Supplementary Information
Interconnect-free parallel logic circuits in a single mechanical resonator

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Supplementary Note 1 Δ dependence

The mechanical oscillator response when excited with a single pump (p) at fixed frequency $f_p = 2f_0$ whilst being swept with a signal (s) excitation at frequency $f_s = f_0 + \delta$ as a function of δ reveals the fundamental oscillation mode along f_s which corresponds to exciting and sampling the resonator at the same frequency i.e. the standard lock-in measurement as shown in Fig. 2a. The interaction of p and s results in the creation of an idler at $f_i = f_p - f_s = f_0 - \delta$. At $f_s = f_0$, the 2 excitations intersect resulting in degenerate parametric amplification. To confirm this interpretation we repeat the above measurement but now with p detuned by Δ so that $f_p = 2f_0 + \Delta$. The resulting response shown in Supplementary Fig. S1a again reveals the fundamental mode along f_s but now as expected, the idler no longer intersects f_s at f_0. Instead the idler appears at $f_i = f_p - f_s = f_0 + \Delta - \delta$ that is the idler is positively offset from f_0 by an amount equal to Δ.

Supplementary Note 2 Multiple signal excitations

The state shown in Fig. 3a is created by injecting multiple signal and pump excitations into the mechanical oscillator. First, the electromechanical resonator is injected with two signals at $f_{s1} = f_0 + \Delta + \delta$ and $f_{s2} = f_0 - \Delta + \delta$ which results in the fundamental mode being excited by both $s1$ and $s2$ as shown in Supplementary Fig. S2a. Activating only pA with $f_{pA} = 2f_0 + \Delta$ (pB with $f_{pB} = 2f_0 - \Delta$) yields 2 first order idler $f_i^{(+s1)}$ and $f_i^{(+s2)}$ ($f_i^{(-s1)}$ and $f_i^{(-s2)}$) as shown in Supplementary Fig. S2b (Supplementary Fig. S2c). From these measurements it can be seen that $f_i^{(-s1)} = f_i^{(+s2)}$ that is the idler from $s1$ and pB overlaps with the idler created by $s2$ and pA. The complete set of idlers created with all the signals and pumps activated is shown in Fig. 3a and the corresponding numerical simulation is shown in Supplementary Fig. S2d.

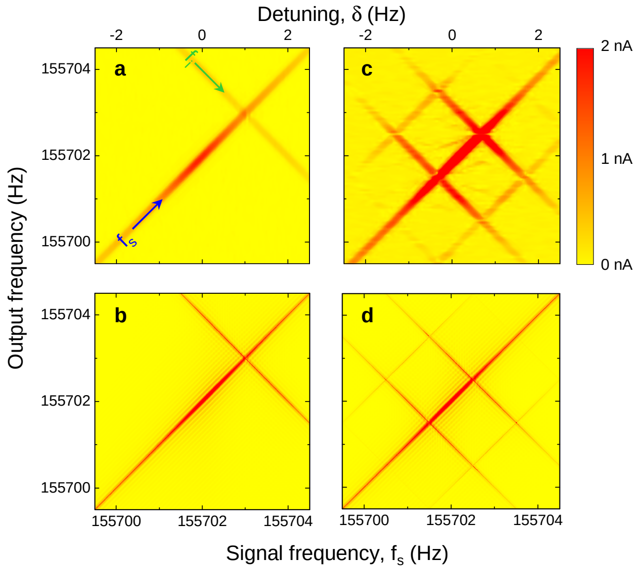
Supplementary Note 3 Idler bandwidth

In order to investigate the bandwidth in which the idlers can be observed; the pump $f_p = 2f + \Delta$ and signal $f_s = f$ are excited and their response is measured at $f_p - f_s = f + \Delta$ whilst f is swept in f_p, f_s and Δ is varied. The result of this measurement shown in Supplementary Fig. S4a and the corresponding numerical simulation shown in Supplementary Fig. S4b reveal the appearance of an idler at $f_0 + \Delta$ which cannot be observed beyond the bandwidth of the fundamental mode i.e. $\Delta \sim$20 Hz. Moreover, an oscillation at f_0 is also observed whose lineshape and amplitude is identical to the idler at $f_0 + \Delta$ as shown in Supplementary Fig. S4c. Both oscillations have the same Q value as the fundamental mode.

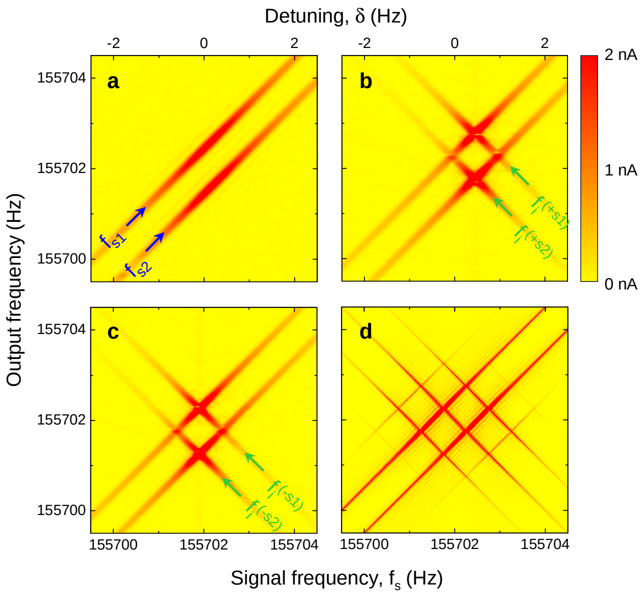
These observations can be interpreted as follows. When $f_s = f_0$ the signal excitation is large as the mechanical oscillator is on resonance and it mixes with pump excitation producing an idler at $f_i = f_p - f_s = f_0 + \Delta$ whose intensity is much less than $f_s = f_0$ as $f_i \neq f_0$ i.e. it is not resonantly enhanced by the mechanical oscillator. Because the mechanical oscillator is sampled at f_i only the *weak* idler excitation is observed and not the resonant signal excitation. Conversely, when $f_s = f_0 - \delta$ the signal excitation is weak because the mechanical oscillator is off resonance by δ. The signal mixes with the pump excitation producing an idler at $f_i = f_p - f_s = 2f_0 - 2\delta + \Delta - f_0 + \delta = f_0 - \delta + \Delta = f_0$ when $\delta = \Delta$ whose intensity is much larger than signal as $f_s \neq f_0$ i.e. it is resonantly enhanced by the mechanical oscillator. Again, because the mechanical oscillator is sampled at f_i only the on resonance idler excitation is observed and not weak off resonant signal. Both processes are identical and produce idlers along f_0 and $f_0 + \Delta$ whose lineshapes and amplitudes are the same.

Supplementary Note 4 Beyond the parametric resonance threshold

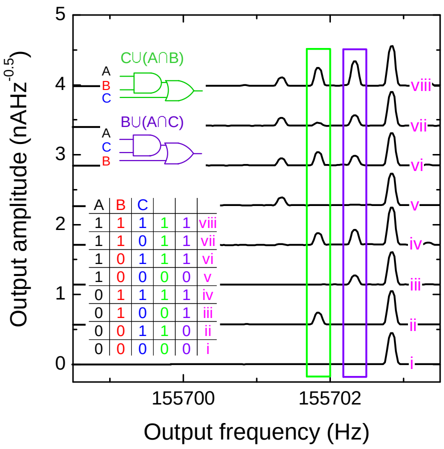
More interestingly still, if the measurement shown in Fig. 2c and Supplementary Fig. S1c is repeated with large pump amplitudes beyond the parametric resonance threshold (i.e. $\Gamma > \gamma$, see Methods) the mechanical resonator exhibits a non-trivial response as shown in Supplementary Fig. S5a. In addition to observing idlers up to the fourth order; parametric resonances along f_0 and $f_0 \pm 2\Delta$ are also observed. Unexpectedly, idlers with a $\pm 2\delta$ dependence can also be observed which are created by the mixing between pumps, signals and idlers via the Duffing nonlinearity (see Methods). This assertion is confirmed by numerically solving equation 9 which can only reproduce the experimental results with a finite nonlinear term i.e. $\beta = 1$ and $\Gamma = 2.5\gamma$ as shown in Supplementary Fig. S5b. Besides reproducing the $\pm 2\delta$ idlers, the simulation also reveals the existence of idlers with a $\pm 3\delta$ and $\pm 4\delta$ dependence which are beyond the detection limit in our experiment. Crucially, this measurement indicates that the parametric idlers can be observed beyond the bandwidth imposed by the fundamental mode.



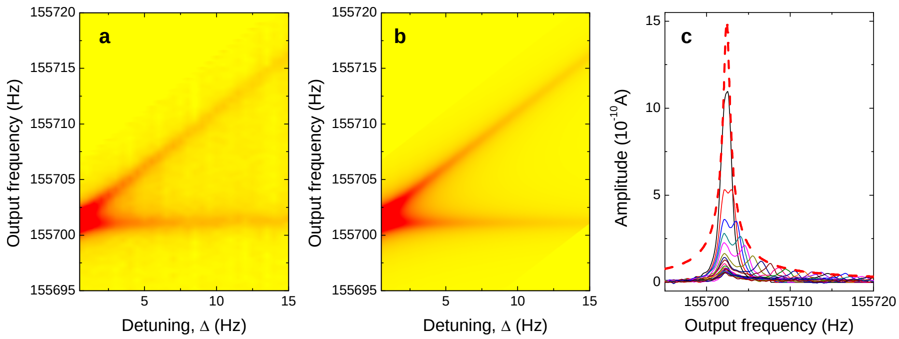
Supplementary Figure S1: **a,** The response of the mechanical oscillator measured via the lock-in amplifier with $f_p = 2f_0 + \Delta$ where $\Delta = 2$ Hz and $f_s = f_0 + \delta$ reveals the fundamental mode f_s and the idler f_i. **b,** The corresponding theoretical response generated from equation 9 (see Methods). **c,** and **d,** The same as Figs. 2**c** and 2**d** except now $\Delta = 1$ Hz reveals the migration of the idlers away from f_0. In all cases the experimental response is broader than the numerical simulations due to an experimental RBW of 50 mHz.



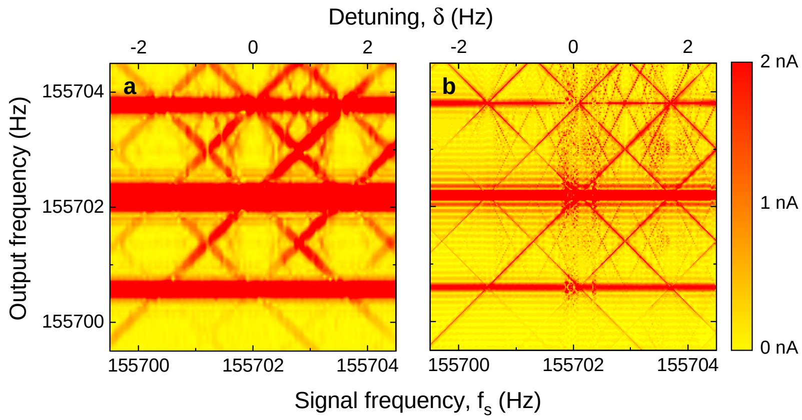
Supplementary Figure S2: **a,** The electromechanical resonator's response measured via the lock-in amplifier when 2 signal excitations at $f_{s1} = f_0 + \Delta + \delta$ and $f_{s2} = f_0 - \Delta + \delta$ with $\Delta = 0.5$ Hz are activated which reveal the fundamental mode being excited by both $s1$ and $s2$. **b,** (**c,**) Activating pA (pB) at $f_{pA} = 2f_0 + \Delta$ ($f_{pB} = 2f_0 - \Delta$) where $\Delta = 0.5$ Hz whilst $s1$ and $s2$ are both active results in 2 first order idlers $f_i^{(+s1)}$ and $f_i^{(+s2)}$ ($f_i^{(-s1)}$ and $f_i^{(-s2)}$) where $f_i^{(-s1)} = f_i^{(+s2)}$. (**d,**) The numerical simulation corresponding to Fig. 3a when the mechanical oscillator is excited by 2 signals and 2 pumps with Δ=0.5 Hz (see Methods).



Supplementary Figure S3: The logic circuits C∪(A∩B) and B∪(A∩C) are constructed with the following pair of first and second order idlers $f_{pC} - f_s$, $f_{pA} - (f_{pB} - f_s)$ and $f_{pB} - f_s$, $f_{pA} - (f_{pC} - f_s)$ respectively where $\Delta_{s1} = \Delta_{s2}$ =0.5 Hz, Δ_{pA}=0.5 Hz, Δ_{pB}=-0.5 Hz and Δ_{pC}=0 Hz (see main text) and are measured via a spectrum analyser with RBW=25 mHz are offset for clarity and numbered (roman numerals) to correlate with the numbered truth combinations in the truth table where the various inputs and circuits have been colour coded.



Supplementary Figure S4: **a,** The electromechanical oscillator excited with a signal and pump excitation at $f_s = f$ and $f_p = 2f + \Delta$ with an actuation amplitude of 50 μV$_{rms}$ and 40 mV$_{rms}$ respectively. The response of the electromechanical oscillator measured as a function of f and Δ at $f+\Delta$ via a lock-in amplifier reveals the existence of two idlers along $f_i = f_0$ and $f_i = f_0+\Delta$. **c,** The corresponding simulation generated from Equation 9 reproduces the experimental response. **c,** The same as **a** except now the data is plotted to more clearly show the amplitude response of the $f_0 + \Delta$ idler as a function of Δ. The dashed line is a Lorentzian centred at f_0 with $Q = 140000$ which clearly indicates that the idler is confined within the bandwidth of the fundamental mode.



Supplementary Figure S5: **a,** The same as Fig. 2c and Supplementary Fig. S1c except now $\Delta = 0.8$ Hz and the pump amplitudes have been increased to 100 mV$_{rms}$ revealing idlers up to the fourth order as well as idlers with a $\pm 2\delta$ dependence. **b,** The corresponding numerical simulation with $\beta = 1$ and $\Gamma = 2.5\gamma$ (see Methods) reproduces the experimental results as well as revealing the existence of idlers with $\pm 3\delta$ and $\pm 4\delta$ dependence.