Supplemenatry material:
Striated acto-myosin fibers can re-organize and register in response to elastic interactions with the matrix

B.M. Friedrich, A. Buxboim, D.E. Discher, S.A. Safran

1 Derivation of strain field induced by striated fiber – equation (2)

A localized point force $F_j\delta(\mathbf{x}')$ acting tangential to an elastic half-space generates a displacement field $u_i(\mathbf{x}) = G_{ij}(\mathbf{x}-\mathbf{x}')F_j$ with a corresponding strain field $u_{ij} = (\partial_i u_j + \partial_i u_j)/2$ where

$$G_{ij} = (1+\nu)/(\pi E_m)\left[(1-\nu)\delta_{ij}/|x| + \nu\, x_i x_j/|x|^3\right] \tag{S1}$$

is the Boussinesq Green's function for the elastic half space. By differentiation, we obtain the strain field due to a single point force dipole $P_{ij}\delta(\mathbf{x}')$ as $u_{ij} = -\partial_l(\partial_j G_{ik} + \partial_i G_{jk})P_{kl}/2$. This is the "building block" for the total strain field of a striated fiber: the parallel component of the strain field induced by the dipole density Π_{ij} is given by a convolution of the density with this Green's function as $u_{11}(x,y) = -\int_{-\infty}^{\infty} dx'\,\partial_{x'}^2 G_{11}(x-x',y)\rho(x')$. In Fourier space, this relation becomes a multiplication $\widetilde{u}_{11}(q,y) = -(iq)^2\widetilde{G}_{11}(q,y)\tilde{\rho}(q)$ where $\widetilde{G}_{11}(q,y)$ is the Fourier transform of the Green's function with respect to the x-coordinate

$$\begin{aligned}\widetilde{G}_{11}(q,y) &= \int G_{11}(x,y)\exp(-iqx)\,dx \\ &= 2(1+\nu)/(\pi E_m)\left[(1-\nu)K_0(q|y|) + \nu(i\partial_q)^2(q/|y|)K_1(q|y|)\right].\end{aligned} \tag{S2}$$

We find $\widetilde{G}_{11}(0,y) = 0$, hence the $q=0$ Fourier component of the strain field is always zero irrespective of the mean dipole density ρ_0. Note $2\tilde{\rho}(q) = \rho_0\delta(q) + \rho_1\delta(q-q_0)$. Now eq. 2 follows with a response factor Φ that is essentially given by the Fourier transform of the Green's function evaluated at the principal wave length $q_0 = 2\pi/a$ of a striated fiber, $\Phi(y/a) = 2\pi^2 E_m\widetilde{G}_{11}(q_0 y)$. More explicitely,

$$\Phi(Y) = 2\pi(1+\nu)\left[(1-\nu)K_0(2\pi Y) - \nu\partial_Y^2\, Y K_1(2\pi Y)/(2\pi)\right]. \tag{S3}$$

Here $Y = y/a$ and K_n denote the modified (hyperbolic) Bessel functions of the second kind.
For sake of completeness, we also report the result for the case that the dipole density $\rho(x)$ is acting inside the bulk of an extended elastic material (as a minimal model of a compliant cytoskeleton surrounding the fiber) instead of on the surface of an elastic half space (the substrate). In this case, eq. 2 holds with a modified propagation factor, $\Phi_{3\mathrm{d}}(Y) = (2\pi/8)(1+\nu)/(1-\nu)[(3-4\nu)K_0(2\pi Y) - \partial_y^2\, Y K_1(2\pi Y)/(2\pi)]$, that shows similar functional dependence on Y and ν as $\Phi(Y)$.

2 Variable Z-body size

For pedagogical reasons, we presented our theory only for the principal Fourier mode of the force dipole density induced by a striated fiber, assuming $\rho(x) = \rho_0 + \rho_1 \cos(2\pi x/a)$. This simplification corresponds to the assumption of an effective size of the crosslinking band of about half a sarcomer size $a/2$. Our theory can be generalized in a straight-forward manner to the case of variable Z-body sizes. To be more specific, we can write the dipole density $\rho(x)$ as a periodic series of peaks of width σ

$$\rho(x) = \sum_n \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-na)^2}{2\sigma^2}\right]. \tag{S4}$$

The corresponding Fourier transformed force dipole density reads $\tilde{\rho}(q) = \sum_n \exp[-q^2\sigma^2/2]\,\delta(q - nq_0)$, and the strain field is given by a superposition of harmonic contributions,

$$u_{11}(x,y) = \frac{2}{E_m a^2} \sum_n \Phi(ny/a)\, \exp[-(2\pi n\sigma/a)^2/2]\, \cos(2\pi nx/a). \tag{S5}$$

A similar formula is then derived for the elastic interaction energy $W_{\text{int}} = \int u_{11}(x,d)\rho(x+\Delta x)$ (using the orthogonality relation for the cosine), which thus generalizes eq. 6. For values $\sigma > 0.2a$, this interaction energy is almost indistinguishable from the simple case studied in the main text. For smaller values of sigma (i.e. if Z-bodies are more localized), simulations show that elastic interactions drive smectic order even more effectively. Additionally, the minimal lateral fiber spacing $d^*(\sigma)$, above which inter-fiber registry is favored by elastic interactions, decreases, if smaller values for σ are chosen. In the limiting case $\sigma \to 0$, inter-fiber registry is favorable for all fiber spacings d, see also (25).