Additional file 1

Mathematical Proofs for Hit Integration

We prove three lemmas used for Equation (7) $\sim$ Equation (10); The first lemma is for Equation (7), the second is for Equation (8) and Equation (9), and the third is for Equation (10).

Lemma 1 *The function I can be solved by a recursive function such as*

$$I^n[i,b] = I^n[i,0b] + I^{(n+1)}[i,1b] - I^{(n+1)}[i,0b].$$

Proof) By applying Equation (3), the right terms are derived directly.

$$\begin{aligned} I^n[i,b] &= \int p^n f[i,b]dp \\ &= \int p^n \left(p \cdot f[i,1b] + (1-p)f[i,0b]\right) dp \\ &= \int p^{(n+1)} f[i,1b]dp + \int p^n f[1,0b]dp \\ &\quad - \int p^{(n+1)} f[i,0b]dp \\ &= I^{(n+1)}[i,1b] + I^n[i,0b] - I^{(n+1)}[i,0b]. \end{aligned}$$

Lemma 2 *If Q hits b, $I^n[i,b]$ is represented as*

$$I^n[i,b] = \begin{cases} \frac{p^{n+1}}{n+1} & \text{if } \ b \in B \ \text{ and } \ |b| = m \\ 0 & \text{if } \ i < m. \end{cases}$$

Proof) Applying Equation (1) and Equation (2) respectively, the right terms are derived.

$$\begin{aligned} I^n[i,b] &= \int p^n f[i,b]dp = \int (p^n \cdot 1)dp = \frac{p^{n+1}}{n+1} \\ I^n[i,b] &= \int p^n f[i,b]dp = \int (p^n \cdot 0)dp = 0. \end{aligned}$$

Lemma 3 $I^n[i,0b] = I^n[i-|b|+|b'|,0b']$ *where* $0b' = B(0b)$

Proof) Applying Equation (4), the right term is derived.

$$\begin{aligned} I^n[i,0b] &- I^n[i-|b|+|b'|,0b'] \\ &= \int p^n f[i,0b]dp - \int p^n f[i-|b|+|b'|,0b']dp \\ &= \int (p^n f[i,0b] - p^n f[i-|b|+|b'|,0b'])dp \\ &= \int p^n (f[i,0b] - f[i-|b|+|b'|,0b'])dp \end{aligned}$$

$$
\begin{aligned}
&= \int p^n \cdot 0 \ dp \\
&= 0.
\end{aligned}
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