Supporting Information

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SI Text

In this supporting information, we first explain how to derive the conditions for persistence, microbial coexistence, and negative plant–soil feedback (PSF) at steady-state (Section 1 and Tables S2 and S3). On the basis of these derivations, we show supplemental results of Figs. 2 and 3 (Fig. S2). Second, we show the robustness of the buffering effect of microbial diversity on nutrient pool size, regardless of microbial food web structures (Section 2 and Fig. S1). Third, we show how parameters related to plant functions (primary production and litter production) and microbial functions (decomposition and immobilization) affect the roles of microbial diversity in facilitating negative PSF (Section 3 and Figs. S3 and S4). Fourth, we show a typical example of the time-evolution of the plant–microbe–soil feedback (PMSF) system, to demonstrate how microbial diversity facilitates plant species coexistence (Section 4 and Fig. S5). Finally, we show the global impact of PMSF on the plant and microbial community compositions (Section 5 and Fig. S6).

Section 1: Deriving Conditions for Persistence, Microbial Coexistence, and Negative PSF. For each equilibrium in a system with a single plant species P_i ($i = L$ or N) and a single microbial group M_j ($j = R$ or S), we obtain the following quadratic equation for the equilibrium nutrient level N^*,

$$0 = \left(1 + \frac{a_i f_i}{m'_{Mj}}\right)N^{*2} + \left\{b_i - T_N + \frac{m'_{Mj}}{c_D}\left(\frac{f_i}{k_{Rj}} + \frac{1-f_i}{k_{Sj}}\right) + \frac{m_P}{m'_{Mj}}\left(b_i - \frac{a_i}{r_i}\right)\right\}N^* - \frac{b_i m_P}{r_i}\left(1 + \frac{m_P}{m'_{Mj}}\right) \equiv F_{P_i M_j}(N^*),$$

which has a unique positive root. This derivation reveals that the ratio of microbial mortality and immobilization efficiency ($1/m'_{Mj} \equiv \mathrm{e}_{Mj}/m_{Mj}$) determines nutrient cycling: both the lower mortality and higher immobilization efficiency imply the higher "immobilization capacity" of microbes. We also obtain the equilibrium values of the other components,

$$P_i^* = 1 - \frac{m_P}{r_i N^*},\ D_R^* = \frac{m'_{Mj} f_i}{c_D k_{Rj}},\ D_S^* = \frac{m'_{Mj}(1-f_i)}{c_D k_{Sj}},\text{ and}$$
$$M_j^* = \frac{b_i m_P + a_i N^*}{m'_{Mj}}\left(1 - \frac{m_P}{r_i N^*}\right)$$

We calculate the persistence condition for the equilibrium with P_i and M_j (i.e., $N^* > m_P/r_i$) from the inequality $F_{P_i M_j}(m_P/r_i) < 0$, as $f_i > f^E_{P_i M_j}$ (Table S2). This is the condition for positive equilibria (i.e., a persistence condition). The system cannot persist when starting from lower litter decomposability than these thresholds, that is, if $f_L < f^E_{P_L M_R}$ (≈ 0.45) or $f_N < f^E_{P_N M_R}$ (≈ 0.44), as shown in Figs. 3 and 4*A*.

For the equilibrium with P_i, M_R, and M_S, we obtain the following quadratic equation for the equilibrium nutrient level N^*,

$$\begin{aligned}0 = &\left[1 + a_i\left\{\frac{(k_{SS}-k_{SR})f_i}{k_{SS}m'_{MR} - k_{SR}m'_{MS}} + \frac{(k_{RR}-k_{RS})(1-f_i)}{-k_{RS}m'_{MR} + k_{RR}m'_{MS}}\right\}\right]N^{*2}\\ &+ \left[b_i - T_N + \frac{(k_{RR}-k_{SR})m'_{MS} - (k_{RS}-k_{SS})m'_{MR}}{c_D(k_{RR}k_{SS} - k_{RS}k_{SR})}\right.\\ &+ m_P\left(b_i - \frac{a_i}{r_i}\right)\left\{\frac{(k_{SS}-k_{SR})f_i}{k_{SS}m'_{MR} - k_{SR}m'_{MS}}\right.\\ &\left.\left.+ \frac{(k_{RR}-k_{RS})(1-f_i)}{-k_{RS}m'_{MR} + k_{RR}m'_{MS}}\right\}\right]N^*\\ &- \frac{b_i m_P}{r_i}\left[1 + m_P\left\{\frac{(k_{SS}-k_{SR})f_i}{k_{SS}m'_{MR} - k_{SR}m'_{MS}}\right.\right.\\ &\left.\left.+ \frac{(k_{RR}-k_{RS})(1-f_i)}{-k_{RS}m'_{MR} + k_{RR}m'_{MS}}\right\}\right] \equiv F_{P_i M_R M_S}(N^*),\end{aligned}$$

noting that it has unique positive root. We also obtain the equilibrium values of the other components,

$$P_i^* = 1 - \frac{m_P}{r_i N^*},\ D_R^* = \frac{k_{SS}m'_{MR} - k_{SR}m'_{MS}}{c_D(k_{RR}k_{SS} - k_{RS}k_{SR})},$$
$$D_S^* = \frac{-k_{RS}m'_{MR} + k_{RR}m'_{MS}}{c_D(k_{RR}k_{SS} - k_{RS}k_{SR})},$$

$$M_R^* = (b_i m_P + a_i N^*)P_i^*\left\{\frac{k_{SS}f_i}{k_{SS}m'_{MR} - k_{SR}m'_{MS}} - \frac{k_{SS}(1-f_i)}{-k_{RS}m'_{MR} + k_{RR}m'_{MS}}\right\},\text{ and}$$

$$M_R^* = (b_i m_P + a_i N^*)P_i^*\left\{-\frac{k_{SR}f_i}{k_{SS}m'_{MR} - k_{SR}m'_{MS}} + \frac{k_{RR}(1-f_i)}{-k_{RS}m'_{MR} + k_{RR}m'_{MS}}\right\}$$

At this equilibrium, the persistence condition for plants ($N^* > m_P/r_i$) calculated from $F_{P_i M_R M_S}(m_P/r_i) < 0$ is equivalent to $\frac{(k_{RR}-k_{SR})m'_{MS} - (k_{RS}-k_{SS})m'_{MR}}{c_D(k_{RR}k_{SS} - k_{RS}k_{SR})} < T_N - \frac{m_P}{r_i}$. For a nonzero biomass of organic nutrients (D_{R}^* and D_{S}^*), $k_{SS}m'_{MR} - k_{SR}m'_{MS}$ and $k_{RS}m'_{MR} - k_{RR}m'_{MS}$ should be positive. For a nonzero biomass of microbes (M_R^* and M_S^*), the inequality $f^{**}_{M_R \to P_i M_S} < f_i < f^{**}_{M_S \to P_i M_R}$ should be satisfied (Table S2), which requires $f^{**}_{M_R \to P_i M_S} < f^{**}_{M_S \to P_i M_R}$. Note that some of these conditions are not satisfied in region $\mathrm{X_M}$ in Fig. S3 and S4.

The condition for P_L or P_N to yield a negative PSF (i.e., $N^* < N_L^{**}$ or $N^* > N_N^{**}$) is obtained from $F_{P_L M_j}(N_L^{**}) > 0$ or $F_{P_N M_j}(N_N^{**}) < 0$, respectively, in a system with a single microbe M_j. We use the same method in a system with two microbes by focusing on $F_{P_i M_R M_S}(N^*)$. With this method, we can obtain the threshold values that separate positive and negative PSF (Tables S2 and S3). From the coefficients of the quadratic equation for N^*, we also find that N^* always increases with the litter decomposability f_i in a system with P_i and M_j. However, for a system with P_i, M_R, and M_S, N^* increases with f_i for $m'_{MR} > m'_{MS}$, decreases with f_i for $m'_{MR} < m'_{MS}$, or is not affected by f_i for $m'_{MR} = m'_{MS}$. This is why the conditions for a negative PSF in a system with P_i, M_R, and M_S depend on the difference between m'_{MR} and m'_{MS}. When both plant species cause a negative PSF, the two species are likely to coexist for a system with one or two microbial groups, respectively (Tables S2 and S3).

To understand how plant litter decomposability affects the competitive outcome in the microbial community, we investigated the condition for D_R-preferring microbes to successfully invade the system under equilibrium with a single plant species P_i and D_S-preferring microbes M_S. The invasion succeeds ($[\frac{1}{M_R}\frac{dM_R}{dt}]_{P_i=P_i^*,\ M_S=M_S^*} > 0$) when a fraction f_i of rapidly decomposable organic nutrient (D_R) in the litter from P_i is sufficiently high ($f_i > f^{**}_{M_R \to P_i M_S}$) (Table S2), which can be analytically derived from the equilibrium. Similarly, D_S-preferring microbes can invade the system under the equilibrium with plant species i and D_R-preferring microbes when a fraction f_i is sufficiently small ($f_i < f^{**}_{M_S \to P_i M_R}$). If we focus on a reasonable situation ($0 < f^{**}_{M_R \to P_i M_S} < f^{**}_{M_S \to P_i M_R} < 1$), we obtain the coexistence condition for microbes (Table S2), noting that this condition is equivalent to the nonzero biomass of two microbes in the equilibrium with plant species i and two microbes.

In addition, by solving $F_{P_L M_j}(N^*) = 0$ and $F_{P_L M_R M_S}(N^*) = 0$, we can plot the relationship between the litter decomposability f_L, nutrient pool size, biomass of detritus, and relative abundance of M_R (Fig. 2 *A* and *B*). The relationship between f_L and N^* with $m'_{MR} \neq m'_{MS}$ is shown in Fig. S2*A*, indicating the buffering effect of microbial diversity on the nutrient pool size. Similarly, by setting a specific parameter set (default values in Table S1), we can calculate the threshold values shown in Tables S2 and S3 and drawn in Fig. 3 (main text). With another set of parameters ($e_{MR} = e_{MS} = 0.45$), we can show that the reduction of N^* by the dominance of M_R shifts the sign of PSF from positive to negative in a system with P_L with high decomposability (shaded range in Fig. S2*B*). However, simultaneously, such a reduction of N^* by the presence of M_R can also shift the sign of PSF from negative to positive in a system of P_N with high decomposability (shaded range in Fig. S2*C*).

Although it would also be valuable to derive the invasion conditions for the competing microbial group in the equilibrium with two plant species, it is difficult to analytically derive simple conditions for this scenario, such as those shown in Table S2. Similarly, it is difficult to analytically derive the equilibrium with P_L, P_N, M_R, and M_S. Realization of these equilibria was checked as a part of the numerical calculations [Fig. 4 (main text) and Fig. S6].

Section 2: Plant Control of Nutrient Pool Size Under Various Microbial Food Web Structures. In Fig. 2*A*, the nutrient pool size is not tightly coupled with plant litter decomposability when two microbial functional groups coexist with a single plant species. We checked the robustness of this prediction under various microbial food web structures. In place of Eqs. **5** and **6**, we used the following equations for the microbial dynamics (M_j), microbial consumers (C_j), and top predator (C_T) on the consumers. Nonassimilated parts of the nutrients by trophic interactions are assumed to be mineralized into the nutrient pool (N), such that the total amount of nutrient in the system is kept constant (i.e., $b_L P_L + b_N P_N + D_R + D_S + M_R + M_S + C_R + C_S + C_T + N = const \equiv T_N$). With these assumptions, we obtain:

$$dM_j/dt = c_D e_{Mj}(k_{Rj} D_R + k_{Sj} D_S) M_j - l_{Mj}\ (j\ =\ R,\ S)$$

$$dC_j/dt = g_{Cj} - l_{Cj}\ (j\ =\ R,\ S)$$

$$dC_T/dt = g_{CT} - l_{CT}$$

where l_{Mj}, l_{Cj}, and l_{CT} are loss terms by predation, and g_{Cj} anc g_{CT} are growth terms.

Four distinct types of microbial food webs are prepared: (*i*) M_R and M_S only, (*ii*) M_R and M_S with a generalist consumer C_R that utilizes both M_R and M_S, (*iii*) M_R and M_S with specialist consumers C_R and C_S for each, and (*iv*) M_R and M_S, specialist consumers C_R and C_S, and generalist top predator C_T. Case (*i*) has the same settings as Fig. 2*A* (main text). For trophic interactions between microbes (M_j) and consumers (C_j), we assume a Lotka-Volterra (LV) or a donor-controlled (DC) interaction type. For LV, the loss term l_{Mj} is proportional to the product of the microbial biomass and the corresponding consumer. For DV, the loss term is proportional to the microbial biomass. The DC interaction would realistically represent the trophic interaction between fungi and their consumers in soil (i.e., for M_S in our model), because fungal biomasses tend to be controlled from the bottom up (1). We prepared all possible combinations for LV and DC interactions (Fig. S1). For example, in "DC-LV" scenarios (Fig. S1), the loss terms l_{MR} and l_{MS} are determined as DC and LV, respectively. For trophic interactions between microbial consumers and the top predator, we assumed LV interactions.

The relative abundance of D_R-preferring microbes ($M_R/[M_R + M_S]$) with a higher preference for readily decomposable litter increases with plant litter quality (f_L), regardless of the microbial food web structure. The nutrient pool size correlates poorly with litter quality in a system with functional microbial diversity compared with a system with single microbial groups (Fig. S1 *A–E*, *I*, *M*) or the availability falls to intermediate levels (Fig. S1 *H* and *L*), suggesting the buffering effect of the microbial diversity. Although exceptions are shown in Fig. S1 *F*, *G*, *J*, and *K*, where the nutrient pool size is largest in a system with microbial diversity, the responses of this microbial community structure are unrealistic; the relative abundance of M_R does not change with litter quality (Fig. S1 *F* and *J*) or M_R does not dominate the community with a very high litter quality (Fig. S1 *G* and *K*). These results demonstrate that the buffering effect of the functional microbial diversity on the plant litter control of the nutrient pool size is a robust prediction under the reasonable assumption that D_S-preferring microbes M_S (e.g., fungi) are donor controlled (Fig. S1 *D*, *E*, *H*, *I*, *L*, and *M*).

Section 3: Parameter Dependence of the Region for Litter Decomposabilities to Cause a Negative PSF in a System with Microbial Diversity. To check whether a system with microbial functional diversity is more likely to cause a negative PSF than a system without microbial diversity, we numerically evaluate the ranges of litter decomposabilities (f_L and f_N) that cause a negative PSF. Using the threshold values shown in Tables S2 and S3, we calculate these ranges for systems with a single microbial group M_R or M_S, and for a system with microbial diversity. For each system with P_i, M_R, and/or M_S, we define $\Delta f_{i,X} = \max[0,\ f_{i,X,\max} - f_{i,X,\min}]$, where plant species i ($= L$ or N) causes a negative PSF when $f_{i,X,\min} < f_i < f_{i,X,\max}$ in a system with M_R only, M_S only, or both M_R and M_S ($X = R$, S, or RS, respectively). A larger $\Delta f_{i,X}$ indicates that a negative PSF is more likely to occur.

We also define two indices for evaluating the roles of microbial diversity in enhancing negative PSF, $\theta_{i,SF} = \Delta f_{i,RS} - \max(\Delta f_{i,R}, \Delta f_{i,S})$ and $\theta_{i,MF} = \Delta f_{i,RS} - (\Delta f_{i,R} + \Delta f_{i,S})/2$, where $i = L$ or N. When $\theta_{i,SF}$ is positive, the microbial functional diversity broadens the region for which f_i can cause a negative PSF in the P_i-dominant plant community, implying a strong facilitation of negative PSF. When $\theta_{i,MF}$ is positive, the likelihood of a negative PSF occurring in a system with microbial diversity ($\Delta f_{i,RS}$) is larger than expected in a hypothetical system whereby two microbial groups equally affect the sign of PSF ($[\Delta f_{i,R} + \Delta f_{i,S}]/2$). We define this state as a moderate level of facilitation. The aim here is to check the parameter sensitivity of $\theta_{i,SF}$ and $\theta_{i,MF}$.

It is ecologically reasonable to exclude parameter values for which the equilibrium with two microbial groups does not exist for either of the plant species with any litter decomposability ($0 \leq f_i \leq 1$). Similarly, it is not meaningful to evaluate $\theta_{i,SF}$ and $\theta_{i,MF}$ when $\Delta f_{i,RS} = \Delta f_{i,R} = \Delta f_{i,S} = 0$ or $\Delta f_{i,RS} = \Delta f_{i,R} = \Delta f_{i,S} = 1$, because the sign of PSF is independent of the litter decomposability and microbial community composition in these cases. The pseudocode for checking the parameter dependence of $\theta_{i,SF}$ and $\theta_{i,MF}$ is:

1. Assign a new parameter set;
2. Check whether "*Persistent conditions for a system with* P_i*,* M_R*, and* M_S" (Table S2) is satisfied for both of P_L and P_N; if satisfied, go to step 3; if not, categorize the parameter set as "$\mathbf{X_M}$" and go back to step 1;
3. Check whether $\Delta f_{L,RS} = \Delta f_{L,R} = \Delta f_{L,S} = 0$ or 1, and $\Delta f_{N,RS} = \Delta f_{N,R} = \Delta f_{N,S} = 0$ or 1; if both are satisfied, categorize the parameter set as "$\mathbf{X_P}$" and go back to step 1; if not, go to step 4;
4. Calculate $\Delta f_{i,X}$ where $i = L$ or N and $X = R$, S, or RS according to the following algorithm;

$$\Delta f_{L,X} = \max\left[0, \min\left(f^{**}_{P_N \to P_L M_X}, 1\right) - \max\left(0, f^{E}_{P_L M_X}\right)\right] \text{ for } X = R \text{ or } S;$$

$$\Delta f_{N,X} = \max\left[0, 1 - \max\left(0, f^{E}_{P_N M_X}, f^{**}_{P_N M_X}\right)\right] \text{ for } X = R \text{ or } S;$$

$$\Delta f^{S}_{L,RS} = \max\left[0, \min\left(f^{**}_{M_R \to P_i M_S}, f^{**}_{P_N \to P_L M_S}, 1\right) - \max\left(0, f^{E}_{P_L M_S}\right)\right];$$

$$\text{If } m'_{MR} < m'_{MS}, \Delta f^{C}_{L,RS} = \max\left[0, \min\left(f^{**}_{M_S \to P_i M_R}, 1\right) - \max\left(0, f^{**}_{P_N \to P_L M_R M_S}, f^{**}_{M_R \to P_i M_S}\right)\right];$$

$$\text{If } m'_{MR} = m'_{MS} \text{ and } \phi^{**}_{P_N \to P_L M_R M_S} < 0,$$

$$\Delta f^{C}_{L,RS} = \max\left[0, \min\left(f^{**}_{M_S \to P_i M_R}, 1\right) - \max\left(0, f^{**}_{M_R \to P_i M_S}\right)\right];$$

$$\text{If } m'_{MR} = m'_{MS} \text{ and } \phi^{**}_{P_N \to P_L M_R M_S} \geq 0, \Delta f^{C}_{L,RS} = 0;$$

$$\text{If } m'_{MR} > m'_{MS}, \Delta f^{C}_{L,RS} = \max\left[0, \min\left(f^{**}_{M_S \to P_i M_R}, f^{**}_{P_N \to P_L M_R M_S}, 1\right) - \max\left(0, f^{**}_{M_R \to P_i M_S}\right)\right];$$

$$\Delta f^{R}_{L,RS} = \max\left[0, \min\left(f^{**}_{P_N \to P_L M_R}, 1\right) - \max\left(0, f^{E}_{P_L M_R}, f^{**}_{M_S \to P_i M_R}\right)\right];$$

$$\Delta f_{L,RS} = \Delta f^{S}_{L,RS} + \Delta f^{C}_{L,RS} + \Delta f^{R}_{L,RS};$$

$$\Delta f^{S}_{N,RS} = \max\left[0, \min\left(f^{**}_{M_R \to P_i M_S}, 1\right) - \max\left(0, f^{E}_{P_N M_S}, f^{**}_{P_L \to P_N M_S}\right)\right];$$

$$\text{If } m'_{MR} < m'_{MS}, \Delta f^{C}_{N,RS} = \max\left[0, \min\left(f^{**}_{M_S \to P_i M_R}, f^{**}_{P_L \to P_N M_R M_S}, 1\right) - \max\left(0, f^{**}_{M_R \to P_i M_S}\right)\right];$$

$$\text{If } m'_{MR} = m'_{MS} \text{ and } \phi^{**}_{P_L \to P_N M_R M_S} < 0$$

$$\Delta f^{C}_{N,RS} = \max\left[0, \min\left(f^{**}_{M_S \to P_i M_R}, 1\right) - \max\left(0, f^{**}_{M_R \to P_i M_S}\right)\right];$$

$$\text{If } m'_{MR} = m'_{MS} \text{ and } \phi^{**}_{P_L \to P_N M_R M_S} \geq 0, \Delta f^{C}_{N,RS} = 0;$$

$$\text{If } m'_{MR} > m'_{MS}, \Delta f^{C}_{N,RS} = \max\left[0, \min\left(f^{**}_{M_S \to P_i M_R}, 1\right) - \max\left(0, f^{**}_{M_R \to P_i M_S}, f^{**}_{P_L \to P_N M_R M_S}\right)\right];$$

$$\Delta f^{R}_{L,RS} = \max\left[0, 1 - \max\left(0, f^{E}_{P_N M_R}, f^{**}_{M_S \to P_i M_R}, f^{**}_{P_L \to P_N M_R}\right)\right];$$

$$\Delta f_{N,RS} = \Delta f^{S}_{N,RS} + \Delta f^{C}_{N,RS} + \Delta f^{R}_{N,RS};$$

5. Calculate $\theta_{i,SF}$ and $\theta_{i,MF}$;
6. Define a small positive constant ZERO = 1.0e-10;
7. If $\theta_{L,SF}$ > ZERO or $\theta_{N,SF}$ > ZERO, categorize the parameter set as "strong facilitation (**SF**)"; else if $\theta_{L,MF}$ > ZERO or $\theta_{N,MF}$ > ZERO, categorize the parameter set as "moderate facilitation (**MF**)"; else, categorize the parameter set as "no facilitation (**NF**)"; go back to step 1.

We assume a small positive constant (step 6) to reduce the numerical error in computing $\theta_{i,SF}$ and $\theta_{i,MF}$.

Using this method, we check whether the microbial functional diversity increases the likelihood of obtaining a negative PSF compared with systems without microbial diversity (Fig. S3). In region **SF** (or region **MF**), microbial diversity strongly (or moderately) facilitates a negative PSF in either in a P_L- or P_N-dominant community. In region **NF**, microbial diversity does not facilitate a negative PSF in either community. Fig. S3*A* shows that a strong facilitation of negative PSF is possible with intermediate values of microbial mortality and assimilation efficiency (i.e., intermediate levels of immobilization capacity) (region **SF**), whereas moderate facilitation is achieved with a wide range of parameter values (region **MF**). Even when differences in immobilization capacity between M_R and M_S are considered, strong or moderate facilitation is realized with a wide range of parameter values (Fig. S3*B*). Region **NF** (Fig. S3 *A* and *B*), in which the facilitation of negative PSF does not occur, is realized in a very limited range of parameter values, suggesting the prevalence of the facilitation of a negative PSF by microbial diversity. Strong or moderate facilitation is also observed in a wide range of parameter values that are related to plant production (Fig. S3*C*, region **SF** + region **MF**). A similar pattern is observed even when interspecific differences in nutrient uptake ability (b_i and a_i) between two plant species are considered (Fig. S3*D*).

We also check whether microbial diversity broadens the region where both plant species cause a negative PSF, by focusing on the quantity $\Delta f_{L,X} \Delta f_{N,X}$ ($X = R$, S, or RS). This quantity roughly corresponds to the size of the plant coexistence region C in Fig. 4 (main text). We define two indices for evaluating the roles of microbial diversity in enhancing the occurrence of negative PSF, $\theta_{SFC} = \Delta f_{L,RS} \Delta f_{N,RS} - \max(\Delta f_{L,R} \Delta f_{N,R}, \Delta f_{L,S} \Delta f_{N,S})$, and $\theta_{SSC} = \Delta f_{L,RS} \Delta f_{N,RS} - \min(\Delta f_{L,R} \Delta f_{N,R}, \Delta f_{L,S} \Delta f_{N,S})$. When θ_{SFC} is positive, the microbial functional diversity broadens the region for the *mutual* negative PSF, which leads to plant coexistence. When θ_{SSC} is negative, the region for *mutual* negative PSF in a system with microbial diversity is smaller than that in a system with either M_R or M_S alone, implying that microbial diversity hinders plant coexistence. The aim here is to check the parameter sensitivity of θ_{SFC} and θ_{SSC}.

Steps 1–4 of the pseudocode for checking the parameter dependence of θ_{SFC} and θ_{SSC} are the same as those of the pseudocode for drawing Fig. S3. The rest of the steps are:

5. Calculate θ_{SFC} and θ_{SSC};

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6. Define a small positive constant ZERO = 1.0e-10;
7. If $\theta_{SFC}$ > ZERO, categorize the parameter set as "strong facilitation of coexistence (SFC)"; else if $\theta_{SSC}$ < −ZERO, categorize the parameter set as "substantial suppression of coexistence (SSC)"; else if $-\text{ZERO} < \Delta f_{L,RS}\Delta f_{N,RS} - \Delta f_{L,R}\Delta f_{N,R} < \text{ZERO}$, categorize the parameter set as "$\text{M}_\text{R}$-controlling PSF (RC)"; else if $-\text{ZERO} < \Delta f_{L,RS}\Delta f_{N,RS} - \Delta f_{L,S}\Delta f_{N,S} < \text{ZERO}$, categorize the parameter set as "$\text{M}_\text{S}$-controlling PSF (SC)"; else, categorize the parameter set as "middle case (MC)"; go back to step 1;
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With this method, we can also check whether microbial functional diversity can promote plant coexistence and examine its parameter dependence (Fig. S4). In region **SFC**, the region for which both f_L and f_N cause a negative PSF in a system with microbial diversity is larger than that in a system with M_R or M_S alone. In region **RC** (or region **SC**), the region for which both f_L and f_N cause a negative PSF in a system with microbial diversity has almost the same size as that in a system with M_R only (or M_S only), implying that PSF is controlled by a single microbial group (see also Fig. S6 *A* and *E*). In region **MC**, the likelihood of plant coexistence is between that in a system with M_R only and that in a system with M_S only. Microbial diversity never appears to hinder plant coexistence, because region SSC was not seen in our range of parameter values (Fig. S4).

Fig. S4*A* indicates that a strong facilitation of plant coexistence is possible with intermediate values of microbial mortality and assimilation efficiency (i.e., intermediate levels of immobilization capacity) (region **SFC**). Even when differences in immobilization capacity between M_R and M_S are considered, such facilitation is realized in a wide range of parameter values (Fig. S4*B*). Fig. S4*C* shows that a strong facilitation of microbial diversity on plant coexistence is observed in a wide range of parameter values (region **SFC**). A similar pattern is observed even when we consider interspecific differences in nutrient uptake ability (b_i and a_i) between two plant species (Fig. S4*D*).

Section 4: Facilitation of Plant Coexistence Through Microbial Diversity. The facilitation of plant coexistence by microbial diversity can be easily understood through temporal dynamics with a single microbial group (Fig. S5). Here we show examples with a specific combination of litter decomposabilities ($f_L = 0.6, f_N = 0.5$). As shown in Fig. 4 *A* and *B* (main text), either P_N or P_L dominates the community with a single microbial group M_R or M_S, respectively. However, the coexistence of two microbial groups enables the coexistence of plants (Fig. 4*C*). Consider a system that starts from M_R only. Without microbial diversity, P_N causes a positive PSF and prevents the invasion of P_L (at $t = 1{,}000$; Fig. S5*A*). This is because f_N (= 0.5) is less than$f^{**}_{P_L \to P_N M_S}$ (≈0.59); f_N does not satisfy the "*Condition for negative PSF in a system with* P_i *and* M_j" (Table S2). However, because f_N satisfies the "*Coexistence condition for microbes in a system with* P_i" (Table S2) ($0.33 = f^{**}_{M_R \to P_i M_S} < f_\text{L} < f^{**}_{M_S \to P_i M_R} = 0.66$), M_S can invade the system and increase the nutrient pool size ($t > 2500$). With the same parameter sets as those in Fig. 4, because the "*Conditions for negative PSF a system with* P_i, M_R, *and* M_S" are satisfied ($\varphi^{**}_{P_L \to P_N M_R M_S} \approx -102.2 < 0$), P_L can invade the system and coexist with P_N ($t > 4{,}000$). Similarly, in a system that starts from M_S only, P_L causes positive PSF and prevents the invasion of P_N (at $t = 1{,}000$, Fig. S5*B*). This is because f_L (= 0.6) is greater than $f^{**}_{P_N \to P_L M_S}$ (≈0.49) and does not satisfy the "*Condition for negative PSF in a system with* P_i *and* M_j" (Table S2). However, once M_R invades the system ($t > 2{,}500$), it reduces the nutrient pool size and P_N can invade the system and coexist with P_L ($t > 4{,}000$). These examples demonstrate how microbial diversity facilitates plant coexistence as a consequence of PMSF.

Section 5: Consequences of PMSF on Plant and Microbial Community Compositions. In Fig. 4*C* (main text), the realized composition of the microbial community is not shown. In this section, we describe all of the consequences of PMSF on the plant and microbial community compositions, on the basis of numerical calculations (Fig. S6). The pseudocode for checking the dependence of the realized composition of plant and microbial communities on litter decomposability is:

```
1. Assign a new parameter set ($f_L, f_N$);
2. Assign one of the two initial conditions:

   $(P_L(0), P_N(0)) = (0.2, 0.0)$ or $(0.0, 0.2)$, with $(N(0), D_R(0),$
   $D_S(0), M_R(0), M_S(0)) = (0.01, 0.1, 0.1, 0.5*[T_N - N(0)$
   $- b_L P_L(0) - b_N P_N(0) - D_R(0) - D_S(0)], 0.5*[T_N - N(0)$
   $- b_L P_L(0) - b_N P_N(0) - D_R(0) - D_S(0)]);$

3. Assign a minimum threshold: $n_{MIN}$ = 1.0e-6;
4. Simulate the time evolution of the system governed by Eqs. 1–6 by a numerical integration method (e.g., fourth-order Runge-Kutta method with a fixed interval) from $t = 0$ to $t$ = 5.0e+4;
5. When $10{,}000.0 < t < 10{,}000.1$ and when $20{,}000.0 < t < 20{,}000.1$, disturb the system as follows:

   $P_L(t)$ += $n_{MIN}$; $P_N(t)$ += $n_{MIN}$; $M_R(\text{t})$ += $n_{MIN}$;
   $M_S(t)$ += $n_{MIN}$; If $N(t) > 4*n_{MIN}$, then
   $N(t)$ -= $4*n_{MIN}$; Else if $D_R(t) > 4*n_{MIN}$, then
   $D_R(t)$ -= $4*n_{MIN}$; Else, $D_S(t)$ -= $4*n_{MIN}$;

6. Calculate the "long-term averages" for all of the variables from $t = 45{,}000$ to $t = 50{,}000$;
7. Evaluate the community compositions of plants and microbes by checking whether the average values of $P_L$, $P_N$, $M_R$, $M_S$ are greater than $n_{MIN}$ or not, respectively;
8. Repeat steps 2–7 for another initial condition
```

Starting from the initial condition with a single plant species (P_L or P_N) (step 1), the system first approaches equilibrium with dominance of the single species at $t = 10{,}000$. By adding a small disturbance (step 5), the invasibility of other combinations of plants and microbes in the system can be checked. The same type of the disturbance is added twice to avoid numerical errors and to reduce the dependence of invasion success on the invasion timing. The values of N, D_R, and D_S are adjusted to keep the total nutrient constant (step 5). By comparing the realized community compositions starting from two different initial conditions (step 2), we can check for the occurrence of bistability. This method allows us to draw Fig. 4 (main text) and Fig. S6. For a system with only M_R or M_S (Fig. 4 *A* and *B*, main text), the initial conditions (step 2) and invasion event (step 5) should be appropriately modified.

Additional explanations are needed to understand the dynamics in some regions of Fig. 4 (main text). In region **X** (Fig. 4*A*, main text) with small f_L and f_N, neither plant species can maintain its population because the litter decomposability is too low to satisfy the persistence condition for a system with P_i and M_j (see Section 1 and Table S2). In region **X** with a very small f_L but with a high f_N, P_N maintains its population but allows the invasion of P_L with very low litter decomposability, resulting in collapse of the system. When f_N is lower than $f^{**}_{P_L \to P_N M_S}$ (≈0.09) (Fig. 4*C*, main text), once P_N dominates plant community, it prevents the invasion of P_L and the system with only P_N is stable. This is why each parameter set falls into region **N**, region **C or N**, or region **L or N**: the dominance of P_N is always one of the stable steady states. When f_N is less than $f^{**}_{P_L \to P_N M_S}$ (≈0.09) and

f_L is lower than $f^{**}_{P_N \to P_L M_R}$ ($\approx$0.72), the system with P_L allows the invasion of P_N. With (f_L,f_N) = (0.6, 0.05), the coexistence of P_L and P_N is possible after the invasion of P_N to the system starting from only P_L, through the periodic fluctuation of nutrient pool size. However, with a slightly different parameter set, (f_L, f_N) = (0.6, 0.0), the invasion of P_N finally excludes P_L and leads to an apparently stable equilibrium. The boundaries between region **C or N** and region **N** are not simple, probably owing to the presence or absence of periodic fluctuations.

Parts *A* and *B*, parts *C* and *D*, and parts *E* and *F* in Fig. S6 correspond to the model with $e_{MR} = e_{MS} = 0.45$, 0.6, and 0.75, respectively. Parts *A*, *C*, and *E* and parts *B*, *D*, and *F* represent the realized community compositions of plants and microbes, respectively. The condition for negative PSF in a system with M_R or M_S roughly determines the possibility of plant coexistence in Fig. S6 *A* or *E*, respectively. The coexistence region for P_L and P_N is separated from other regions by $f_L = f^{**}_{P_N \to P_L M_R}$ ($\approx$0.81) and $f_N = f^{**}_{P_L \to P_N M_R}$ ($\approx$0.72) in Fig. S4*A*, and by $f_L = f^{**}_{P_N \to P_L M_S}$ ($\approx$0.19) and $f_N = f^{**}_{P_L \to P_N M_S}$ (≈ -0.29) in Fig. S6*E*. This result implies that a single microbial group governs the consequences of PMSF, even if two microbial groups coexist. Conversely, in Fig. S6*C* the coexistence region is separated by $f_L = f^{**}_{P_N \to P_L M_R}$ ($\approx$0.72) and $f_N = f^{**}_{P_L \to P_N M_S}$ ($\approx$0.09), implying that both of the microbial groups govern the consequences of PMSF and facilitate the coexistence of plant species, as is shown in Fig. 4*C* (main text).

1. Wardle DA (2002) *Community and Ecosystems: Linking the Aboveground and Belowground Components* (Princeton University Press, Princeton).

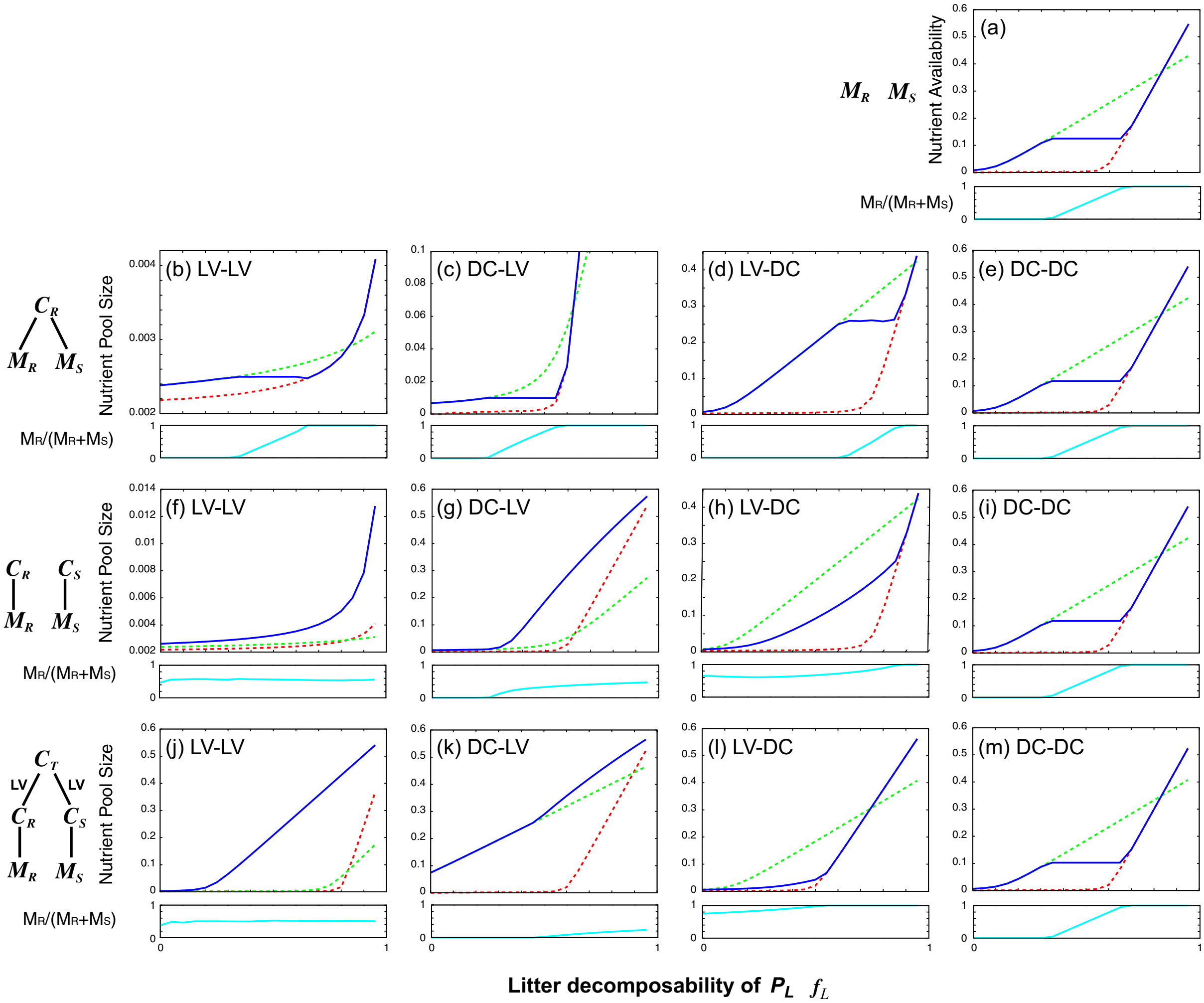

Fig. S1. Roles of the functional microbial diversity in the plant litter control of nutrient pool size. Nutrient pool size is calculated as the long-term average of *N*, because the results include those of the system under stable equilibrium and those under periodic fluctuations. Dashed red or green lines represent system results with the single microbial group M_R or M_S, respectively. System results with M_R and M_S are shown as blue lines. The equation settings and parameter values are as follows: (*A*) $l_{Mj} = m_{Mj}M_j$ and $m_{Mj} = 0.1$ for $j = R, S$. (*B*) $l_{Mj} = m_{Mj}M_jC_L$ for $j = R$ and *S*, $g_{CR} = e_C(l_{MR} + l_{MS})$, $l_{CR} = m_{CR}C_R$ and $m_{Mj} = 100$, $e_C = 0.6$, $m_{CR} = 0.1$. (*C*) $l_{MR} = m_{MR}M_R$, $l_{MS} = m_{MS}M_SC_S$, and $m_{MR} = 0.1$, $m_{MS} = 20.0$. (*D*) $l_{MR} = m_{MR}M_RC_R$, $l_{MS} = m_{MS}M_S$, and $m_{MR} = 20.0$, $m_{MS} = 0.1$. (*E*) $l_{Mj} = m_{Mj}M_{kj}$ and $m_{Mj} = 0.1$ for $j = R, S$. Other settings in *C*–*E* are the same as in *B*. (*F*) $l_{Mj} = m_{Mj}M_jC_j$, $g_{Cj} = e_Cl_{Mj}$, $l_{Cj} = m_{Cj}C_j$ and $m_{Mj} = 100$, $e_C = 0.6$, $m_{Cj} = 0.1$ for $j = R, S$. (*G*) $l_{MR} = m_{MR}M_R$, $l_{MS} = m_{MS}M_SC_S$, and $m_{MR} = 0.1$, $m_{MS} = 20.0$. (*H*) $l_{MR} = m_{MR}M_RC_R$, $l_{MS} = m_{MS}M_S$, and $m_{MR} = 20.0$, $m_{MS} = 0.1$. (*I*) $l_{Mj} = m_{Mj}M_j$ and $\mathrm{m}_{Mj} = 0.1$ for $j = R, S$. Other settings in *G*–*I* are the same as in *F*. (*J*) $l_{Mj} = m_{Mj}M_jC_j$, $g_{Cj} = e_Cl_{Mj}$, $l_{Cj} = m_{Cj}C_jC_T$, $g_{CT} = e_C(l_{CR} + l_{CS})$, $l_{CT} = m_{CT}C_T$ and $m_{Mj} = 100$, $e_C = 0.6$, $m_{Cj} = 20.0$, $m_{CT} = 0.02$ for $j = R, S$. (*K*) $l_{MR} = m_{MR}M_R$, $l_{MS} = m_{MS}M_SC_S$, and $m_{MR} = 0.1$, $m_{MS} = 50.0$. (*L*) $l_{MR} = m_{MR}M_RC_R$, $l_{MS} = m_{MS}M_S$, and $m_{MR} = 50.0$, $m_{MS} = 0.1$. (*M*) $l_{Mj} = m_{Mj}M_j$ and $m_{Mj} = 0.1$ for $j = R, S$. Other settings in *K*–*M* are the same as in *J*.

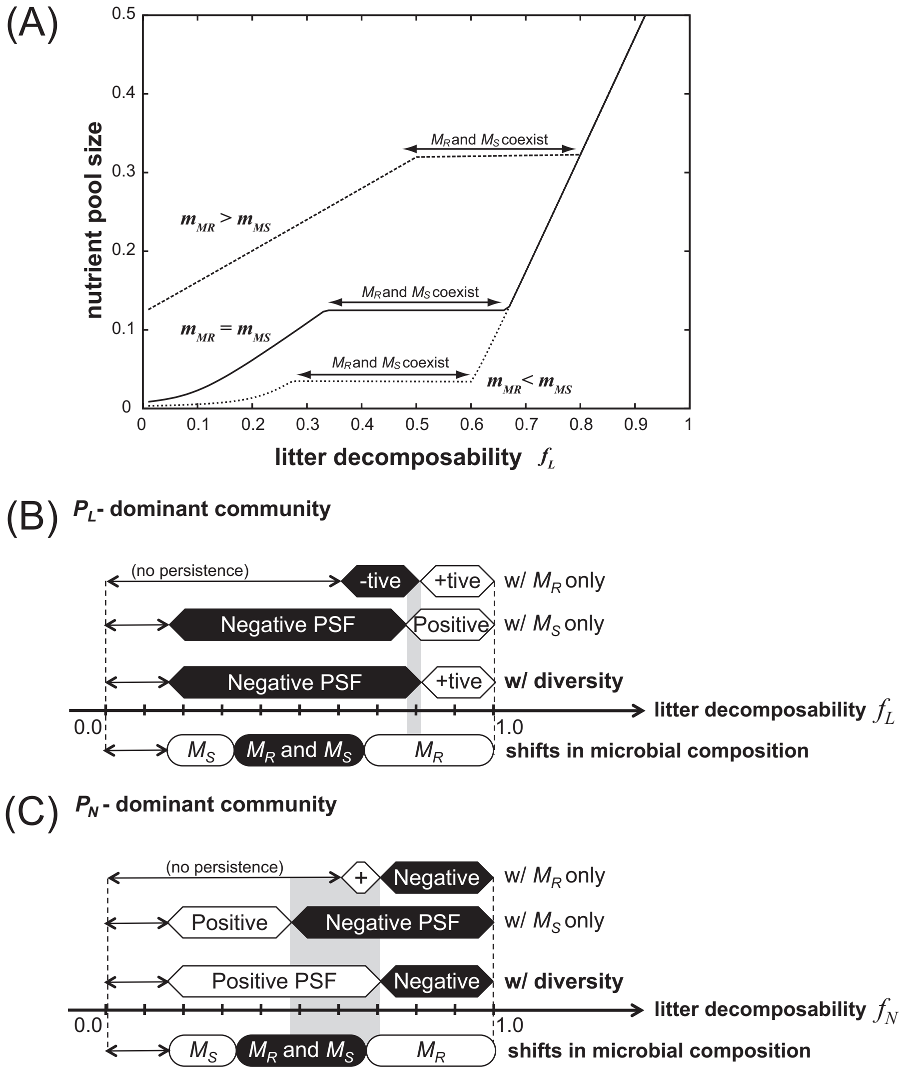

Fig. S2. Supplemental results corresponding to Figs. 2*A* and 3 (main text). (*A*) Dependence of the relationship between litter decomposability (f_L) and equilibrium nutrient pool size on the group-specific mortality of microbes. Parameters are $m_{MS} = 0.08$ (for $m_{MR} > m_{MS}$), 0.1 (for $m_{MR} = m_{MS}$), and 0.11 (for $m_{MR} < m_{MS}$). In the region with coexisting M_R and M_S, the slopes of the lines are positive (≈0.01), negative (≈–0.002), and zero for $m_{MR} > m_{MS}$, $m_{MR} < m_{MS}$, and $m_{MR} = m_{MS}$, respectively (*SI Text*, Section 1). (*B* and *C*) Roles of microbial diversity in determining the sign of PSF in a P_L-dominant (*B*) or P_N-dominant community (*C*). The sign of PSF is determined by "*Conditions for negative PSF in a system with P_i, M_R, and M_S*" (Table S2) in region M_R and M_S, whereas the sign of PSF is determined by "*Condition for negative PSF in a system with P_i and M_j*" (Table S2) in region M_S or region M_R. See Fig. 3 legend (main text) for more details. Parameters are $e_{MR} = e_{MS} = 0.45$. Others are the same as default values (Table S1).

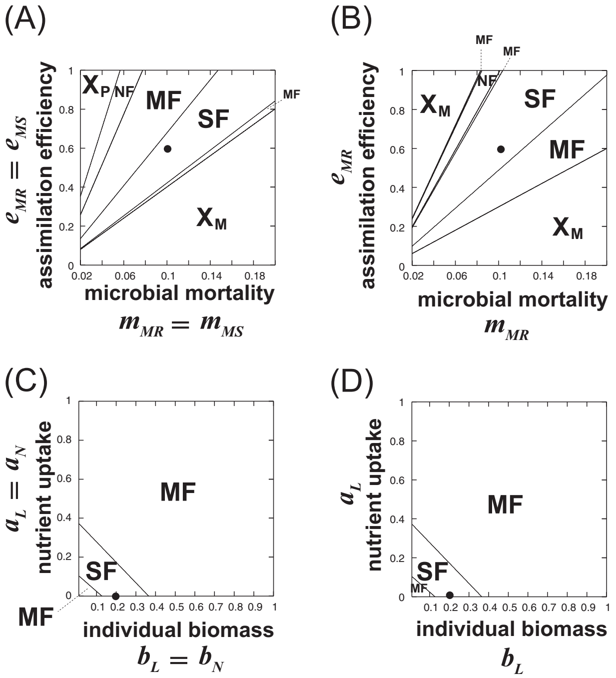

Fig. S3. Roles of microbial diversity in determining the sign of PSF and their dependence on parameters. The definition for each region is described in *SI Text*, Section 3. (*A*) The microbial mortality and assimilation efficiency of M_R are equal to those of M_S. (*C*) Individual biomass and nutrient uptake coefficient of P_L are equal to those of P_N. In other panels, only focal parameter values (on *x* and *y* axes) are changed, whereas other parameters are set to the default values (Table S1). For the focal parameters on *x* and *y* axes, the set of default values is shown as a closed circle in each panel.

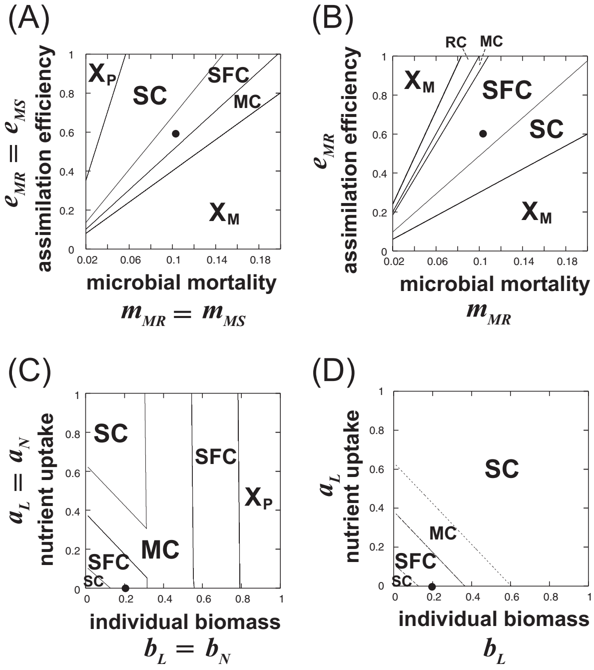

Fig. S4. Parameter dependence of the plant community composition during the coexistence of microbial groups. The definition for each region is described in *SI Text*, Section 3. (*A*) The microbial mortality and assimilation efficiency of M_R are equal to those of M_S. (*C*) The individual biomass and nutrient uptake coefficient of P_L are equal to those of P_N. In other panels, only focal parameter values (on *x* and *y* axes) are changed, whereas other parameters are set as the default values (Table S1). For focal parameters on *x* and *y* axes, the set of default values is shown as a closed circle in each panel.

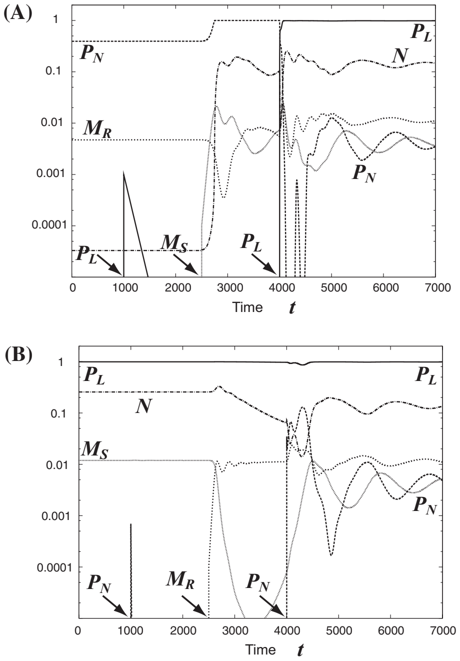

Fig. S5. Plant coexistence realized with microbial diversity. (*A*) The system starts from the equilibrium with P_N and M_R, which prevents the invasion of P_L. (*B*) The system starts from the equilibrium with P_L and M_S, which prevents the invasion of P_N. A combination of species-specific litter decomposability is set as (f_L, f_N) = (0.6, 0.5). Others parameters are the same as default values (Table S1).

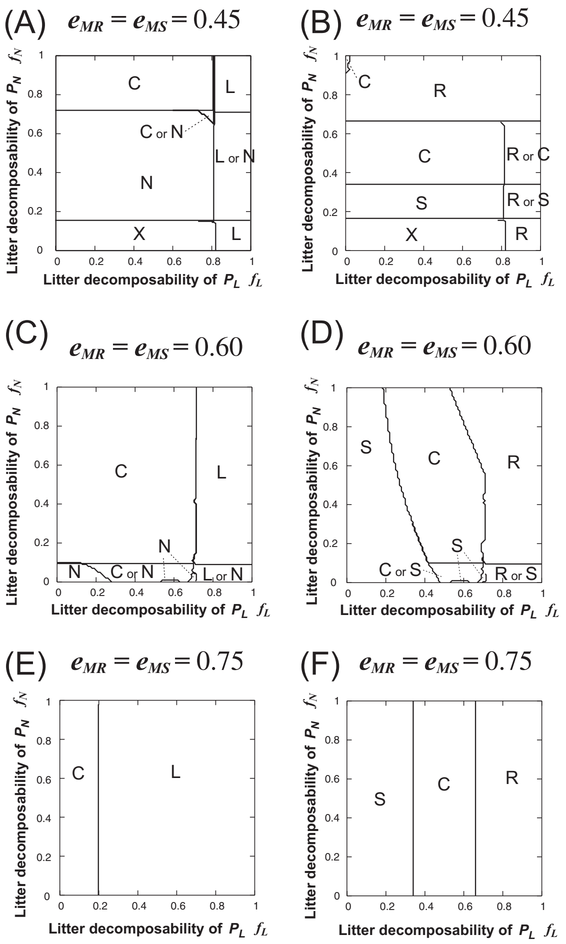

Fig. S6. Consequences of PMSF in plant and microbial communities, depending on the plant litter decomposability (f_L, and f_N). Regions **X**, **L**, **N**, **C**, **C or N**, and **L or N** in *A*, *C*, and *E* correspond to the regions defined in Fig. 4 (main text). A distinct plant community composition is realized in each region. Regions **X**, **R**, **S**, **C**, **C or S**, and **R or S** in *B*, *D*, and *F* represent no persistence of microbial community, dominance of M_R, dominance of M_S, coexistence of M_S and M_R, microbial coexistence or dominance of M_S depending on initial conditions, and dominance of M_R or dominance of M_S depending on initial conditions, respectively. In bistable regions (e.g., in a combination of region **L or N** and region **R or C**), either dominance of P_L with dominance of M_R or dominance of P_N with coexistence of M_R and M_S is realized, depending on initial conditions. Parameters are $e_{MR} = e_{MS} = 0.45$, 0.6, and 0.75 for *A* and *B*, for *C* and *D*, and for *E* and *F*, respectively. Others are the same as in Table S1.

Table S1. List of parameters, assumptions, and default values

Symbol	Definition	Assumption	Unit	Default value
r_L	Recruitment rate of P_L per unit nutrient	$r_L < r_N$	Per unit nutrient per unit time	5.0
r_N	Recruitment rate of P_N per unit nutrient			500.0
b_L	Individual biomass (nutrient content) of P_L	b_L, $b_N < T_N$	Unit nutrient	0.2
b_N	Individual biomass (nutrient content) of P_N			0.2
a_L	Nutrient uptake coefficient of P_L	Negligible	Per unit nutrient per unit time	0.0
a_N	Nutrient uptake coefficient of P_N			0.0
m_P	Mortality rate of plants	m_{MR}, $m_{MS} > m_P$	Per unit time	0.01
f_L	Fraction of D_R in detritus of P_L	$0 \leq f_L, f_N \leq 1$	Dimensionless	Changing
f_N	Fraction of D_R in detritus of P_N			Changing
c_D	Decomposition coefficient	—	Dimensionless	1.0
k_{RR}	Decomposition efficiency of D_R by M_R	Normalized as 1.0	Per unit nutrient per unit time	1.0
k_{RS}	Decomposition efficiency of D_R by M_S	$k_{RR} > k_{RS} > k_{SS} > k_{SR}$		0.5
k_{SR}	Decomposition efficiency of D_S by M_R			0.1
k_{SS}	Decomposition efficiency of D_S by M_S			0.2
e_{MR}	Nutrient assimilation efficiency of M_R	Independent of litter quality	Dimensionless	0.6
e_{MS}	Nutrient assimilation efficiency of M_S			0.6
m_{MR}	Mortality rate of M_R	m_{MR}, $m_{MS} > m_P$	Per unit time	0.1
m_{MS}	Mortality rate of M_S			0.1
T_N	Total nutrient in the model ecosystem	Normalized as 1.0	Unit nutrient	1.0

Table S2. Summary of conditions for persistence, microbial coexistence, and negative PSF

Persistent condition for a system with P_i *and* M_j	$f_i > f^E_{P_iM_j}$ *i* = *L* and *N*, *j* = *R* and *S*.
Persistent conditions for a system with P_i, M_R, and M_S	$\frac{(k_{RR}-k_{SR})m'_{MS}-(k_{RS}-k_{SS})m'_{MR}}{c_D(k_{RR}k_{SS}-k_{RS}k_{SR})} < T_N - \frac{m_P}{r_i}$ $k_{SS}m'_{MR}-k_{SR}m'_{MS}>0$ and $-k_{RS}m'_{MR}+k_{RR}m'_{MS}>0$ $f^{**}_{M_R\to P_iM_S} < f^{**}_{M_S\to P_iM_R}$
Condition for negative PSF in a system with P_i *and* M_j	$f_L < f^{**}_{P_N\to P_LM_j}$ and $f^{**}_{P_L\to P_NM_j} < f_N$
Conditions for negative PSF in a system with P_i, M_R, and M_S	$f_L > f^{**}_{P_N\to P_LM_RM_S}$ and $f_N < f^{**}_{P_L\to P_NM_RM_S}$ if $m'_{MR} < m'_{MS}$ $\phi^{**}_{P_N\to P_LM_RM_S} < 0$ and $\phi^{**}_{P_L\to P_NM_RM_S} < 0$ if $m'_{MR} = m'_{MS}$ $f_L < f^{**}_{P_N\to P_LM_RM_S}$ and $f_N > f^{**}_{P_L\to P_NM_RM_S}$ if $m'_{MR} > m'_{MS}$
Coexistence condition for microbes in a system with P_i	$f^{**}_{M_R\to P_iM_S} < f^{**}_{M_S\to P_iM_R}$ and $f^{**}_{M_R\to P_iM_S} < f_i < f^{**}_{M_S\to P_iM_R}$ and *i* = *L* and *N*

Persistent conditions, conditions for causing negative PSF in a system with a single microbial group or in a system with two microbial groups, and the condition for microbes to coexist are shown. Note that $m'_{Mj} \equiv m_{Mj}/e_{Mj}$. The exact mathematical expression for each threshold value is summarized in Table S3, also noting that $\phi^{**}_{P_N\to P_LM_RM_S}$ and $\phi^{**}_{P_L\to P_NM_RM_S}$ do not include f_L and f_N (Table S3).

Table S3. Summary of the threshold litter decomposability and additional conditions

Symbol	Definition
$f^E_{P_iM_j}$	$\frac{k_{Rj}k_{Sj}}{k_{Rj}-k_{Sj}}\left(\frac{1}{k_{Sj}}-\left(T_N-\frac{m_P}{r_i}\right)\frac{c_D}{m'_{Mj}}\right)$
$f^{**}_{P_N\to P_LM_j}$	$\frac{k_{Rj}k_{Sj}}{k_{Rj}-k_{Sj}}\left[\frac{1}{k_{Sj}}+\frac{c_D}{m'_{Mj}}\left\{-T_N-\frac{a_L}{r_L}+\frac{r_Nm_P}{r_L^2}\left(1+\frac{a_L}{m'_{Mj}}\right)+b_L\left(1-\frac{r_L}{r_N}\right)\left(1+\frac{m_P}{m'_{Mj}}\right)\right\}\right]$
$f^{**}_{P_L\to P_NM_j}$	$\frac{k_{Rj}k_{Sj}}{k_{Rj}-k_{Sj}}\left[\frac{1}{k_{Sj}}+\frac{c_D}{m'_{Mj}}\left\{-T_N-\frac{a_N}{r_N}+\frac{m_P}{r_L}\left(1+\frac{a_N}{m'_{Mj}}\right)+b_N\left(1-\frac{r_L}{r_N}\right)\left(1+\frac{m_P}{m'_{Mj}}\right)\right\}\right]$
$\phi^{**}_{P_N\to P_LM_RM_S}$	$-\frac{k_{RR}-k_{RS}}{-k_{RS}m'_{MR}+k_{RR}m'_{MS}}+\frac{r_L^2r_N}{m_P(a_Lr_N+b_Lr_L^2)(r_N-r_L)}\left\{T_N-\frac{r_Nm_P}{r_L^2}-b_L\left(1-\frac{r_L}{r_N}\right)-\frac{(k_{RR}-k_{SR})m'_{MS}-(k_{RS}-k_{SS})m'_{MR}}{c_D(k_{RR}k_{SS}-k_{RS}k_{SR})}\right\}$
$\phi^{**}_{P_L\to P_NM_RM_S}$	$\frac{k_{RR}-k_{RS}}{-k_{RS}m'_{MR}+k_{RR}m'_{MS}}-\frac{r_Lr_N}{m_P(a_N+b_Nr_L)(r_N-r_L)}\left\{T_N-\frac{m_P}{r_L}-b_N(1-\frac{r_L}{r_N})-\frac{(k_{RR}-k_{SR})m'_{MS}-(k_{RS}-k_{SS})m'_{MR}}{c_D(k_{RR}k_{SS}-k_{RS}k_{SR})}\right\}$
$f^{**}_{P_N\to P_LM_RM_S}$	$\frac{(k_{SS}m'_{MR}-k_{SR}m'_{MS})(-k_{RS}m'_{MR}+k_{RR}m'_{MS})}{(k_{RR}k_{SS}-k_{RS}k_{SR})(m'_{MS}-m'_{MR})}\,\phi^{**}_{P_N\to P_LM_RM_S}$
$f^{**}_{P_L\to P_NM_RM_S}$	$\frac{(k_{SS}m'_{MR}-k_{SR}m'_{MS})(-k_{RS}m'_{MR}+k_{RR}m'_{MS})}{(k_{RR}k_{SS}-k_{RS}k_{SR})(m'_{MS}-m'_{MR})}\,(-\phi^{**}_{P_L\to P_NM_RM_S})$
$f^{**}_{M_R\to P_iM_S}$	$\frac{k_{SS}k_{RS}}{k_{RR}k_{SS}-k_{RS}k_{SR}}\left(\frac{m'_{MR}}{m'_{MS}}-\frac{k_{SR}}{k_{SS}}\right)$
$f^{**}_{M_S\to P_iM_R}$	$\frac{k_{RR}k_{SR}}{k_{RR}k_{SS}-k_{RS}k_{SR}}\left(\frac{k_{SS}}{k_{SR}}-\frac{m'_{MS}}{m'_{MR}}\right)$

These threshold values determine the possibility of the coexistence of plants and microbes (Table S2) where $m'_{Mj} \equiv m_{Mj}/e_j$.