Accounting for Data Errors Discovered from an Audit in Multiple Linear Regression

Supplementary Material

Confidence intervals using $\hat{\lambda}_2$ and $\hat{\lambda}_3$

Construction of confidence intervals for β_1 when using $\hat{\lambda}_2$ is similar to that described in Section 2.1 for $\hat{\lambda}_1$ except $\theta = (\gamma_0, \gamma_1, \mu_w, \sigma_w^2, p, \sigma_u^2)$, lines 3 and 4 of the estimating equations are $(W_i - \mu_w)$ and $((W_i - \mu_w)^2 - \sigma_w^2)$ respectively, and

$$\frac{\partial g(\theta)}{\partial \theta} = \left(0, \frac{\sigma_w^2}{\sigma_w^2 - \sigma_u^2 p}, 0, -\frac{\gamma_1 \sigma_u^2 p}{(\sigma_w^2 - \sigma_u^2 p)^2}, \frac{\gamma_1 \sigma_w^2 \sigma_u^2}{(\sigma_w^2 - \sigma_u^2 p)^2}, \frac{\gamma_1 \sigma_w^2 p}{(\sigma_w^2 - \sigma_u^2 p)^2}\right).$$

Constructing confidence intervals for β_1 when using $\hat{\lambda}_3$ is also similar to that described in Section 2.1 for $\hat{\lambda}_1$ except $\theta = (\gamma_0, \gamma_1, \mu_x, \sigma_x^2, \sigma_t^2)$, lines 5 and 6 of the estimating equations are replaced with the single line $(T_i^2 - \sigma_t^2)V_i$, and

$$\frac{\partial g(\theta)}{\partial \theta} = \left(0, \frac{\sigma_x^2 + \sigma_t^2}{\sigma_x^2}, 0, -\frac{\gamma_1 \sigma_t^2}{(\sigma_x^2)^2}, \frac{\gamma_1}{\sigma_x^2}\right).$$

Bias when outcome and predictor are sometimes incorrect in the database

Under models (1), (2), and (5), and the assumptions given in Section 2.3, γ_1 in the model $E(Y^*|W) = \gamma_0 + \gamma_1 W$ is equal to the following:

$$\begin{aligned}
\gamma_1 &= \frac{Cov(Y^*, W)}{Var(W)} \\
&= \frac{Cov(X + SU, Y + S^y U^y + SU^*)}{\sigma_x^2 + p\sigma_u^2} \\
&= \frac{Cov(X, Y) + Cov(SU, SU^*)}{\sigma_x^2 + p\sigma_u^2} \\
&= \frac{Cov(X, Y)}{\sigma_x^2} \frac{\sigma_x^2}{\sigma_x^2 + p\sigma_u^2} + \frac{E_S\left\{E_{U|S}\left[(SU - E(SU))(SU^* - E(SU^*))\right]\right\}}{\sigma_x^2 + p\sigma_u^2} \\
&= \beta_1 \frac{\sigma_x^2}{\sigma_x^2 + p\sigma_u^2} + \frac{E_S\left\{S^2 Cov(U, U^*)\right\}}{\sigma_x^2 + p\sigma_u^2} \\
&= \beta_1 \frac{\sigma_x^2}{\sigma_x^2 + p\sigma_u^2} + \frac{p\sigma_{u,u^*}}{\sigma_x^2 + p\sigma_u^2} \\
&= \beta_1 \lambda + \nu.
\end{aligned}$$

Second Audit Size Selection for Data Example

After the first set of audits, we performed simulations using estimated parameters in order to decide whether to perform additional audits to improve estimates and if yes, how many. Simulations were similar to those described in Section 3 of the manuscript except data were generated using estimated parameters 1) computed separately for each site, and 2) computed by combining first audit results from all sites. For each site we performed simulations using the estimated error rate and the upper and lower levels of the 80% confidence intervals around the error rate. We simulated using multiple values for the correlation ρ_{u,u^*} We considered audit sizes of 0, 25, 50, 75, 100, and N_z where N_z represents the total number of patients in the site.

Table 25 summarizes our simulation results. Site-specific paramater estimates for all sites except Site C suggested no additional audits were needed based on mean squared errors that remained nearly constant when more records were audited. This result is due to low error rates (p), low error magnitudes (σ_u), and/or low correlation between errors (ρ_{u,u^*}) in the first audit. Using site-specific estimates from Site C, even with 100 audits, the MSE was much larger than if data were re-entered for all records. However, these site specific parameter estimates were based on a relatively small number of audits and Site C's were driven by a single extreme value. The simulations using the combined-site parameter estimates for σ_u and ρ_{u,u^*} were still influenced by the outlier, but to a lesser extent because there was more data. We felt that these simulations using the combined-site parameter estimates could represent worst-case scenarios for all sites except Site C and perhaps a more realistic situation for Site C.

The MSE exponentially decreases as the audit size increases. Ideally, we would like to choose an audit size where the slope of the curve is as close to 0 as possible and below the MSE of the naive estimate. For example, Site C's MSE using across-site parameter estimates dropped below the naive estimate's MSE with an audit size of 50, and continued dropping at a slower rate until it reached the MSE if all data were audited. In contrast, Site D's MSE using across-site parameter estimates dropped below the naive estimate's MSE with an audit size of 100. However, the slope of the MSE curve appeared to be fairly level after 50 audits, implying that there was not much to be gained in terms of MSE by auditing many additional records beyond 50. These sorts of observations as well as practical feasibility guided our audit size considerations. It should also be noted that these simulations were based on parameter estimates using non-transformed CD4 measurements.

Based on feasibility and the high error rate and magnitude in the first set of audits, we chose to aim for 75 total audits in Site C. For most of the other sites, with the combined, worst-case scenario parameter estimates, the MSE was roughly similar to the naive estimate at 50 audited charts. As Sites B and E had higher error rates, we chose to perform 25-30 additional audits at these sites, bringing our total to around 50 at each. Site F had no errors in 25 initial chart reviews, so we felt comfortable performing no additional audits for this site. Sites A and D had identical low error rates in the

first audit and we felt it would be best to perform additional audits for both, but also reasonable not to perform additional audits (particularly because the results shown in Table 25 for across-site parameter estimates represent what we considered worst-case scenarios for these sites). In the end, we chose not to perform audits in Site D because of recent personnel turnover at that site, but we performed additional audits in Site A because we had an approaching site-visit.

Code Used for Analyses and Simulations

The R code used for the analyses and simulations in this manuscript is posted at http://biostat.mc.vanderbilt.edu/DataAuditSimulationCode.

Tables in Supplementary Appendix

Table	Simulation Scenario	N	σ_u	$\rho_{u,u*}$
1	Errors in Predictor	1000	50	
2	Errors in Predictor	1000	20	
3	Errors in Predictor	500	50	
4	Errors in Predictor	500	20	
5	Errors in Predictor	100	50	
6	Errors in Predictor	100	20	
7	Errors in Predictor and Outcome	1000	50	$-0.5, 0, 0.5$
8	Errors in Predictor and Outcome	1000	20	$-0.5, 0, 0.5$
9	Errors in Predictor and Outcome	500	50	$-0.5, 0, 0.5$
10	Errors in Predictor and Outcome	500	20	$-0.5, 0, 0.5$
11	Errors in Predictor and Outcome	100	50	$-0.5, 0, 0.5$
12	Errors in Predictor and Outcome	100	20	$-0.5, 0, 0.5$
13	Errors in Predictor and Outcome with Covariate	1000	50	-0.5
14	Errors in Predictor and Outcome with Covariate	1000	50	0
15	Errors in Predictor and Outcome with Covariate	1000	50	0.5
16	Errors in Predictor and Outcome with Covariate	1000	20	-0.5
17	Errors in Predictor and Outcome with Covariate	1000	20	0
18	Errors in Predictor and Outcome with Covariate	1000	20	0.5
19	Errors in Predictor and Outcome with Covariate	500	50	-0.5
20	Errors in Predictor and Outcome with Covariate	500	50	0
21	Errors in Predictor and Outcome with Covariate	500	50	0.5
22	Errors in Predictor and Outcome with Covariate	500	20	-0.5
23	Errors in Predictor and Outcome with Covariate	500	20	0
24	Errors in Predictor and Outcome with Covariate	500	20	0.5
25	Site-specific Simulations to determine Size of Second Audit			

Table 1: Simulation Results when Predictor is Sometimes Incorrect (N=1000,$\sigma_u = 50$)

						$\hat{\lambda}_1$				$\hat{\lambda}_2$				$\hat{\lambda}_3$			
p	n_v	%-Bias[a]	$\overline{\text{SE}}$[b] ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)
0.05	0	-4.78	0.032	0.663	3.4												
	25					0.38	0.066	0.847	10	3.1	1.058	0.849	1872.7	0.06	0.068	0.834	26.9
	50					0.26	0.056	0.898	5.1	0.52	0.061	0.9	6.9	0.09	0.056	0.901	6.7
	100					0.13	0.047	0.934	2.6	0.2	0.049	0.938	2.8	0.13	0.047	0.943	2.5
	200					-0.03	0.04	0.947	1.6	0	0.041	0.948	1.7	-0.01	0.04	0.951	1.6
	300					-0.03	0.037	0.953	1.4	-0.02	0.038	0.954	1.4	-0.02	0.037	0.954	1.4
	1000	-0.03	0.032	0.952	1												
0.1	0	-9.03	0.032	0.206	9.4												
	25					0.71	0.09	0.766	17.6	0.75	0.674	0.765	1031.6	0.83	0.091	0.762	31.4
	50					0.31	0.071	0.873	7.4	0.67	0.08	0.869	9.4	0.3	0.071	0.878	7.6
	100					0.23	0.057	0.926	3.8	0.39	0.062	0.924	4.5	0.23	0.057	0.93	3.6
	200					0.08	0.046	0.94	2.3	0.15	0.049	0.944	2.6	0.08	0.046	0.942	2.3
	300					0.06	0.042	0.944	1.9	0.1	0.043	0.945	2	0.06	0.042	0.946	1.9
	1000	0.03	0.032	0.947	1												
0.2	0	-16.66	0.031	0.001	28.9												
	25					1.43	0.126	0.798	30.1	4.85	0.577	0.77	737.3	1.43	0.125	0.801	27.7
	50					0.64	0.094	0.88	11.3	1.42	0.116	0.867	17.8	0.63	0.094	0.886	10.6
	100					0.28	0.072	0.928	5.7	0.63	0.082	0.928	7.6	0.27	0.072	0.929	5.5
	200					0.1	0.055	0.944	3.2	0.26	0.061	0.947	3.9	0.09	0.055	0.944	3.1
	300					0.03	0.048	0.946	2.4	0.13	0.052	0.948	2.8	0.03	0.048	0.946	2.4
	1000	-0.01	0.032	0.948	1												
0.3	0	-23.09	0.031	0	54.5												
	25					2.59	0.151	0.839	36.8	-7.18	41.946	0.803	77951.9	2.58	0.151	0.844	34.1
	50					1.14	0.11	0.895	15.5	2.22	0.145	0.877	29.6	1.13	0.11	0.899	15
	100					0.53	0.081	0.927	7.2	0.96	0.098	0.921	10.8	0.52	0.081	0.928	7.1
	200					0.26	0.061	0.942	3.9	0.45	0.071	0.946	5.3	0.25	0.061	0.943	3.9
	300					0.18	0.053	0.951	2.8	0.33	0.06	0.952	3.6	0.18	0.053	0.95	2.8
	1000	0.08	0.032	0.95	1												

[a]Percent bias = $\left(\frac{\hat{\beta}-\beta}{\beta}\right) \times 100\%$.

[b]$\overline{\text{SE}}$= mean standard error of estimates.

Table 2: Simulation Results when Predictor is Sometimes Incorrect (N=1000,$\sigma_u = 20$)

						$\hat{\lambda}_1$				$\hat{\lambda}_2$				$\hat{\lambda}_3$			
p	n_v	%-Bias[a]	$\overline{\text{SE}}$[b] ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)
0.05	0	-0.8	0.032	0.948	1												
	25					0.06	0.034	0.95	1.3	0	0.034	0.949	1.2	0.05	0.035	0.95	2.4
	50					0.01	0.033	0.951	1.1	-0.01	0.033	0.952	1.1	0.03	0.033	0.953	1.7
	100					0.01	0.032	0.952	1	0	0.032	0.953	1	0.01	0.032	0.953	1
	200					0	0.032	0.952	1	0	0.032	0.952	1	0	0.032	0.952	1
	300					0	0.032	0.953	1	0	0.032	0.954	1	0	0.032	0.954	1
	1000	0	0.032	0.954	1												
0.1	0	-1.58	0.032	0.911	1.3												
	25					0.09	0.037	0.94	1.6	-0.02	0.036	0.94	1.5	0.11	0.037	0.941	2.2
	50					0.03	0.034	0.947	1.3	-0.02	0.034	0.945	1.2	0.03	0.034	0.949	1.2
	100					0.01	0.033	0.949	1.1	-0.02	0.033	0.95	1.1	0.01	0.033	0.95	1.1
	200					-0.01	0.033	0.95	1.1	-0.02	0.033	0.95	1.1	-0.01	0.033	0.95	1.1
	300					0	0.032	0.949	1.1	-0.01	0.032	0.949	1.1	-0.01	0.032	0.949	1.1
	1000	-0.01	0.032	0.951	1												
0.2	0	-3.1	0.032	0.835	2												
	25					0.28	0.041	0.943	2	0.07	0.04	0.941	1.8	0.27	0.041	0.947	1.9
	50					0.14	0.037	0.952	1.4	0.05	0.036	0.952	1.4	0.14	0.037	0.954	1.4
	100					0.06	0.035	0.954	1.2	0.02	0.035	0.952	1.2	0.07	0.035	0.954	1.2
	200					0.03	0.033	0.955	1.1	0.01	0.033	0.954	1.1	0.03	0.033	0.956	1.1
	300					0.02	0.033	0.954	1.1	0.01	0.033	0.956	1.1	0.02	0.033	0.954	1.1
	1000	0.01	0.032	0.953	1												
0.3	0	-4.54	0.032	0.696	3.1												
	25					0.39	0.045	0.939	2.4	0.1	0.043	0.937	2.1	0.4	0.045	0.943	2.3
	50					0.21	0.039	0.944	1.7	0.09	0.038	0.942	1.6	0.21	0.039	0.947	1.6
	100					0.12	0.036	0.948	1.3	0.06	0.036	0.945	1.3	0.12	0.036	0.947	1.3
	200					0.07	0.034	0.947	1.2	0.04	0.034	0.948	1.2	0.07	0.034	0.946	1.2
	300					0.06	0.034	0.949	1.2	0.04	0.034	0.948	1.2	0.06	0.034	0.949	1.2
	1000	0.03	0.032	0.951	1												

[a]Percent bias = $\left(\frac{\hat{\beta}-\beta}{\beta}\right) \times 100\%$.

[b]$\overline{\text{SE}}$= mean standard error of estimates.

Table 3: Simulation Results when Predictor is Sometimes Incorrect (N=500,$\sigma_u = 50$)

						$\hat{\lambda}_1$				$\hat{\lambda}_2$				$\hat{\lambda}_3$			
p	n_v	%-Bias[a]	$\overline{\text{SE}}$[b] ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)
0.05	0	-4.75	0.045	0.798	4.6												
	25					0.45	0.075	0.897	11.6	1.21	0.103	0.902	54.9	-0.37	0.076	0.892	19.9
	50					0.19	0.065	0.927	5.8	0.45	0.069	0.932	7.2	-0.09	0.065	0.932	6.5
	100					0.08	0.057	0.945	3.6	0.15	0.058	0.946	3.8	0.04	0.057	0.95	3.4
	200					0.06	0.051	0.945	2.8	0.09	0.051	0.945	2.8	0.05	0.051	0.946	2.7
	300					0.01	0.048	0.947	2.5	0.03	0.049	0.947	2.5	0.01	0.048	0.948	2.4
	500	0	0.045	0.949	2.1												
0.1	0	-9.05	0.045	0.475	10.6												
	25					0.82	0.1	0.855	19.4	2.9	0.177	0.852	100.9	0.89	0.101	0.85	44.6
	50					0.34	0.08	0.911	8.7	0.79	0.09	0.911	12.6	0.35	0.08	0.918	9.2
	100					0.24	0.066	0.932	4.9	0.39	0.069	0.936	5.5	0.24	0.066	0.94	4.7
	200					0.16	0.056	0.943	3.3	0.22	0.057	0.945	3.5	0.17	0.056	0.944	3.3
	300					0.1	0.052	0.945	2.8	0.14	0.053	0.945	2.9	0.1	0.052	0.947	2.8
	500	0.02	0.045	0.951	2												
0.2	0	-16.62	0.044	0.051	30.1												
	25					1.12	0.13	0.824	28.2	6.17	0.802	0.81	1401	1.12	0.13	0.827	29.1
	50					0.71	0.101	0.909	12.5	1.43	0.122	0.901	19.1	0.71	0.101	0.915	11.9
	100					0.38	0.078	0.942	6.7	0.7	0.088	0.943	8.5	0.41	0.078	0.946	6.5
	200					0.16	0.062	0.948	4	0.32	0.067	0.948	4.7	0.17	0.062	0.949	4
	300					0.06	0.056	0.949	3.2	0.16	0.06	0.95	3.7	0.06	0.056	0.949	3.2
	500	0	0.045	0.946	2.1												
0.3	0	-23.01	0.044	0.002	55.3												
	25					1.73	0.152	0.852	36.1	5.29	0.353	0.821	306.7	1.81	0.151	0.86	33.5
	50					0.81	0.113	0.903	16.5	1.82	0.146	0.889	27.4	0.8	0.113	0.909	15.9
	100					0.36	0.086	0.936	8.2	0.76	0.102	0.939	11.4	0.37	0.086	0.937	8.1
	200					0.21	0.067	0.939	4.8	0.38	0.076	0.943	6.1	0.21	0.067	0.941	4.7
	300					0.14	0.059	0.946	3.7	0.24	0.066	0.949	4.5	0.14	0.059	0.944	3.7
	500	-0.02	0.045	0.95	2												

[a]Percent bias = $\left(\frac{\hat{\beta}-\beta}{\beta}\right) \times 100\%$.

[b]$\overline{\text{SE}}$= mean standard error of estimates.

Table 4: Simulation Results when Predictor is Sometimes Incorrect (N=500,$\sigma_u = 20$)

						$\hat{\lambda}_1$				$\hat{\lambda}_2$				$\hat{\lambda}_3$			
p	n_v	%-Bias[a]	$\overline{\text{SE}}$[b] ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)
0.05	0	-0.73	0.045	0.948	2.1												
	25					0.14	0.047	0.95	2.3	0.1	0.047	0.949	2.3	0.23	0.048	0.947	4.7
	50					0.1	0.046	0.952	2.2	0.08	0.046	0.952	2.2	0.13	0.046	0.949	2.7
	100					0.08	0.045	0.951	2.1	0.07	0.045	0.95	2.1	0.07	0.045	0.95	2.1
	200					0.06	0.045	0.951	2.1	0.06	0.045	0.951	2.1	0.07	0.045	0.951	2.1
	300					0.06	0.045	0.951	2.1	0.06	0.045	0.951	2.1	0.06	0.045	0.951	2.1
	500	0.06	0.045	0.953	2												
0.1	0	-1.51	0.045	0.933	2.3												
	25					0.2	0.049	0.947	2.6	0.11	0.048	0.944	2.5	0.26	0.049	0.943	3.9
	50					0.11	0.047	0.946	2.3	0.07	0.047	0.944	2.3	0.11	0.047	0.948	2.3
	100					0.09	0.046	0.947	2.2	0.07	0.046	0.947	2.2	0.09	0.046	0.945	2.2
	200					0.09	0.046	0.946	2.1	0.08	0.046	0.945	2.1	0.09	0.046	0.945	2.1
	300					0.08	0.045	0.946	2.1	0.08	0.045	0.946	2.1	0.08	0.045	0.946	2.1
	500	0.05	0.045	0.951	2.1												
0.2	0	-3.15	0.045	0.894	3												
	25					0.31	0.053	0.949	3	0.08	0.052	0.95	2.8	0.31	0.053	0.949	2.9
	50					0.15	0.049	0.952	2.5	0.05	0.049	0.952	2.4	0.15	0.049	0.954	2.4
	100					0.06	0.047	0.949	2.2	0.02	0.047	0.949	2.2	0.05	0.047	0.95	2.2
	200					0.01	0.046	0.949	2.2	-0.01	0.046	0.949	2.2	0.01	0.046	0.948	2.2
	300					0	0.046	0.948	2.1	-0.01	0.046	0.948	2.1	-0.01	0.046	0.948	2.1
	500	-0.03	0.045	0.951	2												
0.3	0	-4.53	0.045	0.825	4.1												
	25					0.45	0.056	0.942	3.5	0.19	0.055	0.942	3.2	0.47	0.056	0.944	3.5
	50					0.28	0.051	0.946	2.7	0.16	0.051	0.944	2.7	0.28	0.051	0.946	2.7
	100					0.15	0.049	0.946	2.4	0.1	0.048	0.945	2.4	0.15	0.049	0.945	2.4
	200					0.07	0.047	0.945	2.3	0.05	0.047	0.945	2.3	0.06	0.047	0.945	2.3
	300					0.06	0.047	0.944	2.3	0.06	0.047	0.946	2.3	0.06	0.047	0.944	2.3
	500	0.03	0.045	0.947	2												

[a]Percent bias = $\left(\frac{\hat{\beta}-\beta}{\beta}\right) \times 100\%$.

[b]$\overline{\text{SE}}$= mean standard error of estimates.

Table 5: Simulation Results when Predictor is Sometimes Incorrect (N=100,$\sigma_u = 50$)

						$\hat{\lambda}_1$				$\hat{\lambda}_2$				$\hat{\lambda}_3$			
p	n_v	%-Bias[a]	$\overline{\text{SE}}$[b] ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)
0.05	0	-4.47	0.101	0.911	13												
	25					0.52	0.119	0.932	18.1	1.19	0.131	0.937	28.1	0.73	0.122	0.931	70.1
	50					0.36	0.109	0.932	13.3	0.59	0.111	0.935	14.3	0.28	0.109	0.935	21.9
	100	0.12	0.101	0.953	10.1												
0.1	0	-8.89	0.1	0.829	19.8												
	25					0.68	0.136	0.93	24.5	2.22	0.166	0.933	51.8	0.7	0.137	0.931	50
	50					0.51	0.117	0.931	16	0.85	0.123	0.935	18.1	0.46	0.117	0.938	17.9
	100	-0.03	0.101	0.943	10.4												
0.2	0	-16.32	0.1	0.6	38.9												
	25					1.19	0.159	0.933	33	3.16	0.208	0.936	65	1.37	0.158	0.939	37.4
	50					0.63	0.13	0.943	18.6	1.16	0.143	0.95	22.5	0.66	0.13	0.945	18.1
	100	0.13	0.101	0.947	10.2												
0.3	0	-22.79	0.098	0.373	64												
	25					1.69	0.176	0.925	38.4	6.46	0.436	0.925	555.7	1.77	0.176	0.93	36.3
	50					0.71	0.139	0.941	21	1.69	0.162	0.943	30.1	0.73	0.139	0.943	20.7
	100	-0.1	0.101	0.948	10.1												

[a]Percent bias = $\left(\frac{\hat{\beta}-\beta}{\beta}\right) \times 100\%$.

[b]$\overline{\text{SE}}$= mean standard error of estimates.

Table 6: Simulation Results when Predictor is Sometimes Incorrect (N=100,$\sigma_u = 20$)

						$\hat{\lambda}_1$				$\hat{\lambda}_2$				$\hat{\lambda}_3$			
p	n_v	%-Bias[a]	$\overline{\text{SE}}$[b] ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)
0.05	0	-0.8	0.101	0.949	10.2												
	25					0.04	0.101	0.944	10.5	-0.01	0.101	0.945	10.4	0.12	0.101	0.941	12.9
	50					0.01	0.1	0.943	10.3	0	0.1	0.944	10.3	0.11	0.1	0.941	12
	100	0.02	0.101	0.949	10												
0.1	0	-1.58	0.101	0.945	10.5												
	25					0.16	0.102	0.94	11	0.09	0.102	0.94	10.9	0.19	0.102	0.94	11.6
	50					0.06	0.101	0.94	10.6	0.04	0.101	0.94	10.6	0.08	0.101	0.939	10.7
	100	-0.01	0.101	0.949	10.2												
0.2	0	-3.17	0.101	0.936	11.3												
	25					0.26	0.105	0.938	11.7	0.12	0.104	0.936	11.7	0.23	0.104	0.938	11.7
	50					0.06	0.102	0.937	11.2	0.02	0.102	0.935	11.2	0.06	0.102	0.936	11.1
	100	-0.02	0.101	0.947	10.2												
0.3	0	-4.84	0.101	0.919	12.7												
	25					0.05	0.107	0.933	12.3	-0.14	0.106	0.932	12.1	0.06	0.107	0.933	12.2
	50					-0.14	0.104	0.934	11.4	-0.16	0.104	0.933	11.5	-0.14	0.104	0.934	11.4
	100	-0.26	0.101	0.943	10.5												

[a]Percent bias = $\left(\frac{\hat{\beta}-\beta}{\beta}\right) \times 100\%$.

[b]$\overline{\text{SE}}$= mean standard error of estimates.

Table 7: Simulation Results when Predictor and Outcome are Sometimes Incorrect (N=1000, σ_u=50)

		$\rho_{u,u^*} = -0.5$				$\rho_{u,u^*} = 0$				$\rho_{u,u^*} = 0.5$			
p	n_v	%-Bias[a]	$\overline{\text{SE}}$[b] ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)
0.05	0	-2.44	0.042	0.906	2.4	-4.72	0.043	0.791	4.2	-7.21	0.043	0.607	7.3
	25	-2.52	0.073	0.88	15.3	-1.01	0.087	0.852	20	0.66	0.102	0.798	28.4
	50	-2.51	0.061	0.871	7.1	-1.28	0.072	0.878	8.5	0.16	0.085	0.865	12.7
	100	-1.62	0.053	0.913	3.7	-0.85	0.061	0.917	4.5	-0.15	0.07	0.919	6.1
	200	-0.83	0.048	0.941	2.5	-0.48	0.053	0.942	2.9	-0.11	0.059	0.949	3.7
	300	-0.56	0.046	0.945	2.2	-0.29	0.049	0.943	2.5	-0.14	0.053	0.951	2.9
	1000	-0.04	0.032	0.95	1	-0.02	0.032	0.952	1	0.01	0.032	0.95	1
0.1	0	-4.5	0.042	0.799	3.9	-9.17	0.043	0.429	10.4	-13.63	0.044	0.149	20.9
	25	-4.94	0.098	0.79	27.5	-1.54	0.125	0.756	36.6	1.68	0.152	0.723	56.8
	50	-2.87	0.075	0.849	10.1	-1.33	0.096	0.847	13.7	0.38	0.116	0.839	20.3
	100	-1.32	0.062	0.927	4.6	-0.73	0.075	0.919	6.6	0.05	0.09	0.915	9.6
	200	-0.61	0.053	0.945	3	-0.33	0.061	0.943	3.9	-0.01	0.07	0.943	5.1
	300	-0.37	0.05	0.948	2.6	-0.28	0.055	0.952	3.1	0.06	0.062	0.952	3.8
	1000	-0.03	0.032	0.944	1	-0.05	0.032	0.95	1	-0.03	0.032	0.951	1
0.2	0	-8.43	0.041	0.455	8.9	-16.64	0.043	0.037	29.8	-24.91	0.044	0.001	64.5
	25	-6.1	0.133	0.773	39.7	-1.1	0.176	0.765	56.6	3.39	0.222	0.784	90.1
	50	-2.7	0.099	0.872	14.3	-0.75	0.128	0.862	21.5	1.51	0.161	0.866	34.9
	100	-1.33	0.077	0.923	7.1	-0.34	0.096	0.918	10.4	0.76	0.118	0.919	16.1
	200	-0.75	0.062	0.944	4	-0.16	0.074	0.941	5.7	0.48	0.087	0.944	8.1
	300	-0.55	0.056	0.947	3.2	-0.06	0.064	0.949	4.2	0.28	0.073	0.947	5.7
	1000	-0.1	0.032	0.947	1	0.03	0.032	0.955	1	-0.02	0.032	0.953	1
0.3	0	-11.59	0.04	0.178	15.1	-22.99	0.042	0.001	54.9	-34.48	0.044	0	121.4
	25	-5.09	0.162	0.811	46.7	-0.45	0.212	0.81	72.8	3.95	0.264	0.818	117.3
	50	-2.07	0.118	0.896	17.7	-0.1	0.153	0.882	29	1.69	0.188	0.889	43.9
	100	-1.08	0.088	0.93	8.7	-0.06	0.112	0.93	12.9	0.99	0.136	0.932	20
	200	-0.52	0.069	0.946	4.9	-0.03	0.083	0.948	7	0.5	0.098	0.942	10.2
	300	-0.42	0.061	0.946	3.8	-0.06	0.071	0.949	5.1	0.43	0.082	0.949	6.9
	1000	0.03	0.032	0.953	1	0	0.032	0.953	1	-0.03	0.032	0.946	1

[a] Percent bias = $\left(\frac{\hat{\beta}-\beta}{\beta}\right) \times 100\%$.

[b] $\overline{\text{SE}}$= mean standard error of estimates.

Table 8: Simulation Results when Predictor and Outcome are Sometimes Incorrect (N=1000, σ_u=20)

		$\rho_{u,u^*} = -0.5$				$\rho_{u,u^*} = 0$				$\rho_{u,u^*} = 0.5$			
p	n_v	%-Bias[a]	$\overline{\text{SE}}$[b] ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)
0.05	0	0.2	0.043	0.953	1.8	-0.82	0.043	0.953	1.9	-1.79	0.043	0.937	2.2
	25	-0.87	0.049	0.931	4.1	-0.28	0.052	0.935	4.7	0.26	0.054	0.946	4.5
	50	-0.65	0.046	0.933	2.6	-0.16	0.047	0.943	2.7	0.2	0.049	0.95	2.8
	100	-0.36	0.044	0.944	2.1	-0.14	0.045	0.955	2.1	0.08	0.046	0.951	2.2
	200	-0.15	0.043	0.95	1.9	-0.12	0.044	0.954	1.9	0.02	0.045	0.957	2
	300	-0.11	0.043	0.954	1.8	-0.08	0.044	0.951	1.9	0.02	0.044	0.951	1.9
	1000	0.01	0.032	0.939	1.1	0.02	0.032	0.946	1.1	-0.05	0.032	0.953	1
0.1	0	0.36	0.043	0.941	1.9	-1.51	0.044	0.947	2.2	-3.58	0.044	0.863	3.3
	25	-1.63	0.055	0.904	5.6	-0.11	0.059	0.934	5.6	0.9	0.066	0.915	7.6
	50	-0.83	0.049	0.932	2.9	0.04	0.052	0.94	3.3	0.08	0.055	0.931	3.6
	100	-0.36	0.046	0.937	2.3	0.08	0.048	0.952	2.5	-0.01	0.05	0.946	2.6
	200	-0.18	0.044	0.94	2.1	0.1	0.046	0.948	2.2	-0.04	0.047	0.945	2.3
	300	-0.16	0.044	0.939	2.1	0.07	0.045	0.944	2.2	-0.05	0.046	0.949	2.1
	1000	-0.07	0.032	0.945	1	-0.07	0.032	0.947	1.1	-0.01	0.032	0.951	1
0.2	0	0.86	0.044	0.953	1.9	-3.16	0.044	0.886	2.9	-7.14	0.045	0.645	7.2
	25	-2.2	0.065	0.918	7.7	-0.8	0.069	0.909	7.5	1.15	0.084	0.903	10.3
	50	-0.88	0.055	0.945	3.4	-0.44	0.058	0.942	3.7	0.29	0.067	0.933	5.4
	100	-0.37	0.05	0.953	2.5	-0.22	0.052	0.955	2.7	0.07	0.057	0.939	3.5
	200	-0.12	0.047	0.959	2.1	-0.17	0.048	0.954	2.4	-0.05	0.051	0.95	2.8
	300	-0.08	0.046	0.956	2	-0.18	0.047	0.958	2.2	-0.11	0.049	0.943	2.5
	1000	-0.17	0.032	0.952	1	-0.01	0.032	0.953	1	-0.08	0.032	0.952	1
0.3	0	1.12	0.044	0.926	2.1	-4.89	0.045	0.816	4.4	-10.31	0.046	0.404	12.8
	25	-1.91	0.074	0.935	8.2	-0.28	0.084	0.932	10.6	2	0.101	0.908	15.8
	50	-0.77	0.06	0.956	4.1	-0.3	0.067	0.949	5.1	0.66	0.077	0.941	6.8
	100	-0.53	0.053	0.948	2.9	-0.2	0.057	0.953	3.2	0.14	0.063	0.957	4.1
	200	-0.3	0.049	0.951	2.4	-0.2	0.051	0.962	2.5	0.1	0.055	0.957	3
	300	-0.18	0.048	0.945	2.3	-0.33	0.049	0.957	2.3	0.02	0.052	0.943	2.7
	1000	0	0.032	0.952	1	-0.09	0.032	0.936	1.1	-0.06	0.032	0.952	1

[a] Percent bias = $\left(\frac{\hat{\beta}-\beta}{\beta}\right) \times 100\%$.

[b] $\overline{\text{SE}}$= mean standard error of estimates.

Table 9: Simulation Results when Predictor and Outcome are Sometimes Incorrect (N=500, σ_u=50)

		$\rho_{u,u^*} = -0.5$				$\rho_{u,u^*} = 0$				$\rho_{u,u^*} = 0.5$			
p	n_v	%-Bias[a]	$\overline{\text{SE}}$[b] ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)
0.05	0	-1.83	0.059	0.933	4	-4.81	0.06	0.853	6.3	-7.05	0.061	0.759	9.4
	25	-2.62	0.087	0.892	17.4	-1.6	0.097	0.904	16.8	0.95	0.118	0.88	27.9
	50	-2.45	0.075	0.888	9.4	-1.52	0.084	0.911	9.2	0.46	0.098	0.927	13.9
	100	-1.22	0.068	0.932	5.3	-0.94	0.074	0.944	5.8	0.16	0.083	0.946	7.6
	200	-0.26	0.064	0.946	4.3	-0.49	0.067	0.949	4.6	0	0.071	0.951	5
	300	0.06	0.062	0.95	4	-0.41	0.064	0.947	4.2	0.03	0.066	0.951	4.4
	1000	0.3	0.045	0.943	2.1	-0.08	0.045	0.958	1.9	-0.02	0.045	0.935	2.1
0.1	0	-4.27	0.059	0.888	5.6	-8.91	0.06	0.664	12.2	-13.7	0.062	0.394	23.5
	25	-3.81	0.113	0.856	31.3	-2.62	0.127	0.838	28.6	1.63	0.161	0.81	55.8
	50	-1.92	0.089	0.91	10.6	-1.95	0.102	0.907	12.9	0.17	0.124	0.893	20.9
	100	-0.86	0.075	0.939	6.1	-0.84	0.086	0.937	7.9	-0.19	0.098	0.935	10.2
	200	-0.33	0.068	0.941	4.6	-0.6	0.073	0.953	5.2	-0.29	0.079	0.948	6.8
	300	-0.11	0.065	0.947	4.2	-0.24	0.068	0.944	4.8	-0.36	0.072	0.943	5.5
	1000	0.27	0.045	0.947	2	-0.01	0.045	0.951	2.1	-0.1	0.045	0.955	2
0.2	0	-8.5	0.058	0.681	10.9	-16.18	0.06	0.25	30.5	-24.66	0.062	0.032	65.8
	25	-6.88	0.14	0.801	40.6	-0.57	0.184	0.826	58.7	1.81	0.215	0.808	72.6
	50	-3.09	0.107	0.901	15.2	-0.05	0.136	0.908	24.4	1.13	0.164	0.893	35.5
	100	-1.42	0.087	0.947	8.4	0.17	0.104	0.937	12.3	0.61	0.122	0.937	15.4
	200	-0.77	0.074	0.94	5.7	0.08	0.083	0.937	7.4	0.56	0.092	0.963	8.1
	300	-0.54	0.07	0.938	5	0.21	0.075	0.936	6.2	0.27	0.08	0.96	6
	1000	0.01	0.045	0.955	1.9	0.25	0.045	0.931	2.2	0.15	0.045	0.951	1.9
0.3	0	-11.26	0.057	0.496	16.3	-23.17	0.06	0.036	57.7	-34.39	0.063	0.002	124
	25	-5.54	0.169	0.839	46.7	-0.24	0.224	0.831	77.2	4.78	0.265	0.845	102.5
	50	-2.41	0.125	0.914	19.2	-0.61	0.158	0.899	32.1	2.12	0.191	0.915	42.6
	100	-1.24	0.098	0.933	10.8	-0.33	0.118	0.946	14.9	0.88	0.139	0.943	20.4
	200	-0.65	0.081	0.948	6.9	-0.03	0.091	0.952	8	0.53	0.102	0.95	10.6
	300	-0.39	0.074	0.94	5.7	-0.17	0.08	0.958	6.2	0.54	0.086	0.955	7.6
	1000	0.01	0.045	0.939	2.2	0.18	0.045	0.954	1.8	0.02	0.045	0.942	2.1

[a]Percent bias = $\left(\frac{\hat{\beta}-\beta}{\beta}\right) \times 100\%$.

[b]$\overline{\text{SE}}$= mean standard error of estimates.

Table 10: Simulation Results when Predictor and Outcome are Sometimes Incorrect (N=500, σ_u=20)

		$\rho_{u,u^*} = -0.5$				$\rho_{u,u^*} = 0$				$\rho_{u,u^*} = 0.5$			
p	n_v	%-Bias[a]	$\overline{\text{SE}}$[b] ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)
0.05	0	0.26	0.06	0.952	3.8	-0.86	0.061	0.946	3.8	-1.71	0.061	0.943	4
	25	-0.61	0.065	0.941	5.6	-0.34	0.066	0.94	5.6	0.74	0.071	0.944	7.4
	50	-0.48	0.062	0.938	4.5	-0.19	0.063	0.943	4.3	0.47	0.066	0.943	5
	100	-0.31	0.061	0.952	3.9	-0.22	0.062	0.942	4	0.29	0.063	0.95	4.1
	200	-0.12	0.061	0.95	3.9	-0.1	0.061	0.94	3.8	0.15	0.062	0.955	3.8
	300	-0.06	0.06	0.953	3.9	-0.1	0.061	0.938	3.7	0.12	0.061	0.958	3.8
	1000	0.03	0.045	0.955	2	0.14	0.045	0.948	2.1	-0.04	0.045	0.942	2
0.1	0	0.19	0.061	0.943	3.8	-1.67	0.061	0.933	4.2	-3.78	0.062	0.905	5.6
	25	-1.96	0.07	0.92	7.7	-0.43	0.074	0.918	8.9	0.91	0.08	0.938	9.5
	50	-1.11	0.065	0.935	4.9	-0.39	0.068	0.932	5.5	0.5	0.072	0.944	5.9
	100	-0.54	0.063	0.945	4.3	-0.24	0.064	0.949	4.2	-0.02	0.067	0.938	4.8
	200	-0.42	0.062	0.945	4	-0.21	0.063	0.943	4	-0.22	0.064	0.947	4.3
	300	-0.35	0.062	0.945	4	-0.22	0.062	0.946	3.9	-0.25	0.063	0.94	4.2
	1000	0.02	0.045	0.948	2	-0.13	0.045	0.946	2.1	0.03	0.045	0.948	2
0.2	0	0.9	0.062	0.945	3.8	-2.9	0.063	0.925	4.7	-7.43	0.064	0.787	9.8
	25	-2.33	0.08	0.939	9.8	-0.17	0.085	0.942	9.7	0.65	0.096	0.93	13.3
	50	-0.79	0.071	0.95	5.4	0.14	0.074	0.954	6	0.06	0.081	0.939	7.5
	100	-0.33	0.066	0.955	4.4	0.25	0.068	0.95	4.7	-0.23	0.072	0.95	5.3
	200	-0.07	0.064	0.954	4.1	0.27	0.065	0.95	4.1	-0.35	0.068	0.935	4.8
	300	0	0.063	0.958	4	0.26	0.065	0.955	4	-0.42	0.066	0.937	4.6
	1000	0.18	0.045	0.949	2	0.19	0.045	0.948	2.1	-0.12	0.045	0.954	1.9
0.3	0	0.92	0.062	0.95	4	-4.48	0.064	0.911	5.9	-10.39	0.066	0.646	15.5
	25	-2	0.088	0.935	10.9	-0.18	0.095	0.948	11.4	1.78	0.113	0.931	18.7
	50	-0.97	0.076	0.937	6.4	0.05	0.08	0.958	6.6	0.65	0.091	0.944	9.2
	100	-0.57	0.07	0.938	5	-0.03	0.072	0.962	5.1	0.23	0.078	0.946	6.4
	200	-0.41	0.066	0.945	4.4	0.01	0.068	0.957	4.3	0.15	0.072	0.949	5.3
	300	-0.35	0.065	0.953	4.2	0.02	0.067	0.956	4.1	0	0.069	0.943	4.9
	1000	0.13	0.045	0.948	2	0.07	0.045	0.955	1.9	-0.02	0.045	0.94	2.1

[a]Percent bias = $\left(\frac{\hat{\beta}-\beta}{\beta}\right) \times 100\%$.

[b]$\overline{\text{SE}}$= mean standard error of estimates.

Table 11: Simulation Results when Predictor and Outcome are Sometimes Incorrect (N=100, σ_u=50)

		$\rho_{u,u^*} = -0.5$				$\rho_{u,u^*} = 0$				$\rho_{u,u^*} = 0.5$			
p	n_v	%-Bias[a]	$\overline{\text{SE}}$[b] ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)
0.05	0	-2.02	0.133	0.948	17	-4.46	0.136	0.903	23	-7.44	0.137	0.88	27.5
	25	-2.67	0.146	0.933	28.6	-1.08	0.158	0.927	33.9	0.25	0.171	0.918	44.8
	50	-2.28	0.137	0.937	21.3	-0.79	0.144	0.926	25.1	-0.6	0.149	0.922	28.2
	100	-0.21	0.101	0.955	10.1	-0.09	0.101	0.952	9.8	-0.34	0.101	0.941	11
0.1	0	-5.21	0.133	0.927	21.2	-8.32	0.136	0.883	28.6	-13.39	0.138	0.797	43.4
	25	-6.26	0.164	0.89	44.5	-1.37	0.181	0.926	43.5	1.73	0.201	0.917	63.6
	50	-3.99	0.146	0.921	25.6	-0.67	0.156	0.936	26.1	0.63	0.163	0.936	31
	100	-0.39	0.101	0.952	10.1	0.23	0.101	0.956	10.1	-0.42	0.101	0.946	10.6
0.2	0	-8.01	0.13	0.893	25.1	-16.29	0.135	0.735	49.8	-25.49	0.14	0.557	91.9
	25	-5.52	0.187	0.907	54.8	-1.97	0.217	0.914	67.1	3.21	0.255	0.932	82.9
	50	-2.25	0.158	0.941	28.3	-0.81	0.173	0.941	34.2	0.7	0.189	0.952	37.4
	100	0.43	0.101	0.943	10.6	0.04	0.101	0.94	10.6	0.17	0.101	0.943	10.4
0.3	0	-11.33	0.127	0.847	29.8	-22.61	0.135	0.596	72.2	-34.31	0.141	0.35	142.4
	25	-3.7	0.207	0.918	53	0.1	0.246	0.914	84.5	3.98	0.282	0.923	111.2
	50	-1.33	0.17	0.934	32.3	0.1	0.189	0.94	42.1	1.18	0.205	0.936	49.4
	100	-0.05	0.101	0.942	10.4	0.04	0.101	0.947	10.5	-0.29	0.101	0.928	10.8

[a]Percent bias = $\left(\frac{\hat{\beta}-\beta}{\beta}\right) \times 100\%$.

[b]$\overline{\text{SE}}$= mean standard error of estimates.

Table 12: Simulation Results when Predictor and Outcome are Sometimes Incorrect (N=100, σ_u=20)

		$\rho_{u,u^*} = -0.5$				$\rho_{u,u^*} = 0$				$\rho_{u,u^*} = 0.5$			
p	n_v	%-Bias[a]	$\overline{\text{SE}}$[b] ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	$\overline{\text{SE}}$ ($\times 100$)	Coverage	MSE ($\times 10^7$)
0.05	0	1.14	0.136	0.937	20.2	-1.13	0.136	0.954	17.7	-2.07	0.137	0.941	20.3
	25	-0.08	0.136	0.93	22.2	-0.43	0.137	0.945	20.3	0.6	0.139	0.942	22.4
	50	0.13	0.133	0.922	20.8	-0.45	0.134	0.956	18.9	0.18	0.135	0.939	20.5
	100	0.46	0.102	0.937	11.1	-0.45	0.1	0.965	9.7	-0.12	0.101	0.934	10.7
0.1	0	0.94	0.137	0.947	19.3	-1.65	0.138	0.947	19.5	-3.34	0.14	0.937	22.8
	25	-1.2	0.14	0.94	22.6	-0.64	0.142	0.947	22	1.07	0.146	0.937	24.7
	50	-0.35	0.136	0.948	20.3	-0.43	0.138	0.949	20.1	0.6	0.141	0.944	21.4
	100	0.76	0.101	0.957	9.9	-0.55	0.101	0.952	10.2	0.13	0.101	0.952	10.5
0.2	0	0.91	0.138	0.948	20	-2.83	0.142	0.942	21.7	-6.26	0.144	0.919	24.6
	25	-1.8	0.145	0.939	23.3	-0.36	0.151	0.936	25.1	2.12	0.159	0.939	29.1
	50	-0.59	0.14	0.942	21.1	0.09	0.144	0.938	22.5	1.24	0.148	0.946	22.4
	100	0.22	0.101	0.94	11.1	0.1	0.101	0.942	10.6	0.11	0.101	0.959	9.6
0.3	0	0.97	0.14	0.946	20.9	-4.42	0.144	0.933	23.7	-10.03	0.148	0.896	32.3
	25	-2.08	0.151	0.94	27.5	-0.02	0.159	0.932	28.5	1.72	0.17	0.933	31.3
	50	-1.09	0.144	0.938	23.1	0.12	0.149	0.931	24.3	0.78	0.155	0.942	24.4
	100	0.09	0.101	0.946	10.5	0.22	0.101	0.933	10.5	0.04	0.101	0.958	9.3

[a]Percent bias = $\left(\frac{\hat{\beta}-\beta}{\beta}\right) \times 100\%$.

[b]$\overline{\text{SE}}$= mean standard error of estimates.

Table 13: Simulation Results when Predictor (X) and Outcome (Y) are Sometimes Incorrect with always correct Covariate (Z) (N=1000, σ_u=50, $\rho_{u,u^*} = -0.5$)

		β_x				β_z			
p	n_v	%-Bias	SE ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	SE ($\times 10$)	Coverage	MSE ($\times 10^3$)
0.05	0	-2.61	0.044	0.909	2.6	-0.38	0.225	0.947	0.5
	25	0.47	0.082	0.946	12.1	0.09	0.261	0.956	0.7
	50	0.29	0.066	0.953	5.6	0.06	0.241	0.952	0.6
	100	0.1	0.057	0.958	3.5	0.03	0.232	0.95	0.5
	200	0.02	0.051	0.956	2.6	0.02	0.228	0.951	0.5
	300	0.03	0.049	0.957	2.4	0.02	0.227	0.952	0.5
	1000	-0.03	0.033	0.951	1.1	0	0.166	0.954	0.3
0.1	0	-4.95	0.043	0.785	4.4	-0.72	0.227	0.94	0.6
	25	1.4	0.123	0.92	26.3	0.22	0.309	0.958	1.1
	50	0.68	0.087	0.937	9.4	0.12	0.261	0.952	0.7
	100	0.26	0.069	0.954	4.9	0.06	0.242	0.952	0.6
	200	0.13	0.058	0.954	3.3	0.04	0.234	0.952	0.6
	300	0.1	0.053	0.957	2.8	0.03	0.231	0.951	0.5
	1000	0.02	0.033	0.949	1.1	0.03	0.166	0.948	0.3
0.2	0	-8.97	0.042	0.435	10	-1.36	0.232	0.906	0.7
	25	2.43	0.292	0.893	131.8	0.34	0.536	0.953	3.3
	50	0.94	0.116	0.934	15.8	0.13	0.292	0.951	0.9
	100	0.5	0.087	0.95	8.3	0.06	0.261	0.947	0.7
	200	0.27	0.068	0.956	4.9	0.03	0.246	0.946	0.6
	300	0.13	0.061	0.955	3.7	0.01	0.241	0.947	0.6
	1000	0.03	0.033	0.953	1.1	-0.01	0.166	0.945	0.3
0.3	0	-12.45	0.042	0.166	17.4	-1.84	0.237	0.872	0.9
	25	2.74	0.237	0.891	79	0.43	0.455	0.953	2.5
	50	1.05	0.143	0.928	23.6	0.19	0.324	0.954	1.1
	100	0.54	0.102	0.943	11.2	0.1	0.279	0.952	0.8
	200	0.17	0.078	0.952	6.2	0.05	0.257	0.948	0.7
	300	0.05	0.068	0.949	4.7	0.03	0.25	0.946	0.7
	1000	-0.03	0.033	0.948	1.1	0.01	0.166	0.951	0.3

Table 14: Simulation Results when Predictor (X) and Outcome (Y) are Sometimes Incorrect with always correct Covariate (Z) (N=1000, σ_u=50, $\rho_{u,u^*} = 0$)

		β_x				β_z			
p	n_v	%-Bias	SE ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	SE ($\times 10$)	Coverage	MSE ($\times 10^3$)
0.05	0	-5.19	0.045	0.77	4.9	-0.76	0.228	0.939	0.6
	25	0.9	0.105	0.891	20.7	0.15	0.29	0.954	0.9
	50	0.42	0.082	0.922	8.6	0.08	0.258	0.954	0.7
	100	0.2	0.067	0.941	5	0.05	0.242	0.956	0.6
	200	0.06	0.057	0.949	3.4	0.03	0.234	0.954	0.5
	300	0.04	0.053	0.944	2.9	0.02	0.232	0.952	0.5
	1000	0.07	0.033	0.947	1.1	0.01	0.166	0.951	0.3
0.1	0	-9.86	0.044	0.408	12	-1.49	0.233	0.904	0.8
	25	1.61	0.161	0.82	48.5	0.23	0.359	0.947	1.6
	50	0.8	0.112	0.888	16.6	0.11	0.291	0.953	0.9
	100	0.41	0.085	0.932	8.2	0.05	0.261	0.951	0.7
	200	0.24	0.068	0.95	4.7	0.02	0.246	0.95	0.6
	300	0.13	0.06	0.952	3.7	0	0.241	0.95	0.6
	1000	0	0.033	0.951	1.1	0	0.166	0.942	0.3
0.2	0	-18	0.044	0.026	34.7	-2.66	0.242	0.804	1.3
	25	3.95	0.279	0.807	117.5	0.63	0.523	0.946	3.3
	50	1.21	0.158	0.882	29.3	0.22	0.347	0.956	1.2
	100	0.74	0.114	0.918	14.3	0.15	0.295	0.956	0.9
	200	0.39	0.085	0.945	7.5	0.09	0.268	0.95	0.7
	300	0.23	0.073	0.95	5.4	0.07	0.259	0.949	0.7
	1000	0	0.033	0.946	1.1	0.01	0.166	0.951	0.3
0.3	0	-24.84	0.044	0.001	64.1	-3.73	0.25	0.686	2
	25	8.21	0.815	0.824	971.9	1.25	1.462	0.932	30.9
	50	2.14	0.198	0.89	45.6	0.32	0.399	0.948	1.7
	100	1.03	0.137	0.929	19.6	0.15	0.325	0.954	1.1
	200	0.37	0.099	0.953	9.8	0.05	0.288	0.95	0.8
	300	0.22	0.083	0.953	6.9	0.03	0.275	0.948	0.8
	1000	-0.12	0.033	0.947	1.1	-0.04	0.166	0.945	0.3

Table 15: Simulation Results when Predictor (X) and Outcome (Y) are Sometimes Incorrect with always correct Covariate (Z) (N=1000, σ_u=50, $\rho_{u,u^*} = 0.5$)

		β_x				β_z			
p	n_v	%-Bias	SE ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	SE ($\times 10$)	Coverage	MSE ($\times 10^3$)
0.05	0	-7.99	0.045	0.573	8.8	-1.17	0.231	0.921	0.7
	25	1.59	0.14	0.815	49.5	0.26	0.338	0.951	1.5
	50	0.62	0.101	0.875	16	0.12	0.281	0.954	0.8
	100	0.01	0.078	0.918	7.3	0.03	0.253	0.953	0.6
	200	-0.09	0.064	0.941	4.4	0.01	0.241	0.955	0.6
	300	-0.21	0.057	0.941	3.5	-0.01	0.237	0.954	0.6
	1000	-0.14	0.033	0.951	1.1	0.01	0.166	0.952	0.3
0.1	0	-14.81	0.045	0.127	24.5	-2.22	0.238	0.847	1.1
	25	2.79	0.218	0.735	98.1	0.43	0.441	0.939	2.8
	50	1.17	0.142	0.85	28	0.18	0.329	0.949	1.2
	100	0.61	0.105	0.922	12	0.1	0.283	0.951	0.8
	200	0.29	0.079	0.948	6.5	0.05	0.259	0.949	0.7
	300	0.18	0.069	0.952	4.8	0.03	0.251	0.948	0.6
	1000	-0.01	0.033	0.951	1.1	-0.01	0.166	0.946	0.3
0.2	0	-26.87	0.046	0.001	75	-4.03	0.251	0.634	2.3
	25	5.24	0.355	0.776	249.9	0.8	0.632	0.921	6.7
	50	2.36	0.205	0.86	53.4	0.36	0.411	0.942	1.8
	100	1.16	0.143	0.916	22.4	0.17	0.332	0.953	1.1
	200	0.55	0.103	0.942	11	0.08	0.291	0.948	0.8
	300	0.51	0.086	0.95	7.4	0.07	0.277	0.95	0.8
	1000	0.12	0.033	0.952	1.1	0.03	0.166	0.955	0.3
0.3	0	-37.21	0.046	0	141.2	-5.56	0.261	0.438	3.8
	25	11.64	0.84	0.799	1423.9	1.74	1.339	0.908	33.4
	50	4.1	0.266	0.871	100.2	0.65	0.497	0.938	3
	100	1.28	0.173	0.912	32.8	0.22	0.374	0.951	1.5
	200	0.54	0.122	0.941	15.1	0.1	0.32	0.951	1
	300	0.38	0.1	0.951	9.9	0.08	0.301	0.95	0.9
	1000	-0.04	0.033	0.949	1.1	-0.03	0.166	0.95	0.3

Table 16: Simulation Results when Predictor (X) and Outcome (Y) are Sometimes Incorrect with always correct Covariate (Z) (N=1000, σ_u=20, $\rho_{u,u^*} = -0.5$)

		β_x				β_z			
p	n_v	%-Bias	SE ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	SE ($\times 10$)	Coverage	MSE ($\times 10^3$)
0.05	0	0.26	0.045	0.95	2	-0.04	0.225	0.95	0.5
	25	0.02	0.05	0.95	2.6	-0.08	0.227	0.947	0.5
	50	0.05	0.048	0.947	2.3	-0.07	0.225	0.948	0.5
	100	0.03	0.046	0.948	2.2	-0.07	0.225	0.95	0.5
	200	0.04	0.046	0.949	2.1	-0.07	0.224	0.949	0.5
	300	0.04	0.045	0.948	2.1	-0.07	0.224	0.949	0.5
	1000	0	0.033	0.953	1.1	-0.03	0.166	0.947	0.3
0.1	0	0.52	0.045	0.947	2.1	0.07	0.227	0.954	0.5
	25	0.18	0.055	0.953	3.3	0.02	0.232	0.955	0.5
	50	0.12	0.051	0.953	2.7	0.01	0.229	0.954	0.5
	100	0.12	0.048	0.948	2.4	0.01	0.228	0.952	0.5
	200	0.09	0.047	0.947	2.3	0.01	0.227	0.953	0.5
	300	0.09	0.046	0.949	2.2	0.01	0.227	0.953	0.5
	1000	0.03	0.033	0.946	1.1	0	0.166	0.953	0.3
0.2	0	0.79	0.046	0.947	2.1	0.14	0.231	0.952	0.5
	25	-0.01	0.064	0.957	4.6	0.02	0.242	0.949	0.6
	50	-0.14	0.057	0.953	3.3	0	0.237	0.948	0.6
	100	-0.07	0.052	0.955	2.7	0.01	0.234	0.949	0.6
	200	-0.08	0.049	0.95	2.4	0.01	0.232	0.949	0.5
	300	-0.07	0.048	0.948	2.3	0.01	0.232	0.95	0.5
	1000	-0.07	0.033	0.947	1.1	0	0.166	0.947	0.3
0.3	0	1.2	0.046	0.946	2.2	0.22	0.236	0.946	0.6
	25	-0.06	0.073	0.954	6	0.03	0.252	0.945	0.7
	50	-0.08	0.063	0.958	4.1	0.03	0.244	0.944	0.6
	100	-0.07	0.056	0.951	3.1	0.03	0.24	0.946	0.6
	200	-0.12	0.051	0.954	2.6	0.02	0.237	0.947	0.6
	300	-0.1	0.05	0.953	2.4	0.02	0.237	0.944	0.6
	1000	-0.05	0.033	0.947	1.1	0.02	0.166	0.95	0.3

Table 17: Simulation Results when Predictor (X) and Outcome (Y) are Sometimes Incorrect with always correct Covariate (Z) (N=1000, σ_u=20, $\rho_{u,u^*} = 0$)

		β_x				β_z			
p	n_v	%-Bias	SE ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	SE ($\times 10$)	Coverage	MSE ($\times 10^3$)
0.05	0	-0.9	0.045	0.94	2.2	-0.13	0.226	0.952	0.5
	25	-0.05	0.051	0.951	2.9	0	0.229	0.955	0.5
	50	-0.03	0.049	0.95	2.5	0	0.227	0.954	0.5
	100	-0.03	0.047	0.945	2.3	0	0.226	0.955	0.5
	200	-0.04	0.046	0.944	2.2	0	0.225	0.954	0.5
	300	-0.04	0.046	0.945	2.2	0	0.225	0.954	0.5
	1000	-0.03	0.033	0.946	1.2	-0.02	0.166	0.944	0.3
0.1	0	-1.76	0.046	0.929	2.4	-0.29	0.229	0.947	0.5
	25	0.06	0.058	0.948	3.9	-0.01	0.237	0.948	0.6
	50	0.07	0.054	0.951	3.1	-0.01	0.233	0.947	0.5
	100	0.03	0.05	0.95	2.6	-0.02	0.231	0.947	0.5
	200	0	0.048	0.948	2.3	-0.02	0.23	0.947	0.5
	300	-0.03	0.047	0.951	2.2	-0.03	0.229	0.947	0.5
	1000	0.03	0.033	0.948	1.1	-0.01	0.166	0.951	0.3
0.2	0	-3.33	0.047	0.888	3.3	-0.46	0.236	0.949	0.6
	25	0.21	0.071	0.948	5.8	0.07	0.251	0.952	0.6
	50	0.18	0.062	0.943	3.9	0.07	0.244	0.954	0.6
	100	0.16	0.055	0.949	3	0.06	0.24	0.951	0.6
	200	0.1	0.051	0.952	2.6	0.05	0.238	0.95	0.6
	300	0.08	0.05	0.947	2.5	0.05	0.237	0.949	0.6
	1000	-0.02	0.033	0.947	1.1	0.02	0.166	0.951	0.3
0.3	0	-4.94	0.047	0.815	4.8	-0.74	0.242	0.939	0.7
	25	0.14	0.081	0.941	7.7	0.02	0.264	0.947	0.7
	50	0.08	0.068	0.949	4.9	0.01	0.254	0.948	0.7
	100	0.1	0.06	0.952	3.6	0.02	0.248	0.949	0.6
	200	0.1	0.054	0.946	3	0.02	0.245	0.948	0.6
	300	0.09	0.052	0.946	2.8	0.02	0.244	0.947	0.6
	1000	-0.01	0.033	0.947	1.1	0.01	0.166	0.951	0.3

Table 18: Simulation Results when Predictor (X) and Outcome (Y) are Sometimes Incorrect with always correct Covariate (Z) (N=1000, σ_u=20, $\rho_{u,u^*} = 0.5$)

		β_x				β_z			
p	n_v	%-Bias	SE ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	SE ($\times 10$)	Coverage	MSE ($\times 10^3$)
0.05	0	-1.94	0.045	0.925	2.5	-0.31	0.227	0.95	0.5
	25	0.05	0.055	0.951	3.6	-0.01	0.233	0.949	0.6
	50	0.06	0.052	0.95	2.9	-0.01	0.23	0.95	0.5
	100	0.05	0.049	0.955	2.5	-0.01	0.228	0.95	0.5
	200	0.04	0.047	0.953	2.3	-0.01	0.227	0.948	0.5
	300	0.01	0.046	0.954	2.2	-0.01	0.227	0.949	0.5
	1000	0.05	0.033	0.951	1.1	-0.02	0.166	0.95	0.3
0.1	0	-3.9	0.046	0.866	3.7	-0.61	0.232	0.944	0.6
	25	0.17	0.066	0.938	5.2	0	0.245	0.953	0.6
	50	0.04	0.058	0.945	3.6	-0.02	0.238	0.955	0.6
	100	0.05	0.053	0.948	2.9	-0.01	0.235	0.952	0.5
	200	0.01	0.05	0.953	2.5	-0.02	0.233	0.954	0.5
	300	-0.01	0.048	0.951	2.3	-0.02	0.232	0.953	0.5
	1000	0.04	0.033	0.949	1.1	-0.03	0.166	0.95	0.3
0.2	0	-7.56	0.047	0.64	8.1	-1.12	0.241	0.92	0.7
	25	0.35	0.084	0.914	8.9	0.06	0.266	0.947	0.7
	50	0.14	0.07	0.936	5.4	0.03	0.254	0.948	0.7
	100	0.17	0.06	0.943	3.9	0.04	0.247	0.947	0.6
	200	0.13	0.054	0.94	3.1	0.03	0.244	0.95	0.6
	300	0.12	0.052	0.939	2.9	0.03	0.242	0.95	0.6
	1000	0.01	0.033	0.953	1.1	0.01	0.166	0.95	0.3
0.3	0	-11.27	0.049	0.369	15.3	-1.74	0.249	0.891	0.9
	25	0.36	0.098	0.912	11.7	0.01	0.285	0.948	0.9
	50	0.08	0.079	0.938	6.7	-0.04	0.268	0.948	0.7
	100	0.05	0.067	0.952	4.5	-0.04	0.258	0.946	0.7
	200	0.07	0.058	0.947	3.5	-0.04	0.253	0.944	0.7
	300	0.06	0.055	0.946	3.1	-0.04	0.252	0.941	0.7
	1000	0.01	0.033	0.946	1.1	-0.02	0.166	0.946	0.3

Table 19: Simulation Results when Predictor (X) and Outcome (Y) are Sometimes Incorrect with always correct Covariate (Z) (N=500, σ_u=50, $\rho_{u,u^*} = -0.5$)

		β_x				β_z			
p	n_v	%-Bias	SE ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	SE ($\times 10$)	Coverage	MSE ($\times 10^3$)
0.05	0	-2.66	0.062	0.926	4.7	-0.34	0.319	0.947	1.1
	25	0.41	0.096	0.949	13.1	0.12	0.346	0.949	1.3
	50	0.15	0.081	0.949	7.2	0.08	0.329	0.948	1.1
	100	0.1	0.073	0.948	5.4	0.07	0.322	0.945	1.1
	200	0.02	0.068	0.95	4.6	0.06	0.319	0.945	1.1
	300	-0.02	0.066	0.951	4.3	0.05	0.318	0.945	1
	1000	-0.06	0.047	0.955	2.1	0.03	0.235	0.954	0.5
0.1	0	-5.15	0.061	0.858	6.7	-0.8	0.322	0.943	1.1
	25	1.06	0.131	0.936	25.5	0.13	0.385	0.953	1.5
	50	0.26	0.098	0.947	11	0.02	0.345	0.951	1.2
	100	0.02	0.081	0.949	7	-0.02	0.331	0.949	1.1
	200	-0.16	0.072	0.947	5.3	-0.05	0.325	0.948	1.1
	300	-0.2	0.069	0.947	4.8	-0.05	0.324	0.95	1
	1000	-0.18	0.047	0.952	2.2	-0.06	0.235	0.951	0.5
0.2	0	-8.99	0.06	0.665	12.2	-1.34	0.328	0.928	1.3
	25	2.14	0.191	0.927	47.3	0.32	0.454	0.956	2
	50	0.73	0.126	0.947	17.7	0.12	0.374	0.952	1.4
	100	0.26	0.097	0.952	9.8	0.05	0.348	0.945	1.2
	200	0.16	0.081	0.948	6.8	0.03	0.337	0.944	1.2
	300	0.1	0.075	0.946	5.8	0.02	0.333	0.944	1.2
	1000	-0.02	0.047	0.95	2.2	-0.02	0.235	0.945	0.6
0.3	0	-12.38	0.059	0.444	19.2	-1.92	0.335	0.901	1.5
	25	0.54	2.821	0.922	3710.2	-0.05	5.173	0.95	123.6
	50	0.98	0.149	0.945	24.5	0.08	0.402	0.948	1.7
	100	0.33	0.111	0.951	12.6	-0.02	0.365	0.945	1.4
	200	0.23	0.089	0.953	7.9	-0.03	0.349	0.944	1.3
	300	0.19	0.08	0.946	6.5	-0.04	0.344	0.946	1.2
	1000	-0.07	0.047	0.948	2.2	-0.04	0.235	0.949	0.6

Table 20: Simulation Results when Predictor (X) and Outcome (Y) are Sometimes Incorrect with always correct Covariate (Z) (N=500, σ_u=50, $\rho_{u,u^*} = 0$)

		β_x				β_z			
p	n_v	%-Bias	SE ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	SE ($\times 10$)	Coverage	MSE ($\times 10^3$)
0.05	0	-5.14	0.063	0.85	7.3	-0.8	0.323	0.945	1.1
	25	1.31	0.124	0.926	26	0.17	0.381	0.956	1.6
	50	0.51	0.095	0.943	10.7	0.05	0.344	0.953	1.2
	100	0.26	0.081	0.952	6.9	0.01	0.331	0.949	1.1
	200	0.17	0.072	0.944	5.4	0	0.325	0.949	1.1
	300	0.12	0.068	0.944	4.9	-0.01	0.324	0.949	1.1
	1000	0.04	0.047	0.948	2.3	-0.01	0.235	0.946	0.6
0.1	0	-9.84	0.063	0.648	14.1	-1.34	0.33	0.932	1.3
	25	2.29	0.173	0.888	49.8	0.48	0.441	0.956	2.1
	50	0.98	0.123	0.926	18.3	0.29	0.375	0.953	1.4
	100	0.38	0.096	0.951	9.6	0.2	0.349	0.95	1.2
	200	0.11	0.079	0.953	6.4	0.16	0.337	0.944	1.2
	300	0.07	0.073	0.95	5.3	0.15	0.333	0.946	1.1
	1000	0	0.047	0.952	2.2	0.1	0.235	0.951	0.6
0.2	0	-17.92	0.063	0.199	36.7	-2.65	0.343	0.875	1.9
	25	5.31	0.302	0.854	149.6	0.84	0.611	0.952	4.5
	50	1.7	0.168	0.918	33.7	0.31	0.43	0.951	1.9
	100	0.87	0.122	0.944	15.9	0.18	0.381	0.947	1.5
	200	0.51	0.094	0.95	9.1	0.13	0.359	0.946	1.3
	300	0.29	0.082	0.948	6.9	0.09	0.352	0.944	1.3
	1000	-0.01	0.047	0.951	2.2	0.02	0.235	0.945	0.6
0.3	0	-24.83	0.062	0.033	66.4	-3.7	0.354	0.813	2.7
	25	7.03	0.478	0.86	455.9	1.07	0.88	0.947	12.5
	50	2.49	0.207	0.909	54.1	0.38	0.481	0.957	2.5
	100	1.11	0.143	0.942	21.7	0.18	0.411	0.949	1.7
	200	0.36	0.105	0.952	11.2	0.07	0.38	0.944	1.5
	300	0.17	0.091	0.951	8.4	0.04	0.37	0.945	1.4
	1000	-0.06	0.047	0.951	2.3	-0.03	0.235	0.946	0.6

Table 21: Simulation Results when Predictor (X) and Outcome (Y) are Sometimes Incorrect with always correct Covariate (Z) (N=500, σ_u=50, $\rho_{u,u^*} = 0.5$)

		β_x				β_z			
p	n_v	%-Bias	SE ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	SE ($\times 10$)	Coverage	MSE ($\times 10^3$)
0.05	0	-7.8	0.064	0.752	10.7	-1.2	0.327	0.937	1.2
	25	2.02	0.197	0.895	147.2	0.26	0.472	0.954	3.4
	50	0.86	0.113	0.934	16.9	0.1	0.364	0.957	1.3
	100	0.29	0.09	0.947	8.7	0.01	0.341	0.957	1.2
	200	0.06	0.076	0.952	5.8	-0.02	0.332	0.955	1.1
	300	0.05	0.071	0.953	4.9	-0.02	0.329	0.956	1.1
	1000	-0.07	0.047	0.955	2.1	-0.04	0.235	0.952	0.5
0.1	0	-14.91	0.064	0.38	27.7	-2.28	0.337	0.889	1.7
	25	2.76	0.218	0.828	84.5	0.36	0.499	0.946	3
	50	1.12	0.15	0.9	29.7	0.12	0.407	0.949	1.8
	100	0.39	0.112	0.934	13.7	0.02	0.367	0.948	1.4
	200	0.16	0.088	0.952	8.1	-0.02	0.348	0.945	1.3
	300	0.08	0.078	0.943	6.5	-0.03	0.342	0.946	1.2
	1000	-0.06	0.047	0.953	2.2	0.03	0.235	0.945	0.6
0.2	0	-26.78	0.065	0.032	77.3	-4.07	0.355	0.785	2.9
	25	3.33	3.991	0.806	8718	0.44	5.922	0.938	182.6
	50	2.9	0.214	0.895	58.4	0.39	0.492	0.95	2.6
	100	1.27	0.148	0.938	23.9	0.14	0.417	0.948	1.8
	200	0.54	0.108	0.948	11.9	0.03	0.382	0.944	1.5
	300	0.48	0.091	0.952	8.5	0.02	0.371	0.943	1.4
	1000	0.07	0.047	0.95	2.2	0	0.235	0.953	0.6
0.3	0	-37.14	0.065	0.001	143.3	-5.55	0.37	0.671	4.5
	25	9.48	0.53	0.82	448.2	1.45	0.936	0.944	10.8
	50	3.16	0.259	0.886	79.6	0.53	0.557	0.957	3.1
	100	1.6	0.176	0.939	33.9	0.29	0.46	0.956	2.1
	200	0.83	0.124	0.955	15.9	0.17	0.413	0.956	1.7
	300	0.53	0.102	0.951	10.9	0.12	0.398	0.952	1.6
	1000	0.04	0.047	0.947	2.2	-0.04	0.235	0.95	0.5

Table 22: Simulation Results when Predictor (X) and Outcome (Y) are Sometimes Incorrect with always correct Covariate (Z) (N=500, σ_u=20, $\rho_{u,u^*} = -0.5$)

		β_x				β_z			
p	n_v	%-Bias	SE ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	SE ($\times 10$)	Coverage	MSE ($\times 10^3$)
0.05	0	0.05	0.063	0.949	4.1	-0.03	0.319	0.946	1
	25	-0.13	0.067	0.948	4.8	-0.06	0.319	0.947	1
	50	-0.16	0.065	0.945	4.5	-0.06	0.318	0.946	1
	100	-0.14	0.064	0.944	4.3	-0.06	0.317	0.946	1
	200	-0.16	0.064	0.943	4.2	-0.06	0.317	0.945	1
	300	-0.17	0.063	0.945	4.2	-0.06	0.317	0.945	1
	1000	-0.1	0.047	0.957	2.1	-0.05	0.235	0.949	0.6
0.1	0	0.43	0.064	0.948	4.1	-0.01	0.322	0.951	1
	25	0.07	0.072	0.948	5.5	-0.06	0.324	0.948	1.1
	50	0.05	0.069	0.946	4.9	-0.07	0.322	0.947	1
	100	0.03	0.066	0.947	4.5	-0.07	0.32	0.949	1
	200	0.02	0.065	0.945	4.3	-0.07	0.32	0.948	1
	300	0.02	0.065	0.945	4.3	-0.07	0.32	0.949	1
	1000	0.02	0.047	0.948	2.2	-0.05	0.235	0.951	0.5
0.2	0	0.82	0.065	0.943	4.4	0.08	0.328	0.95	1.1
	25	-0.04	0.08	0.949	7	-0.04	0.334	0.948	1.2
	50	-0.01	0.073	0.95	5.6	-0.04	0.33	0.949	1.1
	100	-0.01	0.069	0.949	5	-0.04	0.327	0.947	1.1
	200	-0.01	0.067	0.947	4.6	-0.04	0.326	0.947	1.1
	300	-0.01	0.066	0.947	4.5	-0.04	0.326	0.948	1.1
	1000	-0.05	0.047	0.954	2.2	-0.02	0.235	0.952	0.5
0.3	0	1.3	0.065	0.945	4.5	0.05	0.333	0.947	1.1
	25	0.05	0.088	0.952	8.5	-0.13	0.344	0.945	1.2
	50	0.16	0.078	0.952	6.4	-0.12	0.337	0.946	1.2
	100	0.12	0.072	0.951	5.3	-0.12	0.334	0.946	1.2
	200	0.06	0.069	0.951	4.8	-0.13	0.333	0.945	1.1
	300	0.06	0.068	0.952	4.7	-0.13	0.332	0.946	1.1
	1000	-0.01	0.047	0.952	2.2	-0.08	0.235	0.95	0.6

Table 23: Simulation Results when Predictor (X) and Outcome (Y) are Sometimes Incorrect with always correct Covariate (Z) (N=500, σ_u=20, $\rho_{u,u^*} = 0$)

		β_x				β_z			
p	n_v	%-Bias	SE ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	SE ($\times 10$)	Coverage	MSE ($\times 10^3$)
0.05	0	-0.87	0.064	0.951	4.1	-0.16	0.32	0.951	1
	25	0.06	0.069	0.951	4.9	-0.02	0.32	0.949	1
	50	0.05	0.067	0.953	4.4	-0.02	0.319	0.949	1
	100	0.06	0.065	0.95	4.2	-0.02	0.318	0.948	1
	200	0.01	0.064	0.947	4.1	-0.03	0.318	0.948	1
	300	0.02	0.064	0.95	4.1	-0.02	0.318	0.948	1
	1000	0.01	0.047	0.949	2.2	0.01	0.235	0.955	0.5
0.1	0	-1.83	0.065	0.94	4.5	-0.3	0.325	0.953	1
	25	-0.02	0.075	0.949	6	-0.03	0.33	0.953	1.1
	50	-0.07	0.07	0.944	5.1	-0.04	0.326	0.954	1
	100	-0.07	0.067	0.949	4.6	-0.04	0.325	0.952	1
	200	-0.07	0.066	0.947	4.4	-0.04	0.324	0.953	1
	300	-0.07	0.065	0.948	4.3	-0.04	0.324	0.952	1
	1000	0.02	0.047	0.955	2.2	-0.02	0.235	0.951	0.5
0.2	0	-3.38	0.066	0.919	5.5	-0.48	0.334	0.951	1.1
	25	0.23	0.087	0.951	8.5	0.06	0.345	0.95	1.2
	50	0.19	0.078	0.949	6.3	0.05	0.339	0.954	1.1
	100	0.07	0.072	0.944	5.4	0.03	0.335	0.954	1.1
	200	0.05	0.069	0.944	4.8	0.03	0.334	0.952	1.1
	300	0.03	0.068	0.946	4.7	0.03	0.333	0.952	1.1
	1000	-0.05	0.047	0.951	2.2	0.03	0.235	0.95	0.5
0.3	0	-4.92	0.067	0.888	7	-0.7	0.344	0.947	1.2
	25	0.24	0.096	0.951	10.4	0.08	0.359	0.95	1.3
	50	0.15	0.084	0.95	7.3	0.06	0.35	0.948	1.2
	100	0.1	0.076	0.947	5.9	0.05	0.346	0.946	1.2
	200	0.06	0.072	0.945	5.3	0.05	0.344	0.946	1.2
	300	0.08	0.07	0.943	5	0.05	0.343	0.946	1.2
	1000	0.08	0.047	0.948	2.2	0.01	0.236	0.948	0.6

Table 24: Simulation Results when Predictor (X) and Outcome (Y) are Sometimes Incorrect with always correct Covariate (Z) (N=500, σ_u=20, $\rho_{u,u^*} = 0.5$)

		β_x				β_z			
p	n_v	%-Bias	SE ($\times 100$)	Coverage	MSE ($\times 10^7$)	%-Bias	SE ($\times 10$)	Coverage	MSE ($\times 10^3$)
0.05	0	-1.81	0.064	0.943	4.4	-0.26	0.322	0.944	1.1
	25	0.27	0.072	0.948	5.8	0.05	0.324	0.942	1.1
	50	0.21	0.069	0.948	4.8	0.04	0.321	0.943	1.1
	100	0.22	0.066	0.947	4.4	0.04	0.32	0.942	1.1
	200	0.17	0.065	0.949	4.2	0.03	0.319	0.942	1.1
	300	0.17	0.064	0.95	4.1	0.03	0.319	0.942	1.1
	1000	0.14	0.047	0.952	2.1	0.01	0.235	0.944	0.6
0.1	0	-3.95	0.065	0.907	5.9	-0.61	0.328	0.944	1.1
	25	-0.07	0.081	0.95	7.2	-0.03	0.336	0.949	1.1
	50	-0.05	0.074	0.951	5.6	-0.03	0.331	0.949	1.1
	100	-0.03	0.07	0.948	4.9	-0.02	0.328	0.947	1.1
	200	-0.04	0.067	0.947	4.5	-0.03	0.327	0.946	1.1
	300	-0.05	0.066	0.947	4.4	-0.03	0.326	0.946	1.1
	1000	-0.05	0.047	0.952	2.2	-0.01	0.235	0.945	0.6
0.2	0	-7.46	0.067	0.794	10.2	-1.15	0.341	0.935	1.3
	25	0.27	0.097	0.939	10.9	0.01	0.358	0.949	1.3
	50	0.17	0.084	0.949	7.5	0	0.348	0.947	1.2
	100	0.2	0.076	0.951	6	0	0.343	0.948	1.2
	200	0.2	0.071	0.947	5.2	0	0.34	0.948	1.2
	300	0.21	0.07	0.943	4.9	0	0.34	0.947	1.2
	1000	0.03	0.047	0.946	2.2	0.02	0.235	0.95	0.6
0.3	0	-11.16	0.069	0.633	17.5	-1.7	0.352	0.927	1.5
	25	0.64	0.112	0.93	15	0.07	0.378	0.955	1.5
	50	0.46	0.094	0.946	9.6	0.04	0.364	0.953	1.3
	100	0.25	0.082	0.947	7.1	0.01	0.357	0.951	1.3
	200	0.17	0.075	0.948	5.9	0	0.353	0.951	1.2
	300	0.13	0.073	0.949	5.4	-0.01	0.352	0.952	1.2
	1000	-0.03	0.047	0.95	2.2	0.01	0.235	0.95	0.5

Table 25: Simulation Results using Site-Specific and Combined-Site Parameter Estimates based on the First Audit when Predictor (X) and Outcome (Y) are Sometimes Incorrect

		Site-Specific Parameter Estimates				Across-Site Parameter Estimates			
p	n_v	%-Bias	SE	Coverage	MSE	%-Bias	SE	Coverage	MSE
				Site A					
		$\rho_{u,u^*}=0, \sigma_u=0.02$				$\rho_{u,u^*}=-0.75, \sigma_u=1.0$			
0.07	0	-2.19	4.236	0.951	17.9	21.36	4.048	0.932	18.3
	25	-2.19	4.21	0.949	17.9	-15.11	4.417	0.932	26.8
	50	-2.19	4.206	0.948	17.9	-8.42	4.206	0.942	19.1
	75	-2.19	4.204	0.948	17.9	-5.04	4.15	0.945	18.2
	100	-2.19	4.204	0.948	17.9	-3.27	4.126	0.943	17.9
	260	-2.2	4.236	0.951	17.9	1.58	4.037	0.947	16.9
				Site B					
		$\rho_{u,u^*}=0, \sigma_u=0.07$				$\rho_{u,u^*}=-0.75, \sigma_u=1.0$			
0.18	0	1.91	3.703	0.947	14.1	48.61	3.111	0.886	15.9
	25	1.96	3.677	0.941	14.2	-25.61	4.148	0.946	24.7
	50	1.96	3.677	0.941	14.2	-11.7	3.668	0.955	14.5
	75	1.96	3.677	0.942	14.2	-7.29	3.503	0.958	12.2
	100	1.97	3.677	0.941	14.2	-4.37	3.413	0.962	11.4
	439	1.93	3.704	0.947	14.2	-0.43	3.093	0.959	9.4
				Site C					
		$\rho_{u,u^*}=-0.99, \sigma_u=1.8$				$\rho_{u,u^*}=-0.75, \sigma_u=1.0$			
0.26	0	433.56	4.24	0.001	549.9	44.7	2.495	0.852	11.4
	25	-266.83	18.982	0.843	1163.6	-23.03	3.535	0.95	20.4
	50	-112.41	13.563	0.908	343.5	-9.09	3.065	0.957	10.4
	75	-64.78	11.391	0.928	179	-4.48	2.886	0.957	8.1
	100	-46.52	10.203	0.939	124.9	-2.96	2.807	0.968	7.4
	686	-5.12	4.839	0.941	25.1	0.49	2.48	0.954	5.9
				Site D					
		$\rho_{u,u^*}=0, \sigma_u=0.1$				$\rho_{u,u^*}=-0.75, \sigma_u=1.0$			
0.07	0	0.82	2.667	0.931	7.9	20.14	3.577	0.933	14.7
	25	0.83	2.652	0.929	7.9	-17.63	4.009	0.931	23.3
	50	0.83	2.651	0.928	7.9	-9.62	3.776	0.932	16.3
	75	0.83	2.651	0.928	7.9	-6.3	3.716	0.931	15.1
	100	0.83	2.651	0.928	7.9	-4.12	3.685	0.937	14.5
	332	0.77	2.667	0.932	7.9	0.35	3.567	0.945	13.4
				Site E					
		$\rho_{u,u^*}=0, \sigma_u=0.23$				$\rho_{u,u^*}=-0.75, \sigma_u=1.0$			
0.23	0	14.85	3.338	0.944	11.5	21.48	3.294	0.934	12.6
	25	-73.82	3.33	0.942	11.6	-19.28	3.959	0.936	22.5
	50	-55.87	3.324	0.94	11.6	-9.28	3.624	0.941	14.3
	75	-51.04	3.323	0.941	11.5	-7.27	3.542	0.946	13.4
	100	-48.29	3.322	0.941	11.6	-6.16	3.496	0.94	12.8
	397	-40.42	3.332	0.948	11.4	-2.03	3.272	0.945	10.9
				Site F[a]					
		$\rho_{u,u^*}=0, \sigma_u=0$				$\rho_{u,u^*}=-0.75, \sigma_u=1.0$			
0	0					17.3	2.454	0.928	7.3
	25					-12.61	2.873	0.923	13.3
	50					-9.44	2.685	0.932	9.1
	75					-6.69	2.606	0.934	8
	100					-5.01	2.572	0.931	7.5
	703					0.5	2.448	0.941	6.3

[a] Site F had no errors in date of ART initiation, so no simulations were performed using its site-specific parameter estimates, and simulations using multi-site parameter estimates were based on $p = 0.06$ corresponding to the upper level of the 80% confidence interval for the error rate.