SCIE1000 Project, Semester 1 2009

Topic 3: Blood Flow and the Hagen-Poiseuille Equation

1. Administrative information:

SCIE1000 includes a substantial project, which contributes 20% towards your final assessment. There are four project topics. *If you are enrolled in the Bachelor of Biomedical Science program then you* **must** *choose Topic 4. If you are enrolled in* **any other degree program** *then you* **must choose** *Topic 1, Topic 2 or Topic 3.* The project topics are:

Topic 1: Cryptography and the Hill cipher	Topic 2: Genetics and the logistic map
Topic 3: Blood Flow and the Hagen-Poiseuille Equation	Topic 4: Action Potentials

All project topics include components from each of science, mathematics, Python programming and scientific communication. However, the balance between these areas varies between different topics, so if you have a choice then you may like to choose a topic that suits your expertise and area of interest. Note that all project topics will require you to understand some concepts that are not covered elsewhere in SCIE1000.

If you have a choice of project topics (that is, you are **not** enrolled in the Bachelor of Biomedical Science) then you should put some thought into choosing your project topic, and you might like to read through each of the specification sheets for Topics 1, 2 and 3. Here is some general advice on the focus of the topics, but you must form your own opinions.

- Topic 1 is more mathematical and computing focused, with no experimental work. This topic may well appeal to students who have an interest in quantitative sciences (such as mathematics, physics and computing).
- Topic 2 combines two areas of biology (population models and genetics), and includes a reasonable amount of mathematics and computing, but no experimental work. This topic may well appeal to students who have an interest in biology (particularly ecology and genetics), environmental science, or applications of the quantitative sciences.
- Topic 3 is more experimental in nature, and requires you to design and complete an experiment to investigate fluid flow, and relate this to the circulatory system. The programming component in this topic is a bit smaller than in the other topics (although is still substantial). This topic may well appeal to students who have an interest in biomedical sciences, physics, or experimental work.

All projects will be marked out of 100, and must be submitted in hardcopy to a room monitor in the Science Learning Centre on Level 2 of Building 67 by 4pm on Monday May 18th. You will need to sign your name when you submit your assignment. **If you are up to 30 minutes late then you will suffer no mark penalty, but for each additional minute you are late until 5pm, your final mark will reduce by 3. Please pay attention to this deadline: we really mean it!** No assignments will be accepted after 5pm without a medical certificate or other similar documented reason. Please note that projects are not easy or short. They all contain material that you can complete early in semester. If you leave the work until late then you are running the risk of not completing the material. Remember that you need to receive at least 40% of the marks overall on the assignments/project in order to pass SCIE1000.

All computer programs must be appropriately commented, use meaningful variable names and print useful messages. You must include a printout of all programs and output (including graphs if appropriate) in your assignment submission. Other work may be hand-written, but marks will be deducted if it is illegible. Marks will also be deducted if word limits are not met or are exceeded by more than 10%. You must provide references, in an appropriate scientific style, for any information you use from any other source.

Unless otherwise stated, this project must be completed as individual work. You may discuss questions and content with other people, but the final submission must be your own work. Any plagiarism, either from class mates, the internet, or other sources, may result in a range of penalties, including loss of marks, loss of full credit for the project, or other disciplinary action.

2. Required background:

2.1 Science

Fluid Dynamics: Fluid dynamics is the study of fluids in motion. Two broad classifications of fluid flow are *laminar*, in which the flow is smooth and non-turbulent, and *turbulent*, which involves substantial disturbances in the flow currents, and rapid changes in the movement and properties of the flow. Blood flowing through arteries is an example of laminar flow, whereas water flowing in a fast moving mountain river is an example of turbulent flow.

In lectures we covered the *Hagen-Poiseuille equation*, which can be used to calculate laminar fluid flow rates.

Viscosity: The *viscosity* of a fluid is a measure of the resistance of the fluid to being deformed (by pressure or other forces). Fluids with a high viscosity, such as canola oil or honey, flow slowly compared to fluids with low viscosity such as water or blood. High viscosity fluids tend to display laminar flow at lower flow rates than do low viscosity fluids.

The SI units for viscosity are pascal seconds, written Pa·s. You may also see viscosity expressed in centipoises cP, (named after Poiseuille). The conversion factor is: 1 cP = 0.001 Pa·s.

Reynolds Number: In fluid analysis, an important factor in determining whether the flow is laminar or turbulent is the *Reynolds number* of the system. The Reynolds number is dimensionless (so has no units of measurement). The equation used to calculate the Reynolds number is based on a number of different quantities, including the flow rate, density, and viscosity of the fluid. Large Reynolds numbers (we will assume numbers > 2000) correspond to turbulent flow, in which case the Hagen-Poiseuille equation will not apply. Small Reynolds numbers (we will assume ≤ 2000) correspond to smooth, laminar flow.

For this project you will create an experimental apparatus to model laminar flow, then use the Hagen-Poiseuille equation and Reynolds numbers to evaluate the accuracy of your apparatus.

2.2 Mathematics

The Hagen-Poiseuille equation for calculating flow rates in laminar systems is:

$$Q = \frac{\Delta P \pi r^4}{8 \mu L}$$

where Q is the volumetric flow rate in m^3/s, ΔP is the pressure drop across the system in Pascals, r is the radius of the tube in m, μ is the dynamic viscosity of the fluid in Pa·s, and L is the length of the tube in m.

The equation for calculating the Reynolds number for fluid flow through a cylindrical tube is:

$$R_e = \frac{2 Q r \rho}{\mu \pi r^2}$$

where Q is the volumetric flow rate in m^3/s, r is the radius of the tube in m, ρ is the density of the fluid in kg/m^3, and μ is the dynamic viscosity of the fluid in Pa·s.

2.3 Python

You may find the following command useful when writing your Python program.

- The `axvline()` command will draw a vertical line on a graph at a specified value of x. Use the command as follows: `axvline(x=a)`

3. Experimental work:

An important part of this project is experimentally investigating the impact on flow rates arising from varying the parameters of the system. **You may work individually or in pairs on the experimental components E1 to E5 of this project. However, everyone must complete all other work independently, write their own analyses, and submit an individual report. We will be checking very carefully for plagiarism.**

You must design and use an experimental apparatus which allows you to make measurements of fluid flow through a cylindrical tube. Your apparatus must allow you to control/vary all of the following parameters: pressure difference, length of the tube, radius of the tube, and viscosity of the fluid. You must measure flow rates for three different liquids: water, canola oil, and a third liquid of your choice.

Here are the specific activities you will need to undertake.

E1. Choose your third liquid. In later questions you will need to know the **density** and **dynamic viscosity** of this liquid; ensure that you know these values before you commence the experiment. You may look on the internet or in books for these values, but make sure you choose a reputable source. Also, you might like to choose a liquid that is easy for you to buy or obtain.

E2. For each of the three liquids, measure the flow rate for two tubes with different radii, while keeping all of the other parameters constant. Take at least two measurements for each liquid and radius; if the results are substantially different between the measurements then take additional measurements.

E3. For canola oil, measure flow rates for three different lengths of tube (keeping all other parameters constant), and two different pressure drops (keeping all other parameters constant). Again, take at least two measurements and repeat if there is substantial variation.

E4. Modify your experiment to simulate the partial blockage of a blood vessel by deposits on the inner wall of the vessel. To do this, use some method to block around one half of the internal cross-sectional area of the tube in **one** of your experimental configurations. Then measure the flow rate of canola oil through the half blocked tube, taking at least two measurements and repeating if there is substantial variation.

E5. Take a photograph that clearly shows your apparatus and you (or both you and your partner if you worked in a pair).

There are many possible ways to design and complete this experiment; we suggest you keep it simple. It is sufficient to have a source of liquid, a tube, a receptacle for the liquid, and a simple (but measurable) pressure drop that will cause the liquid to flow from the source to the receptacle in a controllable and measurable manner. Calculating will probably be easier if you ensure the tubes are horizontal.

4. Questions:

4.1 Experimental results

1. (0 marks, but -6 marks if not there) Submit a printed copy of the photograph (if you worked in a pair then you both must submit a copy in your individual assignments; you may both use the same photograph).

2. (3 marks) Submit a table of data showing all of your experimentally measured flow rates from experimental steps E2, E3 and E4, including all repeated measurements with the same conditions. Your table must clearly identify which measurements relate to which experimental conditions, and also include the mean value of the measured flow rates for each set of experimental conditions.

3. (6 marks) Write 400 – 500 words describing and analysing your experimental apparatus and the experimental procedure you followed. Your answer must include a brief description of the apparatus and procedure (including how you simulated the blockage), an explanation of why you selected this method, a critical analysis of any weaknesses with your apparatus and procedure, and a discussion of how these weaknesses could be overcome. Do not discuss or analyse your experimental results in this answer (unless they clearly indicate a strength or weakness of your apparatus).

4.2 Hand calculations

4. **(a)** (1 mark) Verify that the Hagen-Poiseuille equation is dimensionally homogeneous.

(b) (1 mark) In subsequent questions it may be useful to convert flow rates predicted by the Hagen-Poiseuille equation into mL/s. Explain how to do this.

5. (2 marks) Use the Hagen-Poiseuille equation to calculate the **expected** flow rates for each of your experimental results from experimental steps E2 and E3. The following table gives some useful information for canola oil and water; in experimental step E1 you have found equivalent information for your third liquid.

	Canola Oil	Water
Density (kg/m^3)	930	1000
Viscosity (Pa·s)	0.065	0.001

6. (6 marks) Compare your experimentally measured mean flow rates calculated in Question 2 with your theoretically calculated flow rates in Question 5. Present a table showing the absolute and percentage errors in the measured versus expected flow rates. In addition, write 200 – 250 words commenting on the comparative results, highlighting 'good' results and 'bad' results (there is no need to discuss why specific results are 'good' or 'bad'; simply highlight them, and any patterns you observe).

7. (2 marks in total) Using the Hagen-Poiseuille equation, explain the effect on the expected flow rate of doubling the value of each of the following quantities. (In each case, assume that the values of the other quantities are held constant when the identified one is doubled.) Briefly justify your answers.

(a) The length of the tube.

(b) The radius of the tube.

(c) The viscosity of the liquid.

(d) The change in pressure.

8. **(a)** (2 marks) Using the Hagen-Poiseuille equation, explain the effect on the expected flow rate of halving the cross-sectional area of the tube. Briefly justify your answer.

(b) (3 marks) Write 120 – 150 words comparing your experimental results for the half blocked tube in experimental step E4 with the corresponding results for the non-blocked tube in experimental step E2. In particular, how do your results compare with those expected from your calculations in Part (a) of this question, and suggest some reasons for this.

9. Recall that the Reynolds number is denoted Re. Then

$$Re = \frac{2Qr\rho}{\mu\pi r^2}, \qquad Q = \frac{\Delta P \pi r^4}{8\mu L}, \qquad \Delta P = \rho g h.$$

(a) (1 mark in total)

(i) Express Re as a function of $\mu, L, r, \Delta P$ and ρ.

(ii) Express Re as a function of μ, L, r, h and ρ.

(b) (1 mark) Use unit analysis and the equations in Part (a) to confirm that Re is a dimensionless number.

10. **(a)** (2 marks) Use your expression from $\mathbf{9(a)(ii)}$ to calculate Re for each of the configurations in experimental steps E2 and E3 in your experiment.

(b) (3 marks) In Question 6 you highlighted 'good' and 'bad' experimental as compared to theoretically calculated flow rates. Write 150 – 200 words giving reasons **why** specific comparative results were 'good' and 'bad'. You may like to refer to the values of Re calculated in Part (a), if that helps.

11. Consider blood flow along a 20 cm section of artery with a diameter of 4 mm and a pressure difference across the artery of 800 Pa. You may assume that blood has a density of 1060 kg/m^3 and a viscosity of 3 cP.

(a) (1 mark) Is it reasonable to use the Hagen-Poiseuille equation to model blood flow in the artery? Justify your answer.

(b) (4 marks in total) Using an equation you derived in Question 9:

(i) Find the minimum length of an artery with a diameter of 4 mm and pressure drop of 800 Pa for blood flow to be laminar.

(ii) Find the maximum diameter of an artery with a length of 20 cm and pressure drop of 800 Pa for blood flow to be laminar.

(iii) Find the maximum pressure difference for an artery with a diameter of 4 mm and a length of 20 cm for blood flow to be laminar.

(iv) Find the minimum viscosity of blood in an artery with a diameter of 4 mm, length of 20 cm and pressure drop of 800 Pa for blood flow to be laminar.

4.3 Python programming

12. **(a)** (5 by 1 mark = 5 marks) Write each of the following Python functions in a file.

- A function called `getFlowRate` that:
  - Has four input parameters: pressure change, viscosity, length, and radius; and
  - returns the expected Hagen-Poiseuille flow rate for the system.
- A function called `getReynLength` that:
  - Has four input parameters: density, radius, pressure change, and viscosity; and
  - returns the minimum tube length required for flow to be laminar.
- A function called `getReynVisc` that:
  - Has four input parameters: density, radius, pressure change, and length; and
  - returns the minimum fluid viscosity required for flow to be laminar.
- A function called `getReynRad` that:
  - Has four input parameters: density, pressure change, length, and viscosity; and
  - returns the maximum tube radius possible for flow to be laminar.
- A function called `getReynPress` that:
  - Has four input parameters: density, radius, length, and viscosity; and
  - returns the maximum possible pressure change for flow to be laminar.

You do not need to print or submit these functions separately: they are used in Part (b), so you will submit them there.

(b) (15 marks) Write a Python program plots expected flow rates through a cylindrical tube, depending on the viscosity, length, radius and pressure drop in the system. Your program should keep all but one of these values constant (with the values entered from the keyboard), and allow the other value to vary across a given range. Specifically, your program must:

- Prompt the user to enter the following information:
  - the quantity whose value will be allowed to vary. The user should type 0 to plot expected flow rates against viscosity, 1 to plot against length, 2 to plot against radius or 3 to plot against pressure drop. If the user enters any other input then the program must print an error message and prompt the user again.
  - for the quantity that varies, the *lower bound* and *upper bound* for the plot (use sufficient points between these bounds for your plots to look smooth; a value of 200 steps should be a good choice);
  - the (fixed) values of each of the other quantities; and
  - the density of the fluid.
- Plot the expected flow rate on an appropriately labeled set of axes, plot a vertical line at the x value that equals the maximum or minimum value for flow to be laminar, and also print that value with a message.
- Print the maximum possible flow rate for which flow is laminar.
- Have variables and functions with meaningful names, and be appropriately commented.
- Use the five functions from Part (a). (This is compulsory.)

Hint 1: The following lines show input and output from an execution of our solution program. You do not need to adhere to this approach, but you may if you like.

```
Do you want to plot flow rate vs. viscosity (0), length (1),
radius (2) or pressure drop (3)?4

Invalid plot type, please re-enter your selection.
Do you want to plot flow rate vs. viscosity (0), length (1),
radius (2) or pressure drop (3)?1

Enter viscosity in Pa s, length and radius in m,
pressure drop in Pa and density in kg/m^2.

You have chosen to vary length.
What is the minimum value of this? 0.05
What is the maximum value of this? 0.3

What is the density of the fluid (in kg/m^3)? 1060
What is the dynamic viscosity of the fluid (in Pa s)?0.003
What is the radius of the tube (in m)?0.002
What is the pressure drop (in Pa)?800
The minimum tube length for laminar flow is ......  m.
The maximum laminar flow rate is ......  mL/s.
```

Hint 2: Unless you take care, the input statements in your program may become very repetitive. You might like to note that you need to input the value of the viscosity for **all** cases **except** choice 0, you need to input length for all cases except choice 1, and so on.

(c) Test your program in the following ways. (You must include a printed copy of the output from your program in your submission.)

(i) (2 marks) Use your program, or the functions you wrote for it, to repeat Question 5, and verify that the output matches your hand calculations.

(ii) (2 marks) Use your program, or the functions you wrote for it, to repeat Question 10(a), and verify that the output matches your hand calculations.

(ii) (4 marks) Use your program, or the functions you wrote for it, to repeat Question 11(b), and verify that the output matches your hand calculations

(d) (6 marks) Write a brief user guide which explains how to use the program. (Your user guide should **not** assume that the user has read this assignment question sheet.) The guide should contain all necessary information about:

- What the program does.
- What input is requested by the program, and what valid input it can take.
- What output the program gives.
- Any assumptions you have made, or any special cases.

This is a user guide, not a programmer's manual. Do not describe the algorithm you have used or internal details of the program. Instead, if someone with a basic understanding of computers (and a good understanding of the science relevant to your project topic) wanted to run your program, what would they need to know?

4.4 Running the program

(You must submit printed copies of your program and all graphs.)

13. ()4 by 2 marks = 8 marks) For the system described in Question 11:

- Create a plot of *fluid flow rate vs. viscosity* for a suitable range of viscosities. Print the minimum viscosity required for the blood flow to be laminar, and print the maximum possible laminar flow rate.
- Create a plot of *fluid flow rate vs. length* for a suitable range of lengths. Print the minimum length for the blood flow to be laminar, and print the maximum possible laminar flow rate.
- Create a plot of *fluid flow rate vs. radius* for a suitable range of radii. Print the maximum possible radius for the blood flow to be laminar, and print the maximum possible laminar flow rate.
- Create a plot of *fluid flow vs. pressure* change for a suitable range of pressures. Print the maximum possible pressure change for the blood flow to be laminar, and print the maximum possible laminar flow rate.

4.5 Written response

14. (20 marks) Select **one** of the following topics and write a 300 – 350 word response to the topic. Your response must identify which topic you chose, and must be written as an essay in an appropriate scientific style. If your topic asks you to state your opinion then make sure that you do so, with cogent arguments for and against your case. If relevant you can include some diagrams, equations or mathematics (which will not be included in the word limit), but your response should be predominantly text based.

Your submission must be typed (although diagrams and calculations may be hand-written). The SCIE1000 Blackboard site contains a Criteria Sheet for this essay; **you must print a copy of the sheet and attach it as the last page of your project submission**. The sheet shows how marks will be allocated, to a maximum of 20 marks.

- As part of many modern medical and surgical treatments it is necessary for a patient to receive blood via a transfusion. Although the practice of both collecting and transfusing blood is generally very safe and strictly controlled, some individuals have cultural or religious beliefs that make them unwilling to receive such transfusions. For example, in 2005 a 14 year old girl from Canada was ordered by a provincial supreme court to receive a blood transfusion as part of her cancer treatment; she had previously refused transfusions on the grounds of her religious beliefs. She appealed the decision of the court, with the support of her family and church, in another Canadian province. However, the decision to force the girl to receive the blood transfusion was upheld. In this case the girl was legally a minor, and provincial law stated that only persons over 19 could refuse medical treatment.

  This poses some interesting questions. Should the state (represented by governments or courts) have the right to force an individual to receive medical treatment, even if the treatment is considered improper by that individual for cultural/religious reasons? Should the government only take action if the patient is a minor or incapable of making informed decisions for themselves? Should the parents or carers of minors or incapable individuals be allowed to deny such treatment, even if it goes against professional medical advice?

- *Pharmaceutical patents* are routinely granted on newly discovered drugs. Such patents protect the intellectual property of companies and individuals, by restricting production and access to the drugs. Inevitably, some individuals who do not have sufficient financial resources are unable to afford treatment, and hence undergo substantial suffering and may even die.

  This poses some interesting questions. What is the appropriate balance between protecting commercial interests and humanitarian concerns? Should pharmaceutical patents be granted in all cases, sometimes, or never? Should all governments (including those of relatively poor countries) respect patents? Should the legal protections offered by patents be set aside in certain circumstances, such as global pandemics?

The end