Web-based Supplementary Materials for "Longitudinal studies of binary response data following case-control and stratified case-control sampling: design and analysis," by Jonathan S. Schildcrout and Paul J. Rathouz

1. Web Appendix A: Details of the derivation of (15)

To show (15), note from (5) that

$$B_{ij} = \log\left\{1 - (1-r)\lambda_{Pij}(1)\right\} - \log\left\{1 - (1-r)\lambda_{Pij}(0)\right\} \ ,$$

where $r = \pi(1, \boldsymbol{X}_{1i})/\pi(0, \boldsymbol{X}_{1i})$ and $\boldsymbol{X}_i$ has been suppressed from the argument of $\lambda_{Pij}(y)$ for ease of exposition. Therefore,

$$\frac{\partial B_{ij}}{\partial \boldsymbol{\gamma}} = \frac{\partial \lambda_{Pij}(1)}{\partial \boldsymbol{\gamma}}\left\{\frac{-(1-r)}{1-(1-r)\lambda_{Pij}(1)}\right\} - \frac{\partial \lambda_{Pij}(0)}{\partial \boldsymbol{\gamma}}\left\{\frac{-(1-r)}{1-(1-r)\lambda_{Pij}(0)}\right\} \ .$$

From (6),

$$\frac{\partial \lambda_{Pij}(y)}{\partial \boldsymbol{\gamma}} = \begin{pmatrix} \boldsymbol{w}_{1,ij} \\ y \times \boldsymbol{w}_{2,ij} \end{pmatrix} \lambda_{Pij}(y)\{1 - \lambda_{Pij}(y)\} \ ,$$

which yields

$$\frac{\partial B_{ij}}{\partial \boldsymbol{\gamma}} = -\begin{pmatrix} \boldsymbol{w}_{1,ij} \\ \boldsymbol{w}_{2,ij} \end{pmatrix} \lambda_{Pij}(1)\{1 - \lambda_{Pij}(1)\}\left\{\frac{(1-r)}{1-(1-r)\lambda_{Pij}(1)}\right\}$$

$$+ \begin{pmatrix} \boldsymbol{w}_{1,ij} \\ 0 \end{pmatrix} \lambda_{Pij}(0)\{1 - \lambda_{Pij}(0)\}\left\{\frac{(1-r)}{1-(1-r)\lambda_{Pij}(0)}\right\} \ .$$

Collecting components of $(\partial B_{ij}/\partial \boldsymbol{\gamma})$ across j yields (15).

2. Web Appendix B: Details of data generation from Section (4.2)

Using $N = 1,000,000$, we first calculated stratum-specific population sizes, N_z and N_{z,x_1}. At each replication, we generated stratum-specific sample sizes, n_z from $\text{Bin}\{N_z, \pi(z)\}$ for

$S(z)$ and n_{z,x_1} from $\text{Bin}\{N_{z,x_1}, \pi(z, x_1)\}$ for $S(z, x_1)$. We then generated individual subjects'

data following the population model described in section **??**. Based on the value of sampling

variable(s) Z_i or (Z_i, x_{1i}), subjects were included into the sample if the desired stratum-

specific sample size had not yet been reached. Once the stratum specific sample size was

reached, newly generated members of the population belonging to that stratum were rejected

from the sample. This proceeded until all strata sample sizes were obtained.

Note that another way to approach this simulation study is to first generate a population

of size N and then to sample directly from the population. If the population is finite, n_z and

$n_{z,x}$ follow a hypergeometric distribution, and all stratum specific population sizes must be

large for valid inferences to obtain, unless the dependent sampling scheme is acknowledged

in the analysis.