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Supporting Material

Title: Mitotic membrane helps to focus and stabilize the mitotic spindle

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Supplementary Information

We assume that the MTs push against the membrane, and that this force is counterbalanced by that force that is supplied by the membrane's deformation.

The geometry of the problem is assumed to be as described in Figure S4. Here r is the initial length of the mitotic membrane, l is the (half) length of the spindle, s is the overlap between MTs, and m is the MT length. Note that

$$l = m - \tfrac{1}{2}s$$

We assume that the MTs are focused and so all the force pushing on the membrane is along the horizontal. The magnitude of this force is proportional to both the length of the overlap between antiparallel aligned MTs (as motors need to be bound to both MTs), the total number of plus-end motors (N), and the plus-end motor force:

$$F_{\mathrm{motor}} = \alpha s N F_{\mathrm{KinL}}.$$

The coefficient α represents the fraction of motors that are bound to antiparallel aligned MTs. Note that, using $N = 2000$, $s = 1\,\mu\mathrm{m}$ and $F_{\mathrm{KinL}} = 2\,\mathrm{pN}$ we obtain:

$$F_{\mathrm{motor}} = 4000\alpha\,\mathrm{pN}.$$

Note that α unknown.

If the system is at steady-state, then this force has to be counterbalanced by the force from the elastic membrane. This force is given by

$$F_{\mathrm{mem}} = 2k_{\mathrm{mem}}\delta\sin\theta,$$

where k_{mem} is the elastic modulus of membrane and δ is the deformation. The coefficient 2 comes from the symmetry of the problem.

The deformation is approximated as in Figure S4 by

$$\delta = h - r\theta.$$

The length h is

$$\begin{aligned} h &= \sqrt{(r\sin\theta)^2 + (l - r\cos\theta)^2} \\ &= \sqrt{r^2 + l^2 - 2lr\cos\theta} \\ &= \sqrt{r^2 + (m - \tfrac{1}{2}s)^2 - 2(m - \tfrac{1}{2}s)r\cos\theta}, \end{aligned}$$

and thus

$$F_{\mathrm{mem}} = 2k_{\mathrm{mem}}\left(\sqrt{r^2 + (m - \tfrac{1}{2}s)^2 - 2(m - \tfrac{1}{2}s)r\cos\theta} - r\theta\right)\sin\theta,$$

Using values $m = 5\,\mu\mathrm{m}$, $s = 1\,\mu\mathrm{m}$, $\theta = \frac{1}{4}\pi$ and $k_{\mathrm{mem}} = 200\,\mathrm{pN}$ leads to values of force $F_{\mathrm{mem}} \approx 41\,\mathrm{pN}$.

We can use the fact that the forces are balanced at steady-state: $F_{\mathrm{mem}} = F_{\mathrm{motor}}$, to solve for $\alpha \approx 0.01$.

In general, the number of motors, or the length of MTs will influence the spindle length. At steady-state

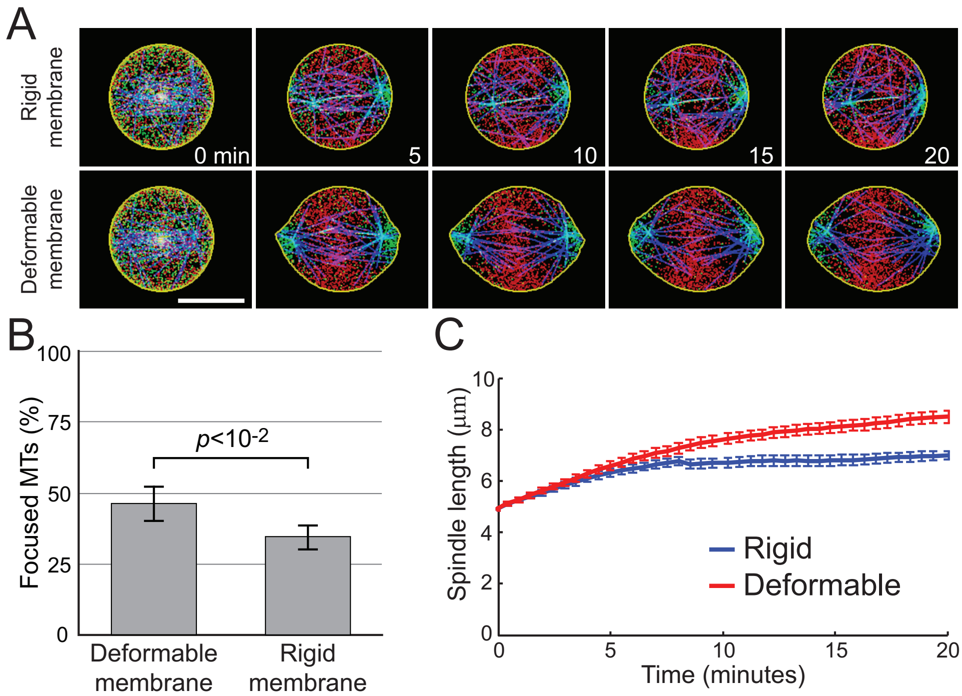
$$2k_{\text{mem}}\left(\sqrt{r^2+(m-\tfrac{1}{2}s)^2-2(m-\tfrac{1}{2}s)r\cos\theta}-r\theta\right)\sin\theta=\alpha sNF_{\text{KinL}},$$

which can be solved, numerically, for s and hence spindle length

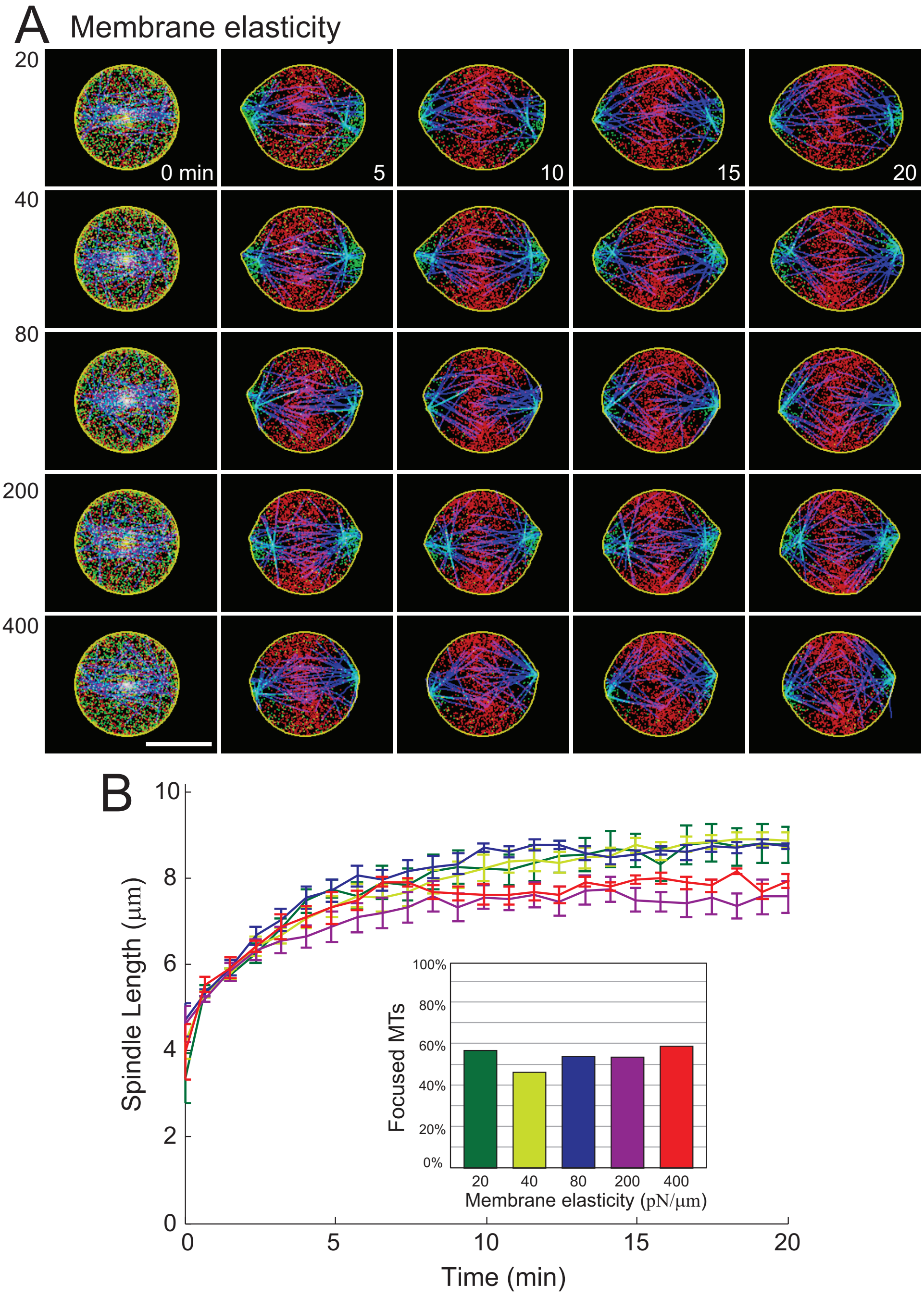
$$2l = 2m - s.$$

This is done in Figure 5 for varying numbers of motors and MT lengths.

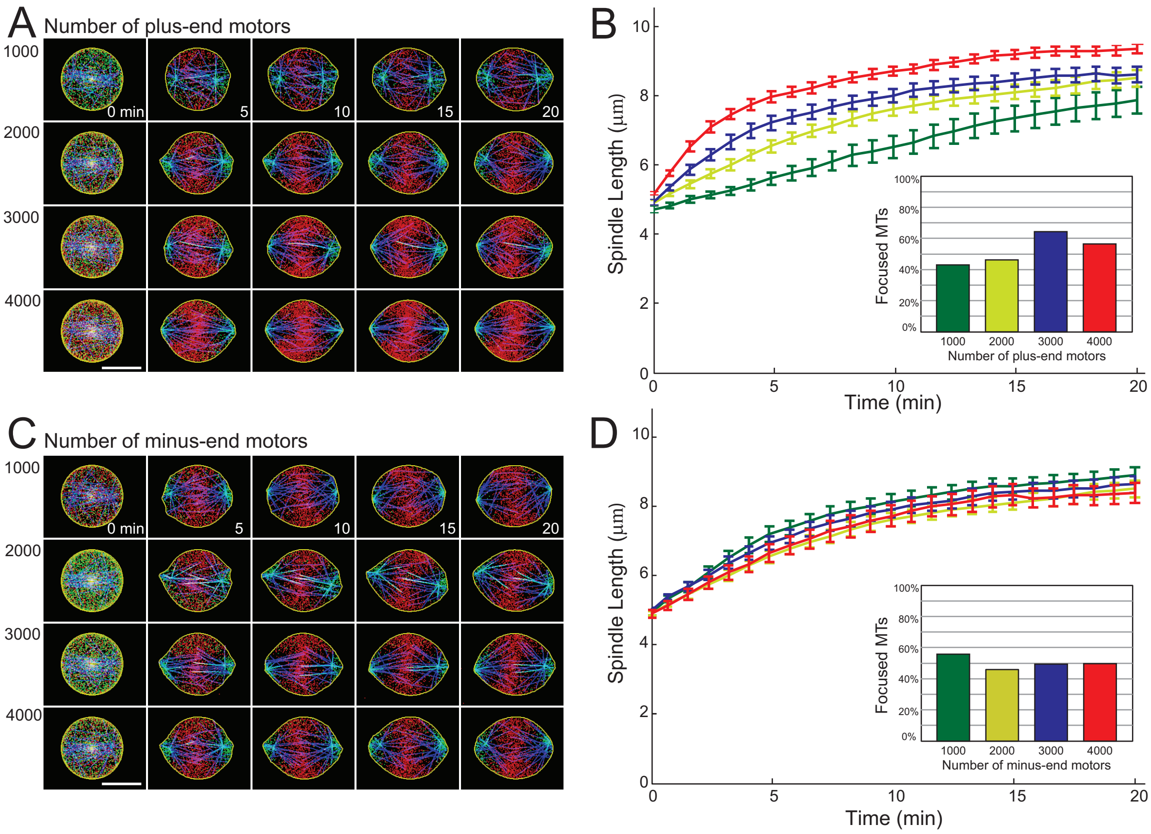
Supplementary Figures



Supplementary Figure S1. In the presence of astral MTs a deformable membrane can still focus MTs. These simulations are the counterparts of those of Fig. 2 but the system has two asters. The degree of focusing is smaller in this case.



Supplementary Figure S2. In the absence of astral MTs, the steady-state length and shape of the mitotic spindle depends on the membrane elasticity. These simulations are the counterparts of those of Fig. 3 but for a system with two asters.



Supplementary Figure S3. In the absence of astral MTs, varying the number of MT motors influences the speed at which the system reaches steady-state. These simulations are the counterparts of those of Fig. 5 but for a system with two asters.

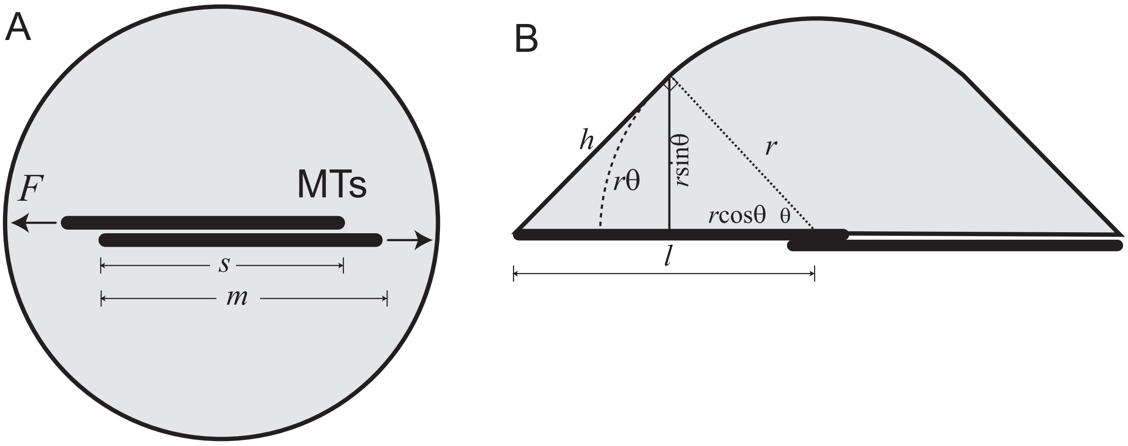


Figure S4. A. The undeformed membrane is assumed to be circular, with radius r. The MTs, of length m are pushed apart by motors that lie in the region of antiparallel overlap, which has length s. B. At steady state, the membrane has been deformed. We assume that the deformation is such that an arc (dashed line) of length $r\theta$ is stretched to the hypotenuse (length h) of a triangle. Note that the amount of antiparallel overlap is shorter. The spindle length is given by $2l$.