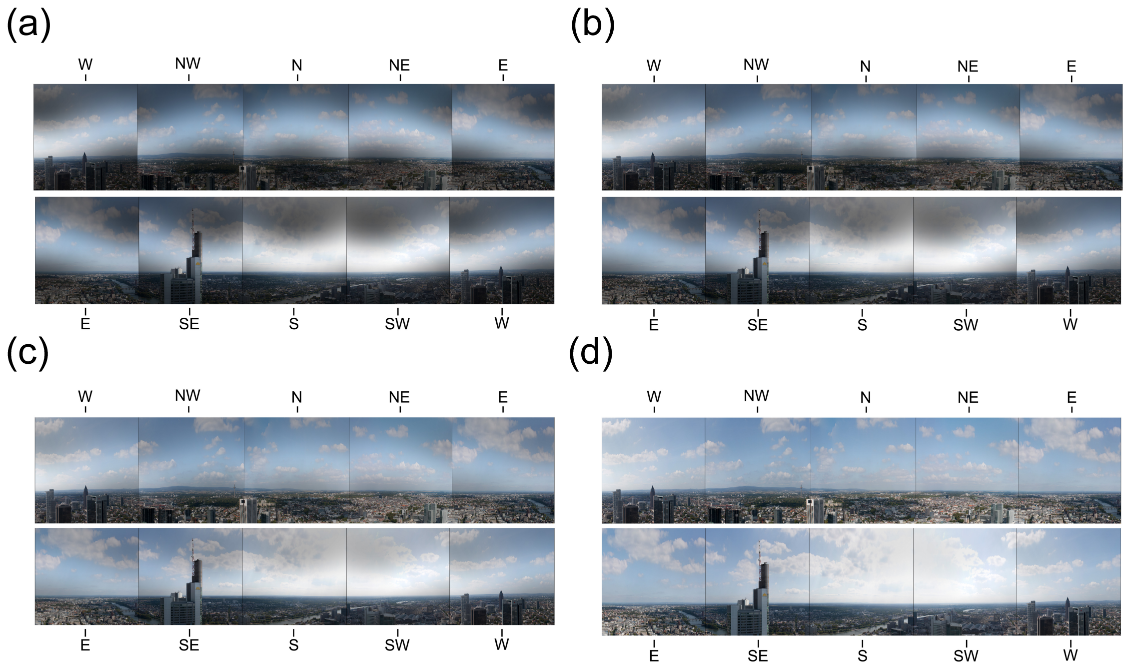

Figure S3: Modulation of the visual field through the geomagnetic field for a bird flying at day time. The modulation patterns correspond to that shown in Fig. 5 and are calculated for different wiggling regimes of cryptochrome, characterized by the parameter ε_0 (see Eq. (5)): (a) $\varepsilon_0 = 100$ (wiggling with small amplitude); (b) $\varepsilon_0 = 10$; (c) $\varepsilon_0 = 3$; (d) $\varepsilon_0 = 1$ (wiggling with large amplitude).

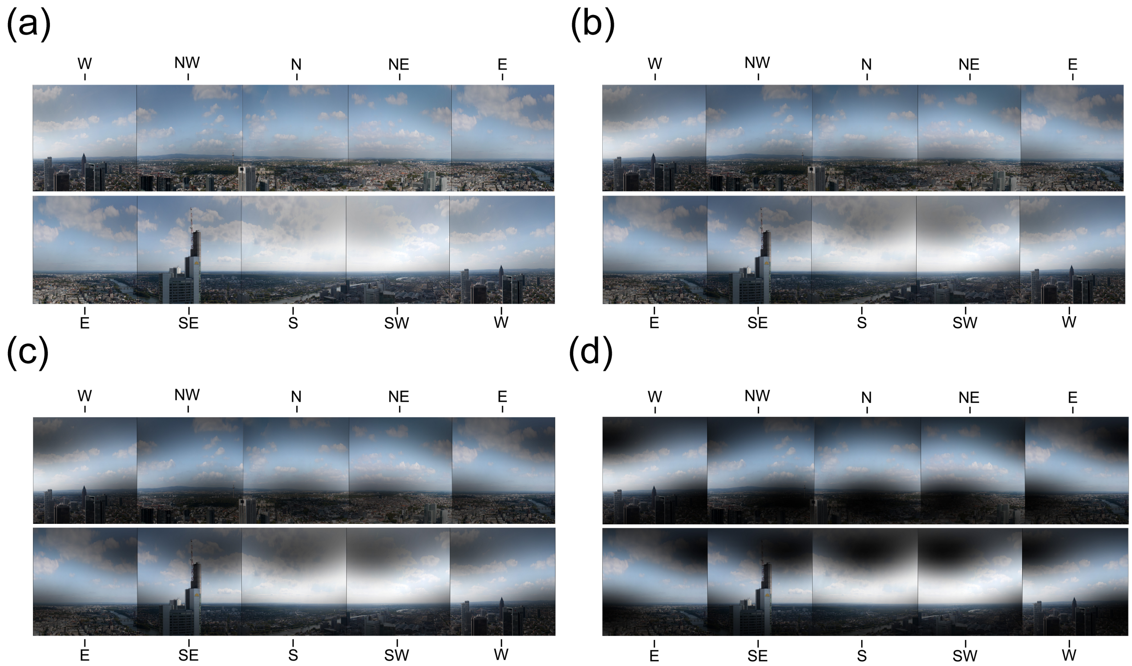

Figure S4: Modulation of the visual field through the geomagnetic field, evaluated as in Fig. 5, for a bird flying at day time, calculated for $\varepsilon_0 = 3$, assuming different numbers of cryptochrome activation cycles: (a) $\eta = 1$; (b) $\eta = 2$; (c) $\eta = 3$; (d) $\eta = 5$.

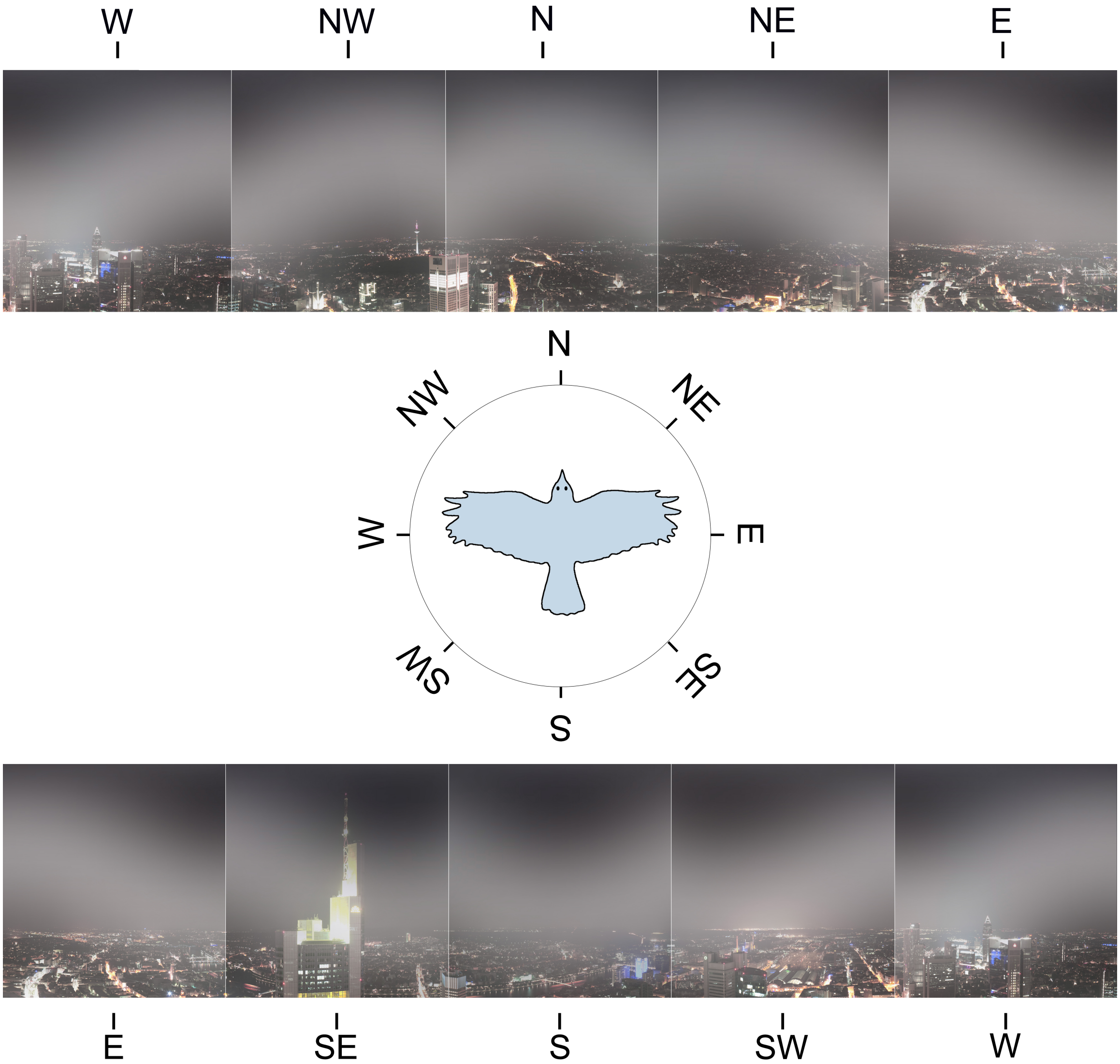

Figure S5: Nighttime panoramic view at Frankfurt am Main, Germany, modified through the magnetic filter function defined in Eq. (S24). The modulation patterns have been evaluated as in Fig. 5.