Supplementary material for Rosenthal *et al*. (March 20, 2001) *Proc. Natl. Acad. Sci. USA*, 10.1073/pnas.071525998

Fig. 5. Neural-network encoding of stimulus distribution. Demonstration of the effect of relative initial tuning width (or fraction of neurons activated by each stimulus, **f**) on interactions among neurons and their formation of attractor units. (*A*) Each synapse in the synaptic matrix is identified by coordinates *i*, *j* denoting pre- and postsynaptic neurons. *i* and *j* also identify the neuron's preferred stimulus, defined as the middle of the range of stimuli that activate it. This notation isomorphism between neurons and (their preferred) stimuli is exploited below, repeatedly. To illustrate formation of synaptic structure, we transform these coordinates to *w* – position along the main diagonal of the matrix and *z* – the distance from the main diagonal ($w = (i+j)/\sqrt{2}$; $z = (i-j)/\sqrt{2}$). (*B*) Ranges of long term potentiation (LTP) and depression (LTD). The diagram presents a subregion of the *w*, *z* matrix shown in *A*. All neuron pairs with average stimulus preference *w* form a line parallel to the *z*-axis. The gray triangular region in *A* and *B* reflects the range of stimuli that activate both neurons and may potentiate synapses between them. The stippled regions reflect ranges of stimuli that activate only one of the pair and may depress synapses between them. Thus, for any pair, *w*, *z*, the set of potentiating stimuli corresponds to the dashed line (parallel to the *w*-axis) within the gray region, while the set of depressing stimuli corresponds to the two ranges along this line within the stippled regions. The fractions *P*(*w*, *z*) and *D*(*w*, *z*) of the presentation events that tend to potentiate and depress synapse *w*, *z*, respectively, determine the asymptotic probability that this synapse will be in a potentiated state, $P_{\mathrm{LTP}}(w, z)$ (see *Appendix* in the body of the paper). Since the peaked distribution used in the simulation, (as in the experiment), is a normalized sum of Gaussians (ignoring edge effects for simplicity), for $z < n\,\mathbf{f}/\sqrt{2}$, *P*(*w*, *z*) is related to the partial area under the Gaussians from $w - n\,\mathbf{f}/\sqrt{2} + z$ to $w + n\,\mathbf{f}/\sqrt{2} - z$. For *M* Gaussians with standard deviation σ_M and inter-peak distance $\Delta\mu$, this area can be expressed as: $P(w,z) = \frac{1}{2M}\sum_{k=1}^{M} S\left(W - \left(k - \frac{1}{2}\right)\cdot\Delta W, \frac{F}{2} - Z\right)$, where *S*(*x*, *y*) is the difference between the error functions *erf*(*x* + *y*) and *erf*(*x* - *y*), and *W*, ΔW, *F*, and *Z* express *w*, $\Delta\mu\sqrt{2}$, $N\mathbf{f}\sqrt{2}$, and *z*, respectively, in units of $2\sigma_M$. Elsewhere, *P*(*w*, *z*) is zero. Similarly:

$$D(w,z) = \frac{1}{2M}\sum_{k=1}^{M}\left[S\left(W - \left(k - \frac{1}{2}\right)\cdot\Delta W + \frac{F}{2}, Z\right) + S\left(W - \left(k - \frac{1}{2}\right)\cdot\Delta W - \frac{F}{2}, Z\right)\right].$$

These expressions make explicit the dependence of the eventual synaptic matrix structure on the relationship between **f** and $\Delta\mu$. (*C*) Effect of tuning width (**f**) on pattern of connectivity: graphs present asymptotic potentiation probability, $P_{\mathrm{LTP}}(w, z)$, as a function of inter-neuronal distance (in units of σ_M), computed as described in text and above (*B*). The three curves in each graph represent P_{LTP} as a function of the presynaptic neuron, *j*, for three different response preferences of the postsynaptic neurons, i.e., $P_{\mathrm{LTP}}(i, j)$ for three choices of *i* [peak: $i = 0$, midpoint: $i = 3/2\sigma$, and minimum: $i = 3\sigma$ (left, center, and right curves, respectively)]. The right and left columns correspond to three- and four-peak distributions, respectively, plotted partially by gray areas. The initial tuning width, indicated by a dark horizontal bar below each graph, corresponds to the interpeak distance for $M = 5$, 4, or 3 (top, middle, and bottom rows, respectively). When the graph is scaled to sigma units, **f** appears different in the three and four peak cases because σ is smaller in the four-peak case. We therefore rescale the abscissa to equate bar size, allowing a direct comparison between the curves. Note that maximal P_{LTP} is always for synapses between closest neighbors (due to high likelihood of coactivation), with P_{LTP} decreasing with interneuron distance. The symmetric connectivity patterns for pairs including a peak or minimum result from the symmetry of the stimulus distributions about these points. To enable strong attractors, probabilities for short-range connections ($\leq 1.5\sigma$) around the peaks have to be higher than around the minima and long-range connectivity probabilities have to be higher around the minima, leading to concave vs. convex curves at peak vs. minimum. Strong attractors also require a connectivity gradient toward the peak, reflected in asymmetric curves for pairs including a neuron with midpoint stimulus preference. When **f** corresponds to the interpeak-interval of the three-peak distribution case ($\mathbf{f} = \Delta\mu_3$, left column, lower graph), strong attractors are possible only with a three-peak distribution, in contrast to cases with narrower tuning, where both three- and four-peak distributions (for $\mathbf{f} = \Delta\mu_4$) or only the four-peak distribution (for $\mathbf{f} = \Delta\mu_5$) induce strong attractors. Only the case of $\mathbf{f} = \Delta\mu_3$ reflects well the behavioral findings of a three-peak distribution advantage in classification learning.