

Sex-Averaged Recombination and Mutation Rates on the X Chromosome: A Comment on Labuda et al. (2010)

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Appendix S1. Correction of the Equations in Appendix A of Labuda et al. (2010):

Genetic Diversity of the X Chromosome and the Autosomes

The sex-averaged X chromosome mutation rate (μ_x) does not depend on the sex-ratio of the population and has been shown to be $\mu_x = (2\mu_f + \mu_m) / 3$, where μ_f and μ_m are the female and male mutation rates, respectively. This expression accounts for the fact that each X chromosome spends 2/3 of its time in females. Importantly, the above expression differs from that used to calculate the sex-averaged recombination rate ($r_x = (2/3)r_{fx}$) since mutation, unlike recombination, can occur in males.

Given the expression for μ_x , the population-scaled mutation rate for the X chromosome Θ_x can be found from the expression

$$\Theta_x = 4N_{ex} \frac{2\mu_f + \mu_m}{3}. \quad (\text{Equation A2})$$

Then, the ratio between genetic diversity on the X chromosome and the autosomes does depend on the ratio of breeding females and males in the population β , and is written as

$$\frac{\Theta_x}{\Theta_A} = \frac{2}{3} \left(\frac{9\beta + 9}{8\beta + 16} \right) \left(\frac{2\mu_f + \mu_m}{\mu_f + \mu_m} \right). \quad (\text{Equation A3})$$

If we define $\alpha = \mu_m / \mu_f$, then Equation A3 can be re-written as

$$\frac{\Theta_x}{\Theta_A} = \frac{2}{3} \left(\frac{9\beta + 9}{8\beta + 16} \right) \left(\frac{2 + \alpha}{1 + \alpha} \right). \quad (\text{Equation A4})$$

Labuda et al. (2010) then use estimates of Θ_x / Θ_A and β to estimate α . The correct expression is

$$\alpha = \frac{4 \frac{\theta_x}{\theta_A} (\beta + 2) - 6(\beta + 1)}{3(\beta + 1) - 4 \frac{\theta_x}{\theta_A} (\beta + 2)}. \quad (\text{Equation A5})$$

If α were known, it could be used to estimate β by

$$\beta = \frac{8 \frac{\theta_x}{\theta_A} (\alpha + 1) - 3(\alpha + 2)}{3(\alpha + 2) - 4 \frac{\theta_x}{\theta_A} (\alpha + 1)}. \quad (\text{Equation A6})$$

Figure S1: Ratio of X to Autosome Genetic Diversity (θ_X/θ_A) as a Function of the Breeding Ratio (β) in the Population for Different Values of α

The curves in this figure are from the corrected Equation A5 (see Appendix S1).

