## Appendix A

Each of the 12 segments of the subject was assumed to be a rigid segment and modeled accordingly. The equations that governed the system are show in equation 1.

$$\begin{aligned}\vec{F}_{contact} + \vec{F}_{dis\tan ce} + \vec{F}_{inertia} &= \vec{0} \\ \vec{M}_{contact} + \vec{M}_{dis\tan ce} + \vec{M}_{inertia} &= \vec{0}\end{aligned} \qquad (1)$$

Here, $\vec{F}$ is a force vector acting on a segment and $\vec{M}$ is the moment vector acting on a segment. The subscripts denote the type of force or moment as being either *contact*, *distance*, or *inertia* which describes forces or moments due to contact, gravity, and inertial forces respectively. The inertial forces are further expanded in equation 2 as:

$$\begin{aligned}\vec{F}_{inertia} &= -m \cdot \vec{a} \\ \vec{M}^{COM}_{inertia} &= \mathbf{R}^{-1} \cdot (-\mathbf{R} \cdot \vec{\alpha} \cdot \mathbf{I} - (\mathbf{R} \cdot \vec{\omega}) \times (\mathbf{I} \cdot \mathbf{R} \cdot \vec{\omega}))\end{aligned} \qquad (2)$$

In equation 2, the inertial moment is taken about the center-of-mass (COM). **R** is the rotation matrix, which describes the segment in the lab reference frame, and **I** is the inertial matrix in the segment reference frame. Since the inertial matrix dotted with the rotation matrix generally results in a matrix with non-zero diagonal terms, rotational dynamic coupling is possible. The mass of the segment is defined by *m* and the angular acceleration and velocity of the segment with respect to the lab reference frame is defined by $\vec{\alpha}$ and $\vec{\omega}$ respectively. The inertial moments acting on the segment can be expressed about any point, *O*, in the lab using equation 3.

$$\vec{M}^{O}_{inerrtia} = \vec{p}^{\,O/COM} \times \vec{F}_{inertia} + M^{COM}_{inertia} \qquad (3)$$

In equation 3, $\vec{p}^{O/COM}$ is the vector from point $O$ in the lab to the COM of the segment, which is necessary to ensure that summation of the contact, distance, and inertial moments occur about the same point with respect to the same reference frame.

Equations 1, 2, and 3 can be applied to any body or group of bodies which are selected as the free body in question.

In order to establish the trunk angular acceleration as a function of the lower extremity joint moments, joint constraints are applied to the ankle, knee, and hip. The equation used to do so is given by equation 4.

$$\vec{a}_D - \vec{a}_P + \vec{\alpha}_d \times \vec{p}_d + \vec{\alpha}_p \times \vec{p}_p + \vec{\omega}_d \times (\vec{\omega}_d \times \vec{p}_d) + \vec{\omega}_p \times (\vec{\omega}_p \times \vec{p}_p) = \vec{0} \tag{4}$$

Equation 4 relates the acceleration of the distal and proximal segment to one another since they are connected by a ball and socket joint. $\vec{p}_d$ and $\vec{p}_p$ are the vectors from a point $O$ in the lab to the COM of the distal and proximal segments, respectively. It is assumed that the moment acting on the proximal end of the distal segment is equal and opposite to the moment acting on the distal end of the proximal segment.

The moments acting at the proximal end of each segment (at each joint) are given by the equations (5).

$$\begin{aligned}
\vec{M}^{F}_{contact} &= -\left(\vec{p}^{F_{COM}} \times (-m_F \cdot \vec{a}_F) + \mathbf{R}_F^{-1} \cdot (-(\mathbf{R}_F \cdot \vec{\alpha}_F) \cdot \mathbf{I}_F - (\mathbf{R}_F \cdot \vec{\omega}_F) \times \mathbf{I}_F \cdot (\mathbf{R}_F \cdot \vec{\omega}_F))\right) - \vec{p}^{F_{COM}} \times (m_F \cdot \vec{g}) - COP \times GRF - GRM \\
\vec{M}^{S}_{contact} &= -\left(\vec{p}^{S_{COM}} \times (-m_S \cdot \vec{a}_S) + \mathbf{R}_S^{-1} \cdot (-(\mathbf{R}_S \cdot \vec{\alpha}_S) \cdot \mathbf{I}_S - (\mathbf{R}_S \cdot \vec{\omega}_S) \times \mathbf{I}_S \cdot (\mathbf{R}_S \cdot \vec{\omega}_S))\right) - \vec{p}^{S_{COM}} \times (m_S \cdot \vec{g}) + \vec{M}^{F}_{contact} \\
\vec{M}^{T}_{contact} &= -\left(\vec{p}^{T_{COM}} \times (-m_T \cdot \vec{a}_T) + \mathbf{R}_T^{-1} \cdot (-(\mathbf{R}_T \cdot \vec{\alpha}_T) \cdot \mathbf{I}_T - (\mathbf{R}_T \cdot \vec{\omega}_T) \times \mathbf{I}_T \cdot (\mathbf{R}_T \cdot \vec{\omega}_T))\right) - \vec{p}^{T_{COM}} \times (m_T \cdot \vec{g}) + \vec{M}^{S}_{contact}
\end{aligned} \tag{5}$$

Similarly the forces acting on the proximal end of each segment are given by equation 6.

$$\begin{aligned}
\vec{F}^{F}_{contact} &= -m_F \cdot \vec{a}_F - m_F \cdot \vec{g} - GRF \\
\vec{F}^{S}_{contact} &= -m_S \cdot \vec{a}_S - m_S \cdot \vec{g} + \vec{F}^{F}_{contact} \\
\vec{F}^{T}_{contact} &= -m_T \cdot \vec{a}_T - m_T \cdot \vec{g} + \vec{F}^{S}_{contact}
\end{aligned} \tag{6}$$

In the equations 5 and 6 the subscripts *F*, *S*, and *T* reflect the foot, shank, and thigh segments, respectively. The above equations of motion can be linearly parameterized into equation 7 which reflects the trunk angular acceleration as a function of the joint moments.

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} & S_{17} & S_{18} & S_{19} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} & S_{27} & S_{28} & S_{29} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} & S_{37} & S_{38} & S_{39} \end{bmatrix} \cdot \begin{bmatrix} \vec{M}_A \\ \vec{M}_K \\ \vec{M}_H \end{bmatrix} + \begin{bmatrix} C_A \\ C_K \\ C_H \end{bmatrix} = \begin{bmatrix} \alpha_X \\ \alpha_Y \\ \alpha_Z \end{bmatrix} \tag{7}$$

For the sake of example, the moment equations for a three-dimensional inverted pendulum is derived and linearly parameterized to express the angular acceleration of the pendulum in terms of the moment at the pivot point. The moment equation for an inverted pendulum is given as:

$$\vec{M}_{contact} + \vec{P} \times \vec{F}_{contact} - \vec{\alpha} \cdot \mathbf{I} - \vec{\omega} \times \mathbf{I} \cdot \vec{\omega} + \vec{P}_{com} \times (-m\vec{a}) + \vec{P}_{com} \times m\vec{g} = \vec{0} \tag{8}$$

This equation can be written in terms of its elements.

$$\begin{aligned} &\mathbf{I}^{-1} \cdot \begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix} + \mathbf{I}^{-1} \cdot \begin{bmatrix} 0 & -P_z & P_y \\ P_z & 0 & -P_x \\ -P_y & P_x & 0 \end{bmatrix} \cdot \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} - \mathbf{I}^{-1} \cdot \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \cdot \mathbf{I} \cdot \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} + \ldots \\ &\ldots + \mathbf{I}^{-1} \cdot \begin{bmatrix} 0 & -P_z & P_y \\ P_z & 0 & -P_x \\ -P_y & P_x & 0 \end{bmatrix}_{com} \cdot \begin{bmatrix} -m\vec{a}_x \\ -m\vec{a}_y \\ -m\vec{a}_z \end{bmatrix} + \mathbf{I}^{-1} \cdot \begin{bmatrix} 0 & -P_z & P_y \\ P_z & 0 & -P_x \\ -P_y & P_x & 0 \end{bmatrix}_{com} \cdot \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} = \begin{bmatrix} \alpha_x \\ \alpha_y \\ \alpha_z \end{bmatrix} \end{aligned} \tag{9}$$

In equation 9, the angular acceleration's sensitivity to the contact moments are the inverse of the inertia matrix in the laboratory reference frame for a given configuration of the pendulum.

The sensitivity matrix *S* (equation 7) is a time varying matrix which is dependant on the geometry of the model. The constant vector C is also dependant on the geometry of the model and the scalars $\alpha_x, \alpha_y$, and $\alpha_z$ in equation 7 are the angular accelerations of the trunk about the *x*, *y* and *z* axes, respectively.