Supplemental color figures for "Nonlinear Extraction of Independent Components of Natural Images Using Radial Gaussianization" by Siwei Lyu and Eero P. Simoncelli, to appear in an upcoming issue of *Neural Computation*.

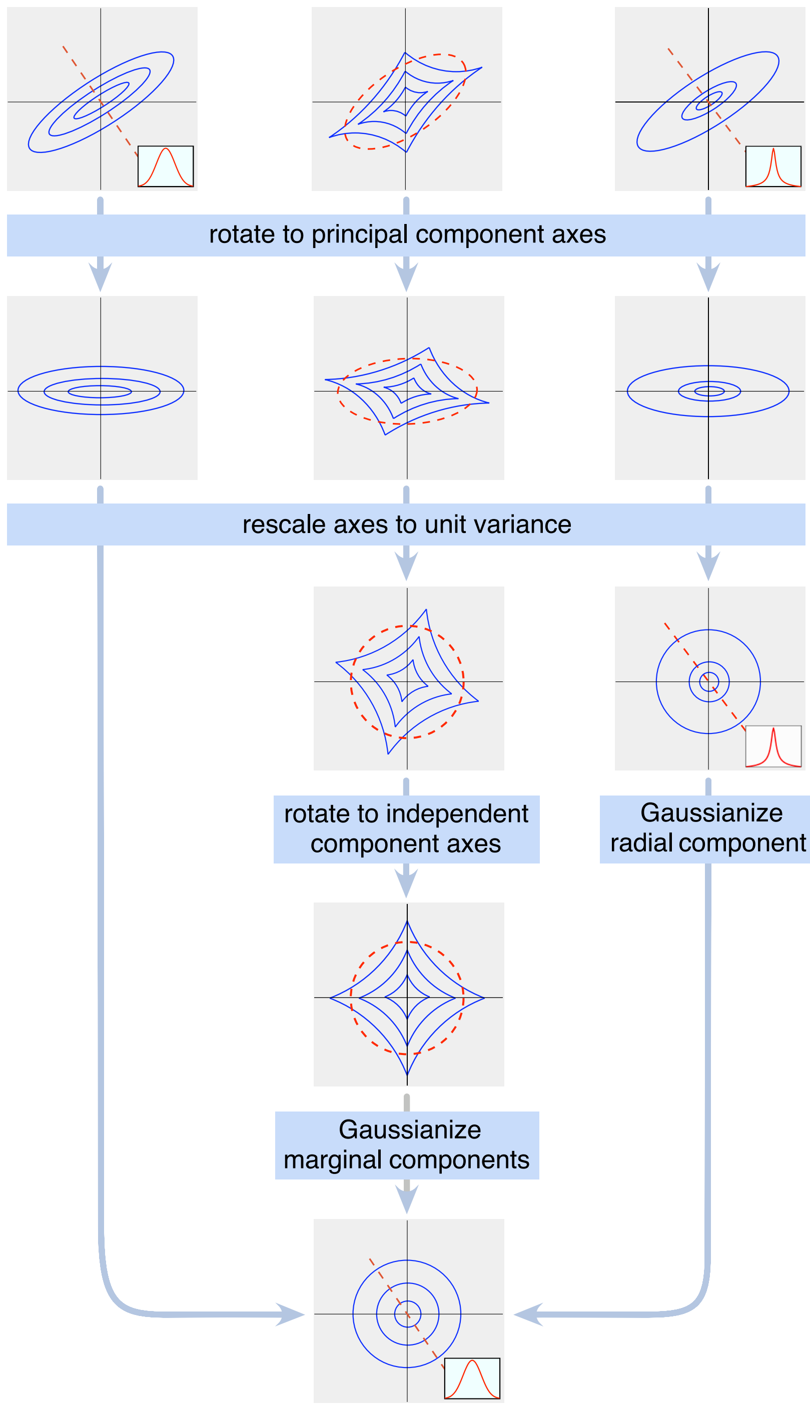

rotate to principal component axes
rescale axes to unit variance
rotate to independent component axes
Gaussianize radial component
Gaussianize marginal components

Figure 1: Three methods of dependency elimination and their associated source models, illustrated in two dimensions. Red dashed ellipses indicate covariance structure. Inset graphs are slices through the density along the indicated (dashed) line. **Left:** PCA/whitening a Gaussian source. The first transformation rotates the coordinates to the principal coordinate axes of the covariance ellipse, and the second rescales each axis by its standard deviation. The output density is a spherical, unit-variance Gaussian. **Middle:** Independent components analysis, applied to a linearly transformed factorial density. After whitening, an additional rotation aligns the source components with the Cartesian axes of the space. Last, an optional nonlinear *marginal Gaussianization* can be applied to each component, resulting in a spherical Gaussian. **Right:** Radial Gaussianization, applied to an elliptically symmetric non-Gaussian density, maps the whitened variable to a spherical Gaussian.

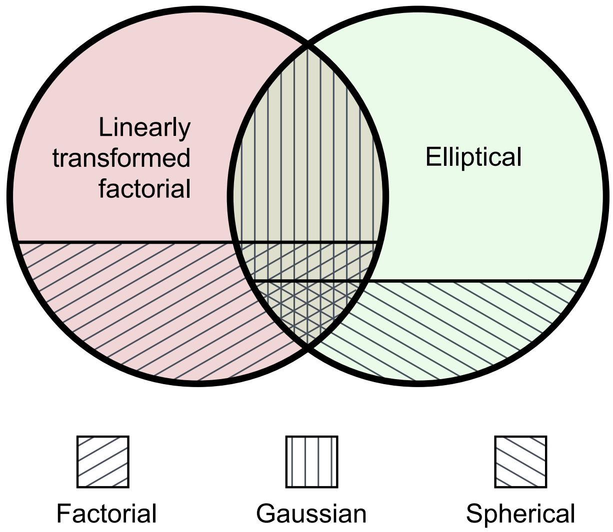


Figure 2: Venn diagram of the relationship between density models. The two circles represent the two primary density classes considered in this article: the linearly transformed factorial densities, and elliptically symmetric densities (ESDs). The intersection of these two classes is the set of all Gaussian densities. The factorial densities (i.e., joint densities whose components are independent) form a subset of the linearly transformed factorial densities and the spherically symmetric densities (SSDs) form a subset of the ESDs.

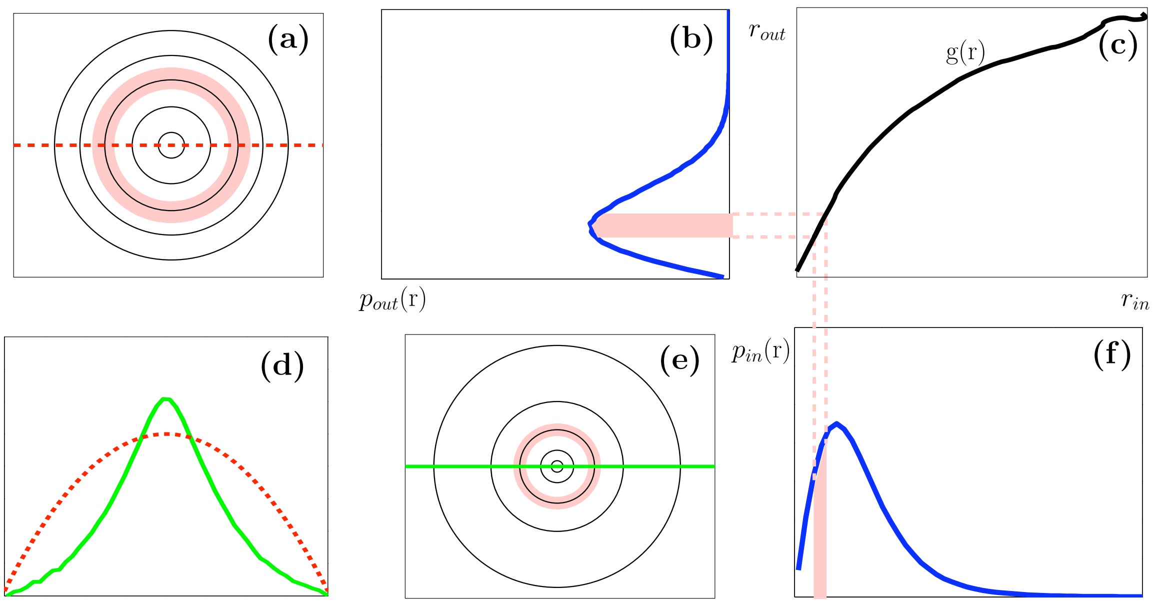
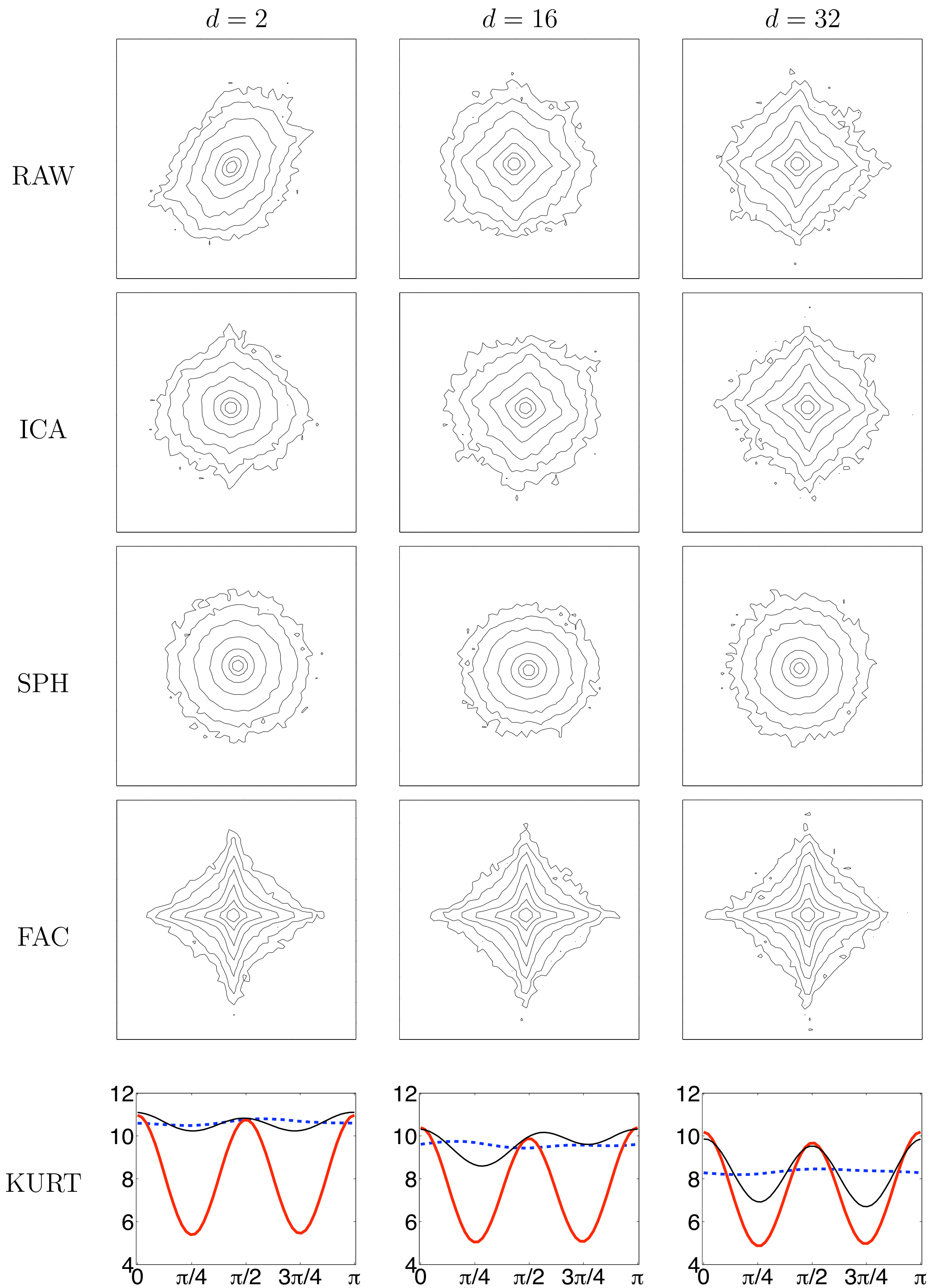


Figure 3: Radial Gaussianization procedure, illustrated for two-dimensional variables. Joint densities of **(a)** a spherical Gaussian, and **(b)** a non-Gaussian SSD (multi-variate Student's t). Plotted levels are chosen such that a spherical Gaussian has equal-spaced contours. **(b,f)**: radial marginal densities of the joint Gaussian and SSD densities in (a,e), respectively. Shaded regions correspond to shaded annuli. **(c)**: radial map of the RG transform. **(d)** log marginal densities of the joint Gaussian (red dashed line) and SSD (green solid line) densities.



$d = 2$
$d = 16$
$d = 32$
RAW
ICA
SPH
FAC
KURT
12
10
8
6
4
0
π/4
π/2
3π/4
π

Figure 4: Joint histograms of pairs of samples from transformed images, at three different spatial separations. Lines indicate level sets of constant probability, chosen such that a Gaussian density will have equi-spaced contours. Data taken from a single test image. RAW: original bandpass filtered image. ICA: data after ICA transformation. SPH: sphericalized synthetic data (randomized directions). FAC: factorialized synthetic data (independently sampled marginal components). KURT: kurtosis of marginal density, as as function of marginal direction, for RAW (black solid line), SPH (blue dashed line), and FAC (red solid line) data.

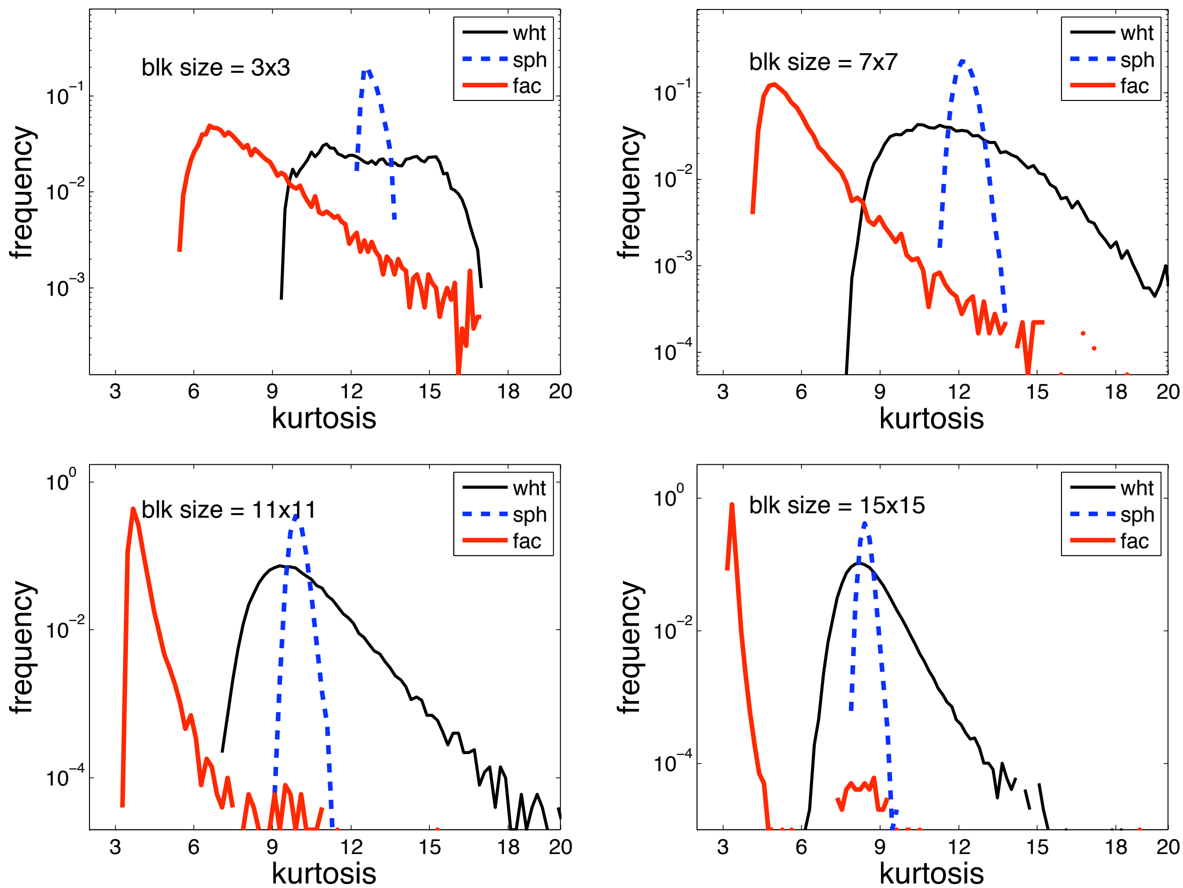


Figure 5: Histograms of kurtosis values for ICA transformed pixel blocks (black), sphericalized synthetic data (blue dashed) and factorialized synthetic data (red). Data are taken from a set of 10 images.

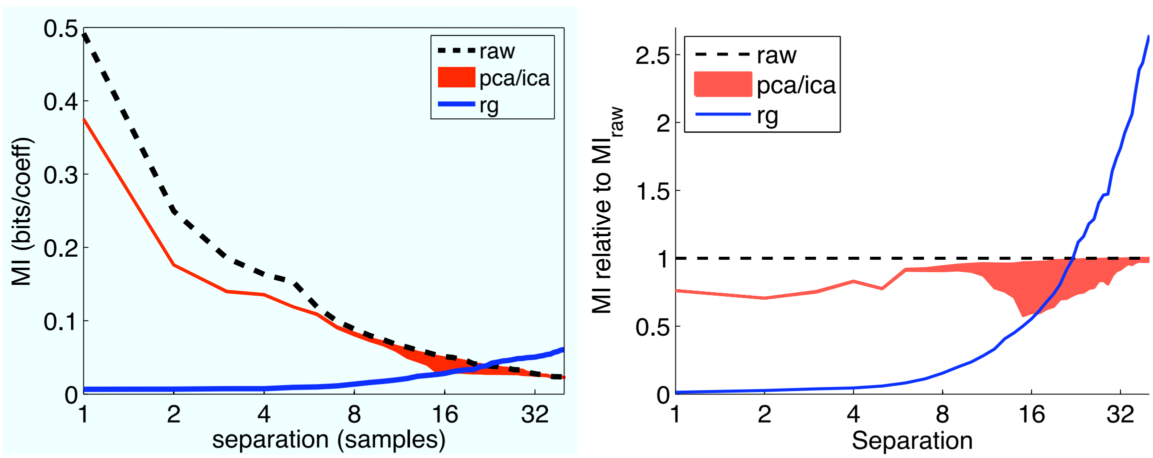


Figure 6: Multi-information for original bandpass filtered pixel pairs (black dashed line), compared with PCA (top of red region), ICA (bottom of red region) and RG (blue solid line) transformations. All values are averages over 10 images.

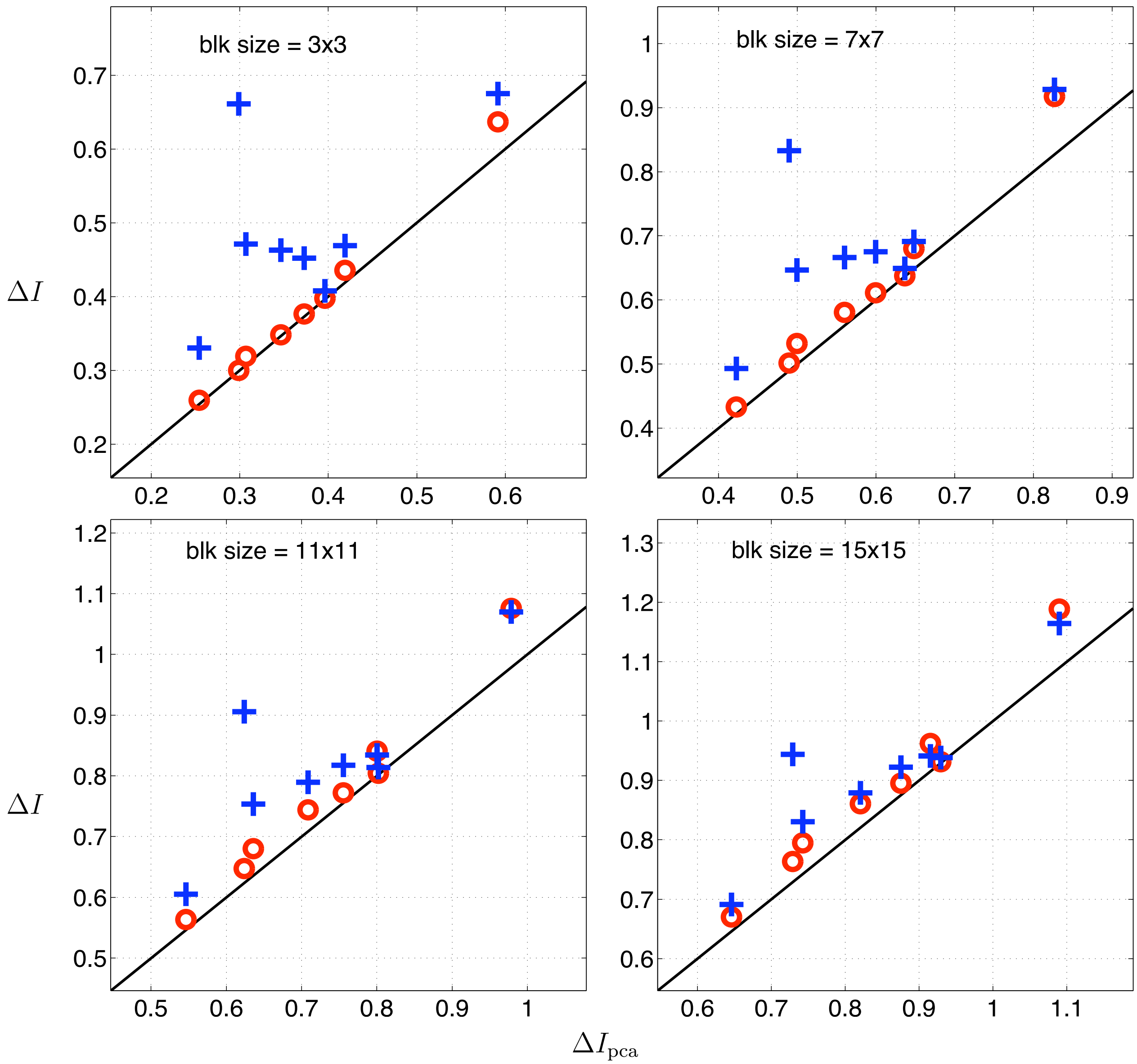


Figure 7: Comparison of reduction of MI achieved by ICA and RG against that achieved by ICA, for pixel blocks of four different sizes. Each symbol corresponds to result from one image in our test set. Blue pluses denote ΔI_{rg}, and red circles denote ΔI_{ica}.

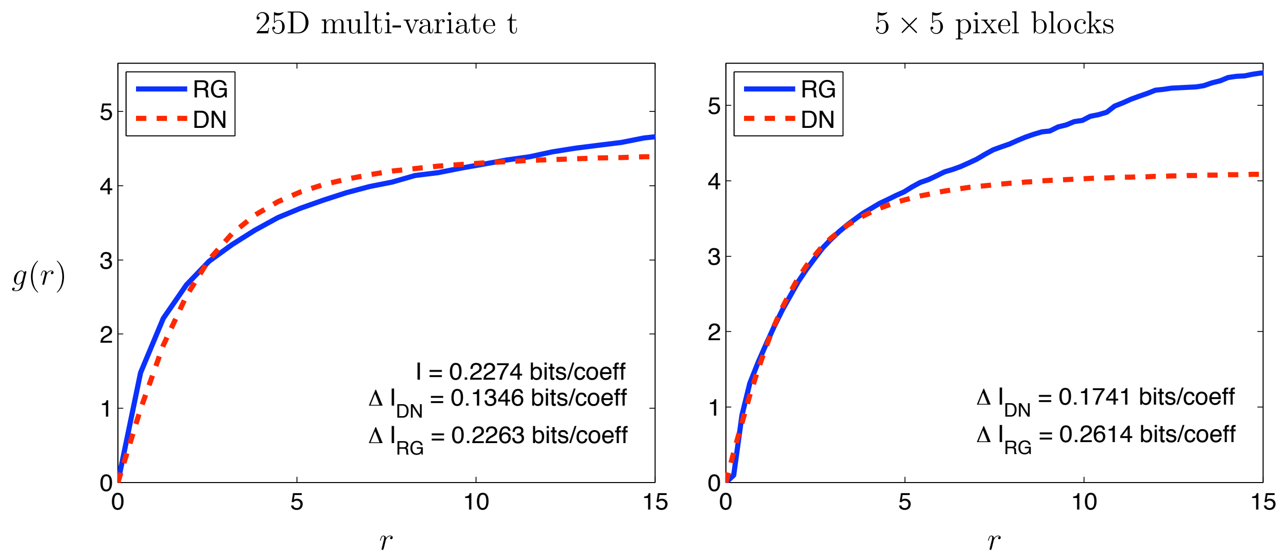


Figure 8: Comparison of radial transforms corresponding to RG (blue solid line) and DN (red dashed line), each optimized to minimize MI for 10^5 samples of a 25-dimensional spherical Student's t density (left), and 5×5 pixel blocks taken from one bandpass filtered test image (right). MI reduction is indicated, inset within each graph.