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Supporting Material

Title: Symmetry, Stability, and Reversibility Properties of Idealized Confined Microtubule Cytoskeletons

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Supplementary Material for *Symmetry, Stability, and Reversibility Properties of Idealized Confined Microtubule Cytoskeletons* by Maly and Maly

1. Calculation of predictions for a specific experimental situation using the nondimensional representation of the model results in the main text

To show the parameter-independent behavior of the model, the results in the main text are presented in the nondimensional form. For example, the force F acting on the centrosome, as a function of the distance of the centrosome from the center Δ, is plotted in Fig. 4. What is actually plotted is the parameter-independent quantity $F\ R^2/(N\ EI)$, which is the average force per microtubule (among the N microtubules in the cell), expressed in units that are the natural units of force in the model. The natural unit of force here is the flexural rigidity of a microtubule EI divided by the square of the cell radius R. Assuming, for example, a rigidity of 25 pN μm^2, which would be near the mid-point of the range of the measured values (see a compendium table in Kikumoto et al., 2006), the force unit will be 1 pN for a cell that is 10 μm in diameter. In this way, the results in the main text show the parameter-independent behavior of the system, and the actual value of the force acting on the centrosome can easily be estimated from the plot, when the parameters (N, R, and EI) are known.

For example, consider an experiment in which an aster of N =20 microtubules, each L = 12 μm long, is assembled inside an approximately spherical chamber of radius R = 10 μm, with a bead replacing the centrosome. (The method of creation of semi-artificial asters and chambers is described in Holy, 1997. The purpose of the present example is to demonstrate how specific values for experimental testing can be derived from the model. It is not asserted that this specific experiment is feasible and that the referenced experimental techniques are suitable for it.) The model predicts (Fig. 2 of the main text) that when L/R = 12 μm / 10 μm = 1.2, the normalized equilibrium distance of the centrosome from the center of the chamber will be $\Delta_{eq}/R \approx 0.4$. In this particular chamber, therefore, the distance of the centrosome from the center will be $\Delta_{eq} = 0.4R = 4$ μm.

Assume now that the chamber was prepared in such a way that it is shallow (Holy et al., 1998), and the microtubules are 12.5 μm long. In this case, using Fig. 4 in the main text and the above calculation strategy, we derive that the distance is predicted to be very close to 2.5 μm. If an optical trap (see Neuman and Block, 2004) is then used to displace the bead (the artificial centrosome) from this position to 2 μm from the center, then the model predicts that the force exerted by the trap on the bead will be very close to 1 $N\ EI/R^2$, as can be read out directly from the plot in figure 4 of the main text. To compare this prediction with the force as measured by the optical trap technique, one should substitute the values of N, EI, and R, which characterize our specific experiment. Using the above experimental estimate for the microtubule rigidity EI, we obtain $1\ N\ EI/R^2 = 20 \cdot 25$ pN μm^2 /(10 μm)^2 = 5 pN.

2. Generalization to a distribution of the microtubule length

The main text presents in detail the results of the model that assumes that all microtubules in the cell have the same length. The generalization to a distribution of lengths in the same cell is straightforward. To preserve the intrinsic symmetry as defined in the main text, the distribution characterized by a density function $q(L)$ should be the same for each orientation of unstressed emanation from the centrosome. Then the only modification to the model will be to integrate with respect to L in addition to integrating with respect to the emanation angle when finding the total force F. Here, an example of the generalization for the three-dimensional case will be presented. The formula for the total force in this case becomes

$$F = \int f(\theta_0, L)\, pq(L) d\Omega dL$$

Let $q(L)$ be the density function of a uniform distribution of L between $1.05R$ and $1.15R$. Then, following the same computational strategy as in the main text, we find $\Delta_{eq} = 0.220R$. In the model with the constant length, when its value was equal to the mean of this distribution ($L = 1.1R$), the equilibrium distance was $0.214R$ (see figure 3 in the main text). Thus, we find that the equilibrium distance of the centrosome from the center with the distribution of lengths is slightly larger.

References for the Supplementary Material

Holy, T. E. 1997. Physical aspects of the assembly and function of microtubules. Dissertation. Princeton University.

Holy, T. E., M. Dogterom, B. Yurke, and S. Leibler. 1997. Assembly and positioning of microtubule asters in microfabricated chambers. Proc. Natl. Acad. Sci. USA. 94:6228–6231.

Kikumoto, M., M. Kurachi, V. Tosa, and H. Tashiro. 2006. Flexural rigidity of individual microtubules measured by a buckling force with optical traps. Biophys. J. 90:1687–1696.

Neuman, K. C., and S. M. Block. 2004. Optical trapping. Rev. Sci. Instrum. 75: 2787–2809.

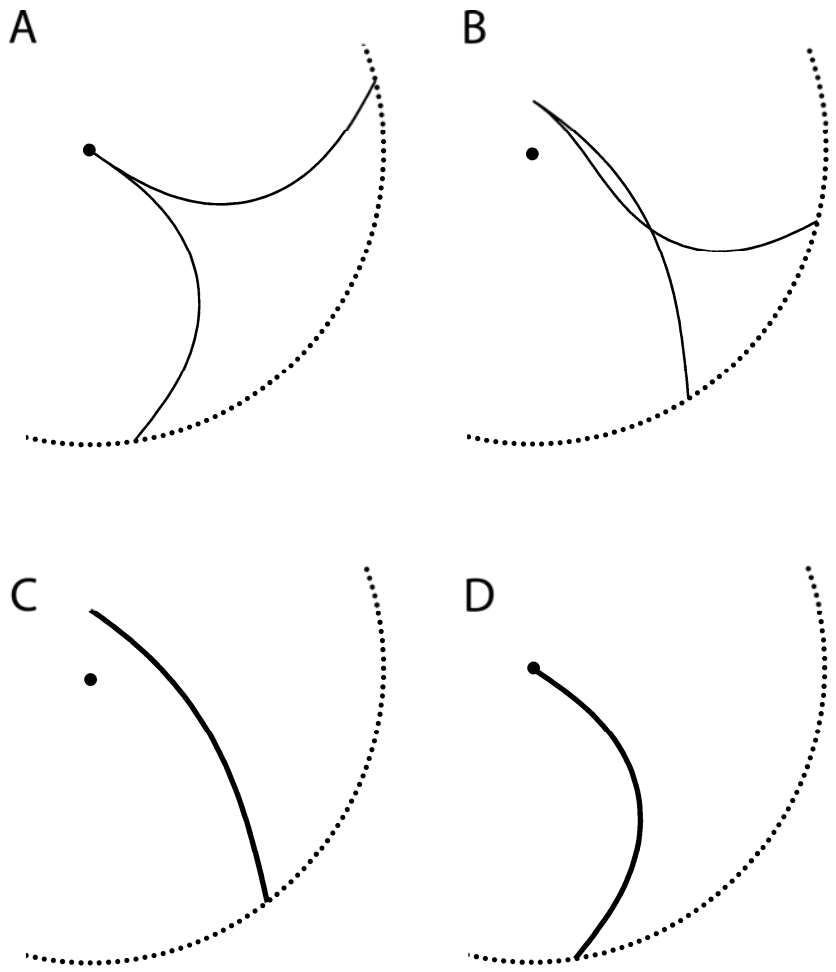

Fig. S1. Sample equilibrium forms of a microtubule. The dotted line represents the cell boundary. The dot marks the cell center. The more centrally positioned end of the microtubule is clamped at the centrosome, which is not specially labeled. In *A*, the centrosome is in the cell center. The curves represent the alternative equilibrium forms of the microtubule that is clamped at the angle selected for the illustration. In *B*, the centrosome has been moved away from the center, and the two microtubule forms are no longer equivalent. In *C*, a further displacement of the centrosome away from the center has led to disappearance of the more highly bent (metastable) equilibrium form. In *D*, moving the centrosome back to the cell center has restored the microtubule shape that was seen in *A*, but only one of the two forms that differ in the direction of buckling is now occupied (the one which is continuously and reversibly connected with the form that was the only one left occupied in the state shown in *C*).

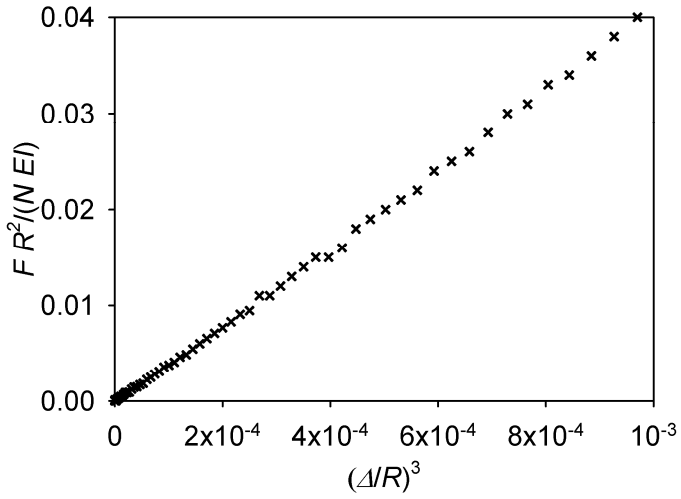

Fig. S2. Total force exerted by the microtubules on the centrosome for small deviations of the centrosome from the center of a flat cell, starting from the fully symmetric cytoskeleton conformation. $L = 1.25\ R$. This is a plot of F as a function of Δ, with the argument and the value of the function normalized to N, R, and EI in order to show the parameter-independent behavior of the nondimensionalized model.

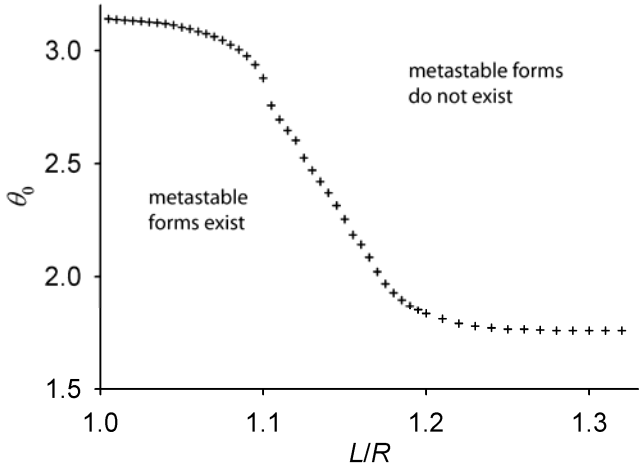

Fig. S3. The angle of the unstressed microtubule direction, beyond which metastable forms no longer exist, in the course of the continuous microtubule elongation in the flat cell.