Supporting Information

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SI Methods

Simulation Results. A star network with 101 nodes was used as a test case to compare our rules of thumb with the outside–in strategy assuming perfect detection. The test network consisted of a central node with five arms, each of length 20 nodes. Baseline parameters for the simulations were $p_{\mathrm{d}} = 0.1$, $p_{\mathrm{m}} = 0.7$, and $p_{\mathrm{e}} = 0$. To test the effect of a network with heterogeneous infection and recovery parameters, the baseline parameters p_{d} and p_{m} were allowed to vary randomly above or below the baseline level in increments of 0.05 and 0.1, respectively, up to a maximum amount in each time step. Different amounts of variation were tested. We tested variations in p_{d} of 0, 0.05, and 0.1 and variations in p_{m} of 0, 0.1, 0.2, and 0.3 (Table S2). The rule of thumb and the outside–in management strategy were simulated 30 times each on 30 randomly generated test networks. It was assumed that all patches were initially infected in each simulation. Each simulation was run for 700 time steps and the average time to eradication was computed. For each parameter combination, we also varied the number of nodes managed per time step. We tested scenarios in which 1, 2, or 5 nodes could be managed per time step (Figs. S1–S3).

Our rules of thumb outperform the outside–in strategy regardless of the probabilities of infection and recovery and regardless of the number of patches that are managed in each time step. Increasing the variation in the probability of recovery made our rule of thumb show slightly increased performance relative to the outside–in strategy, although increasing the variation in the probability of infection did not seem to affect the scale of the improvement for the values tested.

The time to eradication decreases as more patches are able to be managed in each time step. We found that the relative benefits of our rules of thumb are greater when fewer patches are managed in each time step. The reason that our rule of thumb outperforms the outside–in strategy is because reinfections are less likely to occur using our rule of thumb. The longer the time it takes to eradicate the infection, the more opportunity there is for reinfections to take place, and the greater the benefit of our rule of thumb over the outside–in strategy. By managing multiple patches in a time step, the time to eradication is reduced, which makes the performance of the two strategies more similar.

Factored POMDP. Using the Bellman principle of optimality and the previously defined FPOMDP parameters (*Methods* in main text), we can calculate the optimal t-step value function from the $(t-1)$-step value function,

$$V_0^*(b) = \min_{a\in A}\left[\sum_{s\in S} R(s,a)\Pr(s|b)\right], \qquad \textbf{[S1]}$$

$$V_t^*(b) = \min_{a\in A}\left[\sum_{s\in S} R(s,a)\Pr(s|b) + \sum_{s\in S}\sum_{s'\in S}\sum_{z\in Z}\Pr(s|b)P(s'|s,a)O(z|s',a)V_{t-1}^*(b_z^a)\right], \qquad \textbf{[S2]}$$

where $\Pr(s \mid b)$ represents the probability of being in state s given a belief state b, and $b^a{}_z$ is the belief state assuming action a and observation z and P is the state–action transition matrix. Eq. **S1** minimizes the expected sum of instantaneous costs when there is no time left to manage for the infection. Similarly when there are t steps to go, Eq. **S2** minimizes the instantaneous costs and the future expected costs for the remaining $t-1$ steps. Interested readers could refer to Cassandra et al. (1) for further explanations of the dynamic programming equations.

The optimal solution π can be represented in two different ways. We can either apply directly the strategy function for each belief state we are in or represent the optimal strategy as a policy graph. The policy graph automatically generates all of the possible transitions over time given the performed action and the new observation, whereas the use of strategy functions requires updating the belief state using Bayes' rule given the performed action and the new observation. After performing action a and observing z, the updated belief b_z^a can be calculated from the previous belief b:

$$b_z^a(s') = \Pr(s'|b,a,z), \qquad \textbf{[S3]}$$

$$b_z^a(s') = \frac{O(z|a,s')\sum_{s\in S}P(s'|s,a)b(s)}{\Pr(z|a,b)}, \qquad \textbf{[S4]}$$

with

$$\Pr(z|a,b) = \sum_{s\in S}\sum_{s''\in S} O(z|s'',a)P(s''|s,a)b(s). \qquad \textbf{[S5]}$$

Algebraic Decision Diagram. The components of our optimization models (transition probabilities, rewards, and costs) are computationally represented with algebraic decision diagrams (2). Fig. S6 illustrates how the dynamic of node 3 (s_3) connected to node 2 (s_2) can be defined following our discrete-time SIS model assuming a line network and no management occurs (e.g., the action is do nothing). Our decision tool automatically generates the transition probabilities for each node and action on the basis of the structure of the network.

Analytic Approximation. We highlight the similarities between managing invasive species or disease (model 1) and managing a threatened species (model 2). For both POMDP models, the sets of states, actions, observations, and observation functions can be defined in the same way.

The main difference between both models resides in the effect of the manage action on the state transition model (P). Although managing an invasive species or a disease decreases the local probability of persistence ($p_{\mathrm{m1}} < p_{01}$), managing a threatened species increases its local probability of persistence ($p_{\mathrm{m2}} > p_{02}$). Also for an invasive species or disease we would expect a probability of persistence equal to or close to 1 when not managed, whereas a threatened species by definition will have a probability of persistence <1 ($1 \geq p_{01} > p_{02}$). Note that "survey" and do nothing have the same effect on the state dynamic for both models and the ability to detect the state infected is defined in the same way.

In the case of a disease or a pest the optimization objective is to maximize the expected number of susceptible nodes (model 1) or maximize the expected number of infected nodes (model 2) over time. This information is expressed by defining the cost or reward functions $R_1(s, a) = r_1(s) + c_1(a)$ and $R_2(s, a) = r_2(s) + c_2(a)$ with s in S_i, a in A_i, and r_i the immediate reward of being in a state and the cost of performing an action c_i. Actions have the same cost for both models but the cost or reward of being in a state is defined differently for both models. Here we use $r_1(\text{infected}) = -V$ and $r_2(\text{infected}) = V$ and $r_1(\text{susceptible}) = -r_2(\text{susceptible}) = 0$, with V a positive value representing the economic benefits of a persisting threatened species or the economic loss of a persisting disease or invasive species. The objective can also be expressed as the minimization of the expected number of infected nodes (model 1) or the expected number of susceptible nodes in the case of a threatened population (model 2). In absence of detection of the species/disease, these two models share a same solution structure: manage, survey, and do nothing.

From ref. 3 we know that the time we should spend managing is dependent on the belief of persistence of the species at the boundary between managing and surveying $b_{\mathrm{m/s}}$,

$$T_{\mathrm{m}} = \frac{\log(b_{\mathrm{m/s}})}{\log(p_{\mathrm{m}})}$$

with p_{m} the probability of persistence of the species when managed and

$$b_{\mathrm{m/s}} \approx \frac{(c_{\mathrm{m}} - c_{\mathrm{s}})}{[2c_{\mathrm{m}} d p_0 + V(p_{\mathrm{m}} - p_0)(1 + (1-d)p_0(1+p_0) - d p_0 p_{\mathrm{m}})]}$$

with p_0 the probability of persistence of the species when not managed, and d the probability of the species being detected if present when we survey.

Similarly the time we should spend surveying is defined by

$$T_{\mathrm{s}} = \frac{\log\left[b_{\mathrm{s/n}}(1-p_0(1-d(1-b_{\mathrm{m/s}})))/b_{\mathrm{m/s}}(1-p_0(1-d(1-b_{\mathrm{s/n}})))\right]}{\log[(1-d)p_0]}$$

with $b_{\mathrm{s/n}}$ defined as

$$b_{\mathrm{s/n}} \approx \frac{c_{\mathrm{s}}(1-p_{\mathrm{m}})(1-p_0)}{\left[p_0 d(V(p_{\mathrm{m}} - p_0 + p_0^T(1-p_{\mathrm{m}}^T)) - p_{\mathrm{m}}(1-p_0) - c_{\mathrm{m}}(1-p_{\mathrm{m}})(1-p_0)(T-1)))\right]},$$

with T the time we should spend surveying. For an n-population POMDP, the cost of managing (c_{m}), the cost of surveying (c_{s}), and the economic value of the species *(V)* remain unchanged as we manage or survey only one population at a time. Note that V can be easily transformed into the economic cost of losing a species (c). A species persists in the network when it is not extinct from the whole network; therefore we can define p_0 as $p_0 = 1 - p_{\mathrm{e}}{}^n$.

Similarly we can redefine the probability of persistence of the species across the network when one population is managed (p_{m}) as $p_{\mathrm{m}} = 1 - p_{\mathrm{e}}{}^{n-1} p_{\mathrm{m}}$.

We then define the probability of a species being detected, if present, when we survey one node (d). We first define the probability of not detecting the species across the network if one node is surveyed as $(1 - d_{\mathrm{s}})(1 - d_{\mathrm{n}})^{n-1}$. The probability of detecting at least one species across the n populations is $d = 1 - (1 - d_{\mathrm{s}})(1 - d_{\mathrm{n}})^{n-1}$.

We have redefined the key parameters defined in Chadès et al. (3) and provide an approximate solution to how long we should manage and survey n independent populations (Fig. S5).

1. Cassandra AR (1998) Exact and approximate algorithms for partially observable Markov decision processes. PhD thesis (Brown University, Providence, RI).
2. Bahar RI, et al. (1997) Algebraic decision diagrams and their applications. *Form Methods Syst Des* 10:171–206.
3. Chadès I, et al. (2008) When to stop managing or surveying cryptic threatened species. *Proc Natl Acad Sci USA* 105:13936–13940.
4. Canadian Wildlife Service and US Fish and Wildlife Service (2007) *International Recovery Plan for the Whooping Crane* (Recovery of Nationally Endangered Wildlife (RENEW), Ottawa and US Fish and Wildlife Service, Albuquerque, NM), p 162.
5. Loomis J, Ekstrand E (1997) Economic benefits of critical habitat for the Mexican spotted owl: A scope test using a multiple-bounded contingent valuation survey. *J Agric Resour Econ* 22:356–366.

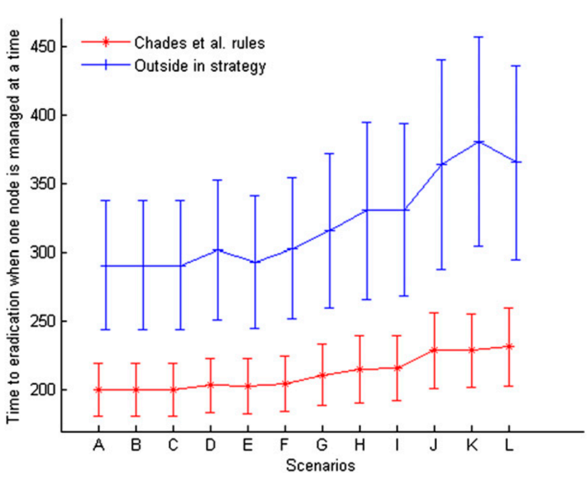


Fig. S1. Time to eradication on a star network of 101 nodes when 1 node can be managed at a time under 12 scenarios (Table S2). Error bars represent SD.

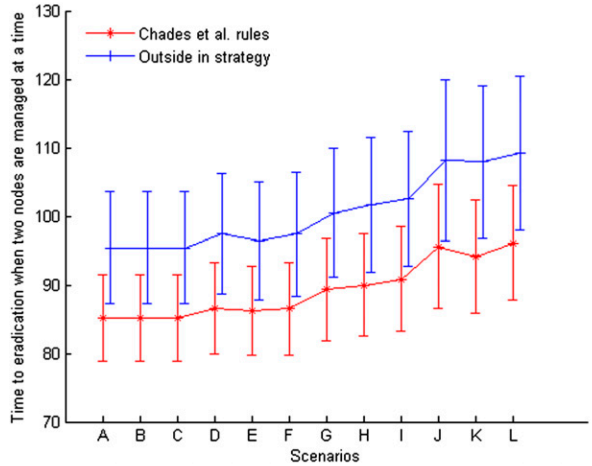


Fig. S2. Time to eradication on a star network of 101 nodes when 2 nodes can be managed at a time under 12 scenarios (Table S2). Error bars represent SD.

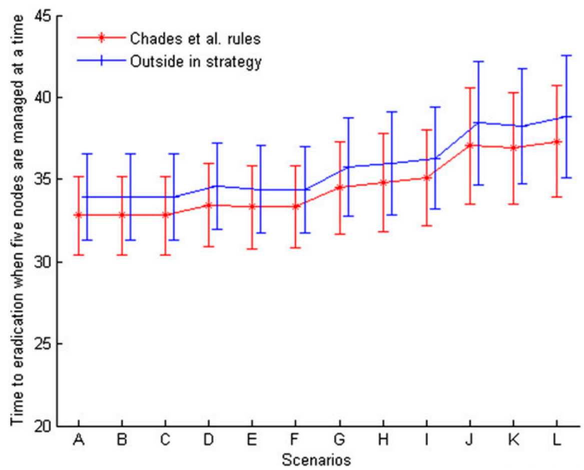


Fig. S3. Time to eradication on a star network of 101 nodes when 5 nodes can be managed at a time under 12 scenarios (Table S2). Error bars represent SD.

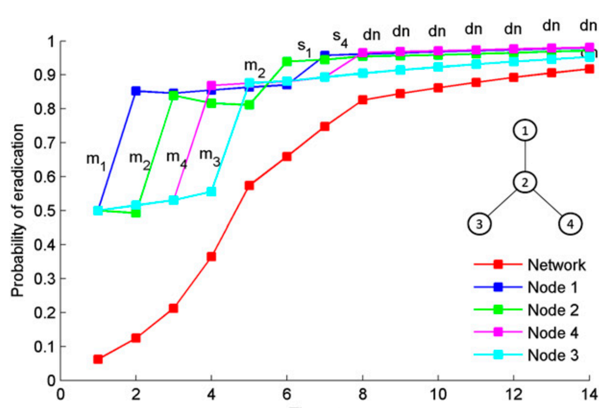


Fig. S4. Management recommended over time under imperfect detection of an infected node on a star network of four nodes (*Inset*) when no prior information is available about the infection of the network. The red line represents the probability of eradication of the infection across the whole network and starts at 1/16. The other colored lines represent the probability of eradication of each node. m_i, manage node *i*; s_i, survey node *i*; and dn, do nothing.

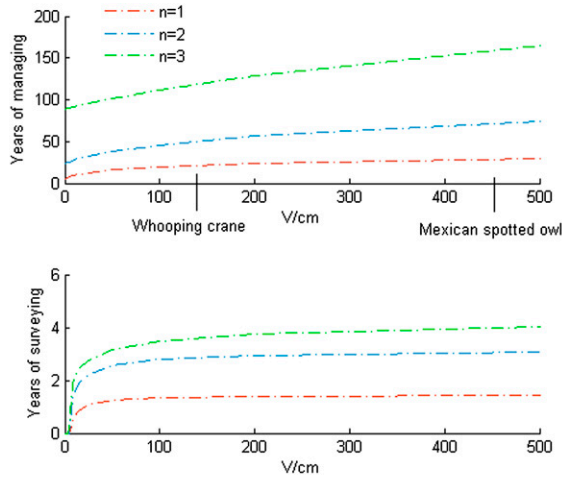


Fig. S5. Comparison of resource allocation for (*Upper*) managing and (*Lower*) surveying of an approximate analytical solution for a hypothetical highly threatened species. *n* represents the number of populations. We used the following parameters: probability of local persistence 0.7 if not managed and 0.85 when managed and probability of detection of 0.2 when managed and 0.78 when surveyed. The whooping crane and Mexican spotted owl are given as examples of potential economic loss (V) and estimated cost of management (c_m) derived from refs. 3–5.

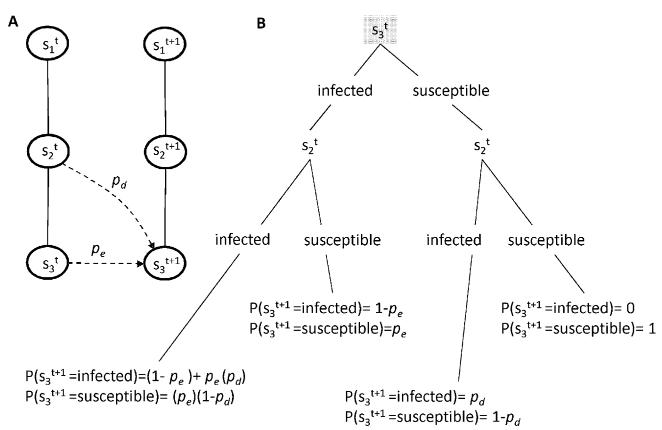


Fig. S6. Graphical representation of the dynamic of node 3 (*B*) connected to node 2 assuming a line network (*A*) and no management. p_e is the probability of local extinction. p_d represents the probability of dispersal.

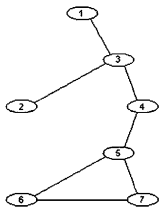
Table S1. Parameters used for our general case study

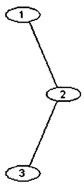
MDP/POMDP parameters	Value
Probability of a node being infected if one neighbor is infected (p_d)	0.1
Probability of a node becoming susceptible if infected and not managed (p_e)	0.01
Probability of a node becoming susceptible if it has been managed (p_m)	0.7
Probability of detecting an infected node when "manage" or "do nothing" is used (d_n)	0.1
Probability of detecting an infected node when "survey" is used (d_s)	0.7
Cost of managing one node (c_m)	100
Cost of surveying one node (c_s)	50
Cost of a node being infected (c)	100

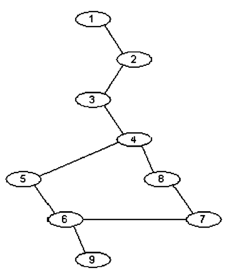
We derived our rules of thumb assuming a hypothetical infection for our SIS network model.

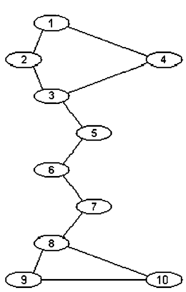
Table S2. Robustness of Chadès et al.'s rules (this study) for a star network of 101 nodes when varying the management option under different uncertain scenarios

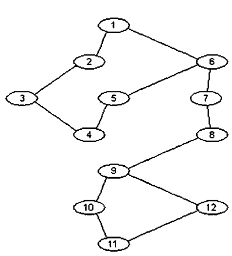
			Improvement under different management options: no. treatments avoided (%)		
Simulation scenarios	Maximal variation in dispersal (p_d)	Maximal variation in recovery (p_m)	1 node	2 nodes	5 nodes
A	0	0	90.68 (31.2)	20.36 (10.7)	5.68 (3.3)
B	0.05	0	90.68 (31.2)	20.36 (10.7)	5.68 (3.3)
C	0.1	0	90.68 (31.2)	20.36 (10.7)	5.68 (3.3)
D	0	0.1	98.32 (32.6)	21.65 (11.1)	5.68 (3.3)
E	0.05	0.1	90.20 (30.8)	20.23 (10.5)	5.33 (3.1)
F	0.1	0.1	98.61 (32.5)	21.79 (11.2)	5.08 (3.0)
G	0	0.2	105.08 (33.2)	22.35 (11.1)	6.40 (3.6)
H	0.05	0.2	115.45 (34.9)	23.51 (11.6)	5.66 (3.2)
I	0.1	0.2	115.29 (34.8)	23.60 (11.5)	5.98 (3.3)
J	0	0.3	135.81 (37.2)	24.97 (11.5)	6.89 (3.6)
K	0.05	0.3	151.81 (39.8)	27.59 (12.8)	6.72 (3.5)
L	0.1	0.3	134.15 (36.7)	26.30 (12.0)	7.65 (3.9)
Average performance of Chadès et al.'s rules (this study)			111.42 (34.1)	22.97 (11.3)	6.07 (3.4)











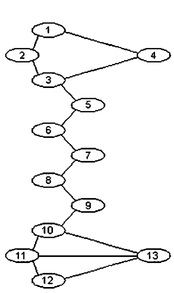
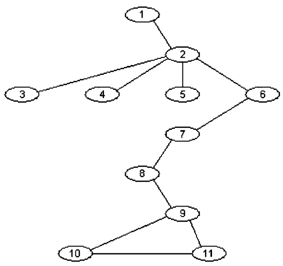
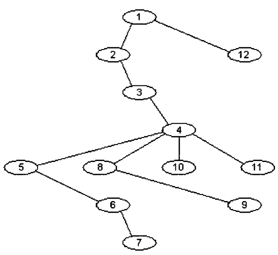


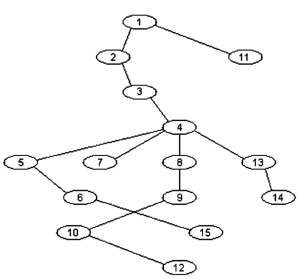
Table S3. Optimal starting node, centrality measures, and connectance for nine small networks

Node/optimal starting node	Closeness centrality	Eigenvector centrality	Subgraph centrality	Betweenness centrality
	Connectance = 0.143			
1*	0.375*	6.19901*E*-06	1.639827167	0
2*	0.375*	6.19901*E*-06	1.639827167	0
3	0.545454545	0.000474185	2.970722293	18
4	0.6	0.020593476	2.405064022	18
5	0.545454545	0.467122568	3.548827737	16
6	0.4	0.446541381	2.785170056	0
7	0.4	0.446541381	2.785170056	0
	Connectance = 0.222			
1*	0.666666666666667*	1.11022*E*-16	1.589091778	0
2	1	0	2.178183557	2
3*	0.666666666666667*	4.44089*E*-16	1.589091778	0
	Connectance = 0.111			
1*	0.266666666666667*	5.47939*E*-08	1.590664613	0
2	0.347826087	6.2538*E*-06	2.230519482	14
3	0.444444444	0.000467986	2.332523605	24
4	0.533333333	0.021049118	3.09935418	32
5	0.470588235	0.020587331	2.406658041	16
6	0.421052632	0.021042808	3.045334013	16
7	0.4	0.020581076	2.354238711	4
8	0.444444444	0.020581132	2.35586695	8
9	0.307692308	0.000455588	1.641877645	0
	Connectance = 0.11			
1	0.25*	6.60899*E*-12	2.387727503	1
2	0.3	3.54822*E*-10	2.43854041	7
3	0.375	5.58534*E*-08	3.184818517	37
4	0.3	3.54822*E*-10	2.43854041	7
5	0.409090909	6.19972*E*-06	2.336976825	40
6	0.409090909	0.000461732	2.283417888	40
7	0.375	0.020581077	2.354295345	36
8	0.321428571	0.467110059	3.547158417	28
9*	0.257142857	0.446535126	2.785135174	0
10*	0.257142857	0.446535126	2.785135174	0
	Connectance = 0.083			
1	0.323529412	0	2.336885376	22
2	0.275	−3.60822*E*-16	2.284499388	10
3	0.239130435	−3.60822*E*-16	2.282898901	4
4	0.275	2.77556*E*-16	2.284499388	10
5	0.323529412	2.77556*E*-17	2.336885376	22
6	0.392857143	1.33227*E*-15	3.079961857	64
7	0.392857143	−5.55112*E*-17	2.335398156	60
8	0.366666667	−1.11022*E*-16	2.338600345	56
9	0.323529412	−1.11022*E*-15	3.1848471	49
10*	0.261904762	−1.66533*E*-15	2.438540723	9
11*	0.22*	−1.22125*E*-15	2.387727512	1
12*	0.261904762	−1.44329*E*-15	2.438540723	9
	Connectance = 0.088			
1*	0.184615384615385*	−1.11128*E*-15	2.387727503	1
2*	0.214285714	2.4434*E*-15	2.438540407	10
3	0.255319149	1.6714*E*-12	3.184818151	55
4*	0.214285714	2.55963*E*-15	2.438540407	10
5	0.285714286	3.54855*E*-10	2.336942307	64
6	0.307692308	5.65615*E*-08	2.281299105	70
7	0.315789474	6.42527*E*-06	2.279650222	72
8	0.307692308	0.000487437	2.28184756	70
9	0.285714286	0.02253151	2.363206665	64
10	0.255319149	0.557314127	3.875563984	54
11	0.218181818	1.005338126	4.335579023	10
12	0.184615384615385*	0.513687999	3.027562192	0
13	0.218181818	1.005338126	4.335579023	10

Table S3. Cont.







Node/optimal starting node	Closeness centrality	Eigenvector centrality	Subgraph centrality	Betweenness centrality
		Connectance = 0.09		
1	0.303030303	3.54845*E*-10	1.748050626	0
2	0.416666667	5.72728*E*-08	4.792724749	60
3	0.303030303	3.54846*E*-10	1.748050626	0
4	0.303030303	3.54846*E*-10	1.748050626	0
5	0.303030303	3.54845*E*-10	1.748050626	0
6	0.434782609	6.20112*E*-06	2.445293933	50
7	0.416666667	0.000461734	2.28673399	48
8	0.37037037	0.020581079	2.354351998	42
9	0.3125	0.46711006	3.547159041	32
10*	0.25*	0.446535126	2.785135179	0
11*	0.25*	0.446535126	2.785135179	0
		Connectance = 0.076		
1	0.282051282	−4.2466*E*-15	2.228942914	20
2	0.35483871	−9.4369*E*-16	2.283073466	36
3	0.44	0	2.448751983	48
4	0.523809524	8.88178*E*-16	4.913204096	90
5	0.407407407	9.99201*E*-16	2.44717798	36
6	0.314285714	1.47105*E*-15	2.23394323	20
7	0.244444444	1.52656*E*-16	1.590721201	0
8	0.379310345	−1.11022*E*-16	2.397908291	20
9	0.282051282	5.68989*E*-16	1.595554054	0
10	0.35483871	1.33227*E*-15	1.751455123	0
11	0.35483871	1.33227*E*-15	1.751455123	0
12*	0.224489795918367*	−1.65146*E*-15	1.590637769	0
		Connectance = 0.062		
1	0.245614035	5.06539*E*-16	2.22894323	26
2	0.304347826	6.93889*E*-16	2.283103048	48
3	0.378378378	6.66134*E*-16	2.450528038	66
4	0.466666667	8.43769*E*-15	4.974437299	150
5	0.358974359	2.10942*E*-15	2.44895403	48
6	0.28	4.71845*E*-16	2.233972806	26
7	0.325581395	8.32667*E*-16	1.753172953	0
8	0.378378378	1.77636*E*-15	2.450528038	66
9	0.304347826	1.69309*E*-15	2.283103048	48
10	0.245614035	2.60209*E*-15	2.22894323	26
11*	0.2*	9.01189*E*-16	1.590637771	0
12*	0.2*	2.68015*E*-16	1.590637771	0
13	0.341463415	1.11022*E*-16	2.399683719	26
14	0.259259259	4.16334*E*-17	1.595583006	0
15	0.222222222	2.84495*E*-16	1.590721512	0

*Optimal starting node.