Supporting Information

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SI Text

Experiments of Ant Rafts in Soap Solutions. We find that even trace amounts will cause the ants to radically change their behavior, as shown in the images of a raft on water with traces of soap (Fig. S1). As soon as ants become even slightly soapy, they immediately release their grip with each other, which is shown by the disintegration of the raft and its submergence underwater. This is in contrast to the closely packed ants in the buoyant raft, as shown in Figs. 1 and 2 in the main text.

Differential Equation Models of Ant-Raft Formation. ***Straight movement in random direction model.*** Let N equal the total number of ants. Let t denote time in seconds. Let $n(t)$ denote the number of ants in the bottom layer at time t equal to the area of the raft at time t. The number of ants in the bottom layer can be converted to square centimeters with conversion factor 34 ants = 1 cm^2. Let h denote the eventual thickness of the raft, experimentally determined to be approximately 2.5 (ant heights). In the straight movement model, we assume that only ants in the top layer move, and they move by picking a random direction and going straight until they hit a boundary. When they hit a boundary, they "bounce" with probability $p = 0.65$ (this value determined by observation), picking a random direction away from the boundary, and "stick" with probability $1 - p$. If they bounce, they go again until they hit a boundary, where again they bounce with probability p and stick with probability $1 - p$. They repeat this process until they stick, at which time they join the bottom h layers there. The expected number of "bounces" before sticking is therefore $\frac{1}{1-p} - 1 = 1.86$.

Lemma 1. *Let x be a random point in a circle of radius r, and let θ be a direction randomly chosen from the interval $[0,2\pi]$. Then the expected distance from x to the circle boundary, in the direction θ, equals $\frac{8r}{3\pi} \approx 0.8488r$. Now let x be a point on the circle boundary and let θ be a direction randomly chosen from the interval $[0,\pi]$ where $\theta = \pi/2$ means the direction toward the circle center. Then the expected distance from x to the circle boundary in the direction θ equals $4r/\pi \approx 1.27r$.*

The Proof is given in *Calculus derivations* so as not to disrupt the exposition of the model.

Let u denote the speed of an ant in centimeters per second. When $N \geq (h+1)n(t)$, there is a full top layer of ants who can move. A single ant in that layer takes on average $[0.849 + (1.86)(1.27)]r/u = 3.21r/u$ s to reach a boundary and stick, where $\pi r^2 = n(t)/34$ so $r = \sqrt{n(t)/34\pi}$ is the radius of the raft in centimeters. There are $n(t)$ ants in the moving layer so, on average, $n(t)u/3.21r$ ants reach and stick to the boundary per second, equaling $\sqrt{34\pi n(t)}u/3.21$ ants per second. These ants form a new exterior boundary h ants thick. So,

$$\frac{dn(t)}{dt} = u\sqrt{34\pi n(t)}/3.21 \text{ h}.$$

When $hn(t) \leq N \leq (h+1)n(t)$, there is a partial layer of $N - hn(t)$ ants who can move. So,

$$\begin{aligned}\frac{dn(t)}{dt} &= u[N - hn(t)]/3.21 \text{ hr} \\ &= u\sqrt{34\pi}[N - hn(t)]/3.21 \text{ h}\sqrt{n(t)}.\end{aligned}$$

Brownian motion model. Let N,t,$n(t)$,h be as in the Straight Motion Model. We assume that only ants in the top layer move, and they move in Brownian motion until they hit a boundary. It is more or less obvious that the expected time to reach a boundary from the center of a circle of radius r is r^2 (up to scaling by ant speed). It is not as obvious what the expected time to reach a boundary is from a random point in the circle.

Lemma 2. *Let x be a random point in a circle of radius r. Execute standard Brownian motion from x. The expected time until the circle boundary is reached is $r^2/4$.*

Proof: From Øksendal (1), the expected first hitting time is $(r^2 - |x|^2)/2$ where $|x|^2$ is the squared distance to the circle center from x. The rest is calculus, given later.

When $N \geq (h+1)n(t)$, there is a full top layer of ants who can move. A single ant in that layer takes $r^2/4 = n(t)/136\pi$ s to reach a boundary, because $\pi r^2 = n(t)/34$. There are $n(t)$ ants in the moving layer so, on average, $136\pi n(t)/n(t) = 136\pi$ ants reach the boundary per second. Notice this number is independent of the size of the raft and of N:

$$\frac{dn(t)}{dt} = 136\pi/h.$$

When $hn(t) \leq N \leq (h+1)n(t)$, we have $N - hn(t)$ ants in the moving layer. On average, $136\pi[N - hn(t)]/n(t)$ reach the boundary per second. For this time period,

$$\frac{dn(t)}{dt} = 136\pi[N - hn(t)]/hn(t).$$

Note, this model has to be scaled by some unknown factor, which is the ant speed, but the ant speed here is not the usual centimeters per second, but rather the Brownian motion scaling compared with standard Brownian motion. This model's predictions do not match the data. In the data, rafts grow faster when N is bigger. We changed the model to let every ant except those in the lowest h layers move. The formula is simply the second formula used for all t, that is,

$$\frac{dn(t)}{dt} = 136\pi[N - hn(t)]/hn(t).$$

However, this model did not fit the data, either.

Calculus derivations. For the Brownian Motion Model, we want the average value of $(r^2 - x^2)/2$ in a circle of radius r, where x^2 is the squared distance to the circle center, which is

$$\frac{1}{\pi r^2}\int_{x=0}^{r} \frac{r^2 - x^2}{2} 2\pi x dx = r^2/4.$$

For the Straight Motion Model, we first need a geometric lemma. Suppose the ant is at distance x from the circle center. Consider the chord that is perpendicular to the line segment from the circle center to the ant and passes through the ant. Let z denote the length of the chord. Then $(z/2)^2 = (r - x)(r + x)$ whence $z/2 = \sqrt{r^2 - x^2}$. Now suppose the ant moves at angle θ

to the chord. If $\theta = 0$ or $\theta = \pi$, the distance to the circle boundary is $z/2$. If $\theta = \pi/2$, the distance is $r + x$ and if $\theta = 3\pi/2$ the distance is $r - x$. We need a formula for the general case of θ. By symmetry, it suffices to consider the cases $0 \le \theta \le \pi/2$; $\pi \le \theta \le 3\pi/2$. The two cases θ, $\theta + \pi$ define a chord passing through the ant. Let y denote the length of the shorter segment of the chord (where the chord intersects the ant). Then the longer segment has length $2x \sin\theta + y$, from which we obtain

$$y(2x\sin\theta + y) = (r - x)(r + x) \Rightarrow y^2 + 2x\sin\theta y - (r^2 - x^2) = 0.$$

One of the roots to this quadratic is negative. Hence,

$$y = -x\sin\theta + \sqrt{x^2\sin^2\theta + (r^2 - x^2)}.$$

If the ant randomly chooses between θ and $\theta + \pi$, the expected distance traveled is $\frac{1}{2}(2x\sin\theta + 2y) = \sqrt{x^2\sin^2\theta + r^2 - x^2} = \sqrt{r^2 - x^2\cos^2\theta}$. (Check this formulation against the two extremes: When $\theta = \pi/2$, we have $\cos\theta = 0$ and the expected distance is r, which is correct. When $\theta = 0$, we have $\cos\theta = 1$ and the expected distance is $\sqrt{r^2 - x^2} = z/2$, which is correct.)

We assume that the ant chooses a random direction. The expected distance traveled by the ant to a boundary is therefore

$$\frac{2}{\pi}\int_0^{\pi/2} \sqrt{r^2 - x^2\cos^2\theta}\, d\theta.$$

Taking the mean value of this individual expected distance as x ranges from 0 to r gives an overall expected distance of

$$\frac{1}{\pi r^2}\int_0^r 2\pi x dx \int_0^{\pi/2} \frac{2}{\pi}\sqrt{r^2 - x^2\cos^2\theta}\, d\theta.$$

Putting the factor of r on the left (which is obvious by scaling; we could have done it earlier) gives

$$r\int_0^1 4x/\pi dx \int_0^{\pi/2} \sqrt{1 - x^2\cos^2\theta}\, d\theta = 8r/3\pi \approx 0.8488r.$$

The integral was calculated both numerically and with Mathematica 7. (Check this integral against the two extremes. At the two extremes of integration, the values are $2r/\pi$ at $x = r$, and r at $x = 0$. The average value ought to be somewhere between $0.64 \approx 2/\pi$ and 1.)

After the ant reaches the boundary, the situation is slightly different. We assume that the ant moves in a random direction *away* from the boundary. In terms of our model, we assume that the ant chooses a random value of θ from the interval $0 \le \theta \le \pi$. The expected distance to the (next) boundary is

$$\frac{2}{\pi}r\int_0^{\pi/2} 2\sqrt{1 - \cos^2\theta}\, d\theta = \frac{4r}{\pi}\int_0^{\pi/2} \sin\theta\, d\theta = \frac{4r}{\pi} = 1.27r.$$

1. Øksendal B, (2003) *Stochastic Differential Equations: An Introduction with Applications* (New York, Springer), 6th Ed, p 125.

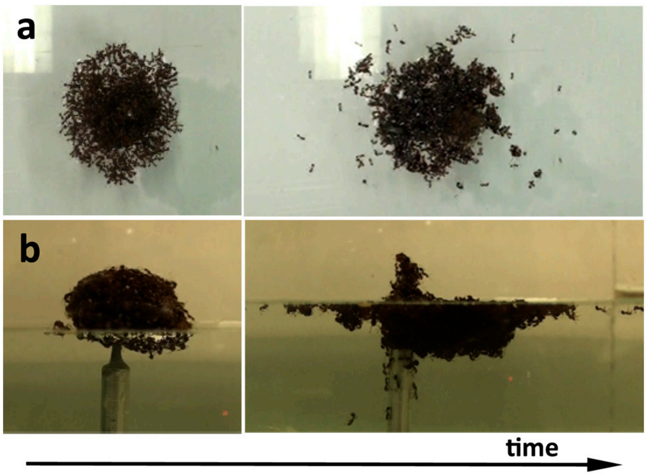
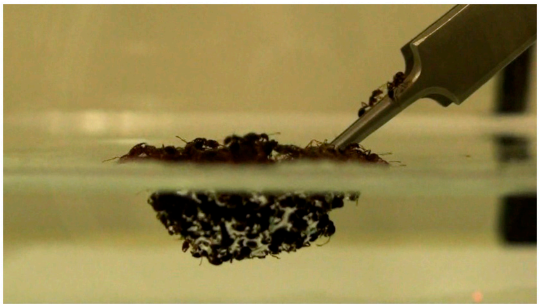
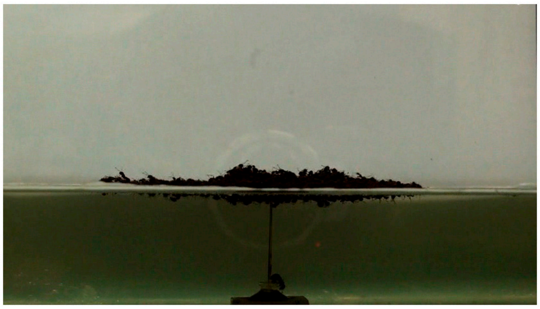


Fig. S1. Raft of 3,000 ants exposed to traces of surfactant: (*A*) top view; (*B*) side view.



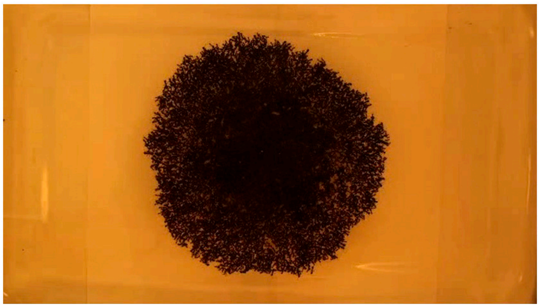
Movie S1. A raft of 500 ants pushed underwater using tweezers. The glistening sheen marks the presence of an air-layer waterproofing the raft.
Movie S1 (MOV)

Movie S2. Panoramic view of an ant raft, showing the ant–water contact line. Time sped up 2×.
Movie S2 (MOV)



Movie S3. Side view of 3,000 ants constructing a raft. The green fluid is water, atop which is air. At all times during the raft construction, the top layer of ants remain dry, floating atop a wetted bottom layer of ants. Time sped up 8×.
Movie S3 (MOV)

Movie S4. Top view of 3,000 ants constructing a raft. A sphere of ants is placed on the water surface. Upon contact with water, ants from the center of the raft scramble radially outward, adding to the raft's edge until generating an equilibrium shape of the raft resembling a pancake. Time sped up 8×.
Movie S4 (MOV)



Movie S5. Top view of 8,000 ants constructing a raft. Although this raft was the largest we studied, rafts of hundreds of thousands of fire ants can be observed in the wild. Time sped up 15×.
Movie S5 (MOV)