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Supporting Material

Anomalous diffusion reports on the interaction of misfolded proteins with the quality control machinery in the endoplasmic reticulum

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Anomalous diffusion reports on the interaction of misfolded proteins with the quality control machinery in the endoplasmic reticulum – Supplement

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I. ANOMALOUS DIFFUSION: FCS VS. SPT SIMULATIONS

One may wonder whether fluorescence correlation spectroscopy (FCS) and single particle tracking (SPT) yield the same information about anomalous diffusion. We concentrate here on the case of obstructed diffusion, a more general discussion on stochastic processes with stationary and non-stationary increments may be found in Ref. [1].

To demonstrate that ensemble-averaged SPT data yield the same information as the accompanying FCS data, we have performed simulations as described in the main text and we calculated FCS curves as described in Ref. [1]. For simplicity, we have concentrated on the extreme cases of a free diffusional motion and strongly obstructed diffusion with immobile obstacles.

Fitting the mean square displacement (MSD) of the SPT data (Fig. S1, left) and using these parameters to calculate an analytical autocorrelation function yields a very good agreement with the numerically obtained FCS data (Fig. S1, right). Slight deviations for large time lags τ in the FCS curve indicate why SPT is the more favorable approach for us: To obtain a reasonable statistics of the autocorrelation function at times $\tau > 0.1$s, one needs to simulate the particle motion for more than 100s. Hence, the FCS simulation is computationally more expensive than analyzing SPT data but both yield the same information. We therefore have restricted our simulations in the main text to SPT analysis.

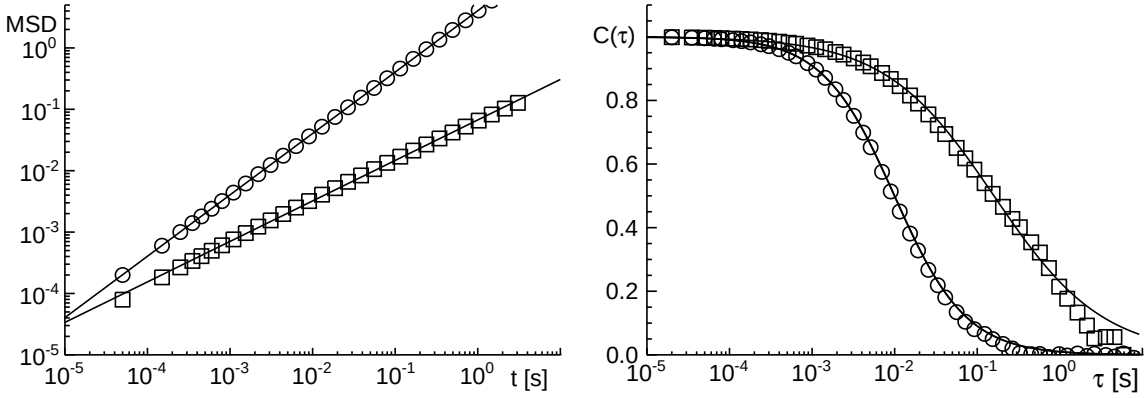

FIG. S1 Left: Mean square displacement (MSD) in μm^2 for free diffusional motion (circles) and obstructed diffusion with fixed obstacles (squares). Full lines are best fits. Right: Using the results of the fits to the MSD as parameters for the autocorrelation function reveals an almost perfect agreement with the numerically obtained FCS curves (symbols as before). Thus, SPT and FCS yield the same information.

II. DERIVATION OF THE FITTING FUNCTION

Here, we would like to sketch out the derivation of the FCS fitting function (Eq.1, main manuscript) and its relation to previous formulations. As stated in in the main text, we assume the random walk to be described by a Gaussian propagator with a time-dependent diffusion coefficient $D = \Gamma t^{\alpha-1}$, i.e.

$$G(x, x', \tau) = \exp\left(-\frac{(x-x')^2}{4\Gamma\tau^\alpha}\right) / \sqrt{4\pi\Gamma\tau^\alpha} \ . \tag{1}$$

This propagator shows a MSD $\langle r^2(t)\rangle = 4\Gamma t^\alpha$ with Γ being the generalized diffusion coefficient with units [length^2/time$^\alpha$].

Calculating the autocorrelation function requires to solve the integrals $C(\tau) = \int dx \int dx' I(x) I(x') G(x, x', \tau)$ for each spatial direction, with $I(x)$ denoting the Gaussian beam profile (width r_0). As a result in two dimensions, one obtains

$$C(\tau) = \frac{1}{1 + 4\Gamma\tau^\alpha / r_0^2} \tag{2}$$

which reduces to Eq.1 when replacing $r_0^2/(4\Gamma)$ (units [time$^\alpha$]) by a characteristic time to the power α, i.e. $\tau_D^\alpha = r_0^2/(4\Gamma)$. With this definition, τ_D measures the half-time of the autocorrelation decay (as in the case of normal diffusion). Moreover, τ_D converges for $\alpha \to 1$ towards the familiar expression $\tau_D = r_0^2/(4D)$.

References

[1] J. Szymanski & M. Weiss, Phys. Rev. Lett. **103**, 038102 (2009).