Formal definitions of graph measures

We employed several commonly used measures that are listed below. We did not include *assortativity* because of the symmetric degree distributions we used for the small-world and Erdös-Rényi networks.

Average degree (k)

The degree of node i is defined as the sum of the row elements j in the adjacency matrix $\boldsymbol{A}$ containing binary values that represent the network's connectivity

$$k_i = \sum_j^N A_{ij}$$

and the average degree k is the average over all nodal degrees

$$\langle k \rangle = \frac{1}{N} \sum_i^N k_i = \frac{1}{N} \sum_{i,j}^N A_{ij} .$$

Edge density

The edge density represents the fraction of existing edges in the network out of the total number of possible edges

$$m = \frac{1}{N(N-1)} \sum_{\substack{i,j \\ i \neq j}}^N A_{ij} .$$

Characteristic path length (L)

L represents the average minimum number of edges between any two points in the network. To account for isolated nodes, we used the so-called global efficiency

$$E = \frac{1}{N(N-1)} \sum_{\substack{i,j \\ i \neq j}}^N \frac{1}{d_{ij}},$$

where d_{ij} is the minimum number of edges between nodes i and j, and defined L as the harmonic mean $L = 1/E$.

Average clustering coefficient (C)

The average clustering coefficient refers to the probability that neighbors of a node are also connected. It measures the occurrence of clusters in the network by means of

$$C = \frac{1}{N} \sum_i^N \frac{2n_i}{k_i(k_i - 1)},$$

where n_i represents the number of existing edges between neighbors of node i.

Small-world index (SW)

The small-world index has often been used to indicate the presence of a small-world network [1-6]. It is defined as

$$SW = \frac{C / C_{\mathrm{rand}}}{L / L_{\mathrm{rand}}},$$

where C_{rand} and L_{rand} are the clustering coefficient and path length of random networks with the same number of nodes and edges and often also the same degree distribution. Since small-world networks are characterized by $C \gg C_{\mathrm{rand}}$ and $L \approx L_{\mathrm{rand}}$, the ratio becomes $SW \gg 1$.

Number of hubs (NHUBS)

We assigned nodes as hubs when their nodal degree exceeded the average degree of the network

$$NHUBS = \sum_{i}^{N} \left[k_i > \langle k \rangle \right].$$

Maximum degree (MAXD)

The largest nodal degree defines the maximum degree of the network in terms of

$$MAXD = \max_{i} \left[k_i \right].$$

Synchronizability (S)

S describes the network's capacity to synchronize [see, e.g., 7,8] and is calculated from the eigenvalues of the graph's Laplacian matrix $\boldsymbol{\Lambda}$

$$\boldsymbol{\Lambda} = \boldsymbol{D} - \boldsymbol{A},$$

where $\boldsymbol{D}$ represents a diagonal matrix containing the nodal degrees. The synchronizability is defined as the ratio between the first non-zero eigenvalue λ_2 and the largest eigenvalue $\lambda_{\max}$ of $\boldsymbol{\Lambda}$, that is,

$$S = \frac{\lambda_2}{\lambda_{\max}}.$$

Central point dominance (CPD)

The central point dominance is a measure that describes the distribution of betweenness centrality scores of nodes B_i, i.e. the fraction of shortest paths from node p to node q that runs through node i. It was introduced by Freeman [9] and defined as follows

$$CPD = \frac{1}{N-1}\sum_{i}^{N}\left(B_{\max} - B_i\right),$$

where

$$B_i = \frac{2}{(N-1)(N-2)}\sum_{i \neq j}^{N}\frac{\sigma(p,i,q)}{\sigma(p,q)}.$$

The number of shortest paths between node p and node q that run through node i is indicated here by $\sigma(p,i,q)$, the number of total shortest paths between node p and q by $\sigma(p,q)$.

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