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Effect of f on Attractor Formation

Fig. 5 describes the computation of the probability that a synapse is eventually in a potentiated state (P_{LTP}). It demonstrates the effect of stimulus distribution (number of peaks) and initial tuning width (**f**) on P_{LTP}, focusing on neuron pairs that include a neuron with preferred stimulus at the distribution peak, its minimum or the midpoint between them.

As long as the stimulus distribution is not too broad, connectivity patterns around the peak and minimum will depend differently on synaptic distance. In particular, the convexities of the curves corresponding to peak and minimum are clearly different (C in Fig. 5). There are more potentiated short-range connections ($<1.5\sigma$) around the peak and more long-range connections around the minimum. This connectivity pattern maintains attractor reverberations around the peaks, preventing escape from them and enabling spread of activity from the minima towards the attractors. The larger the difference in connectivity pattern at peak and minimum, the more stable the attractors, the narrower their boundaries, and the fewer the presentations needed for their formation. In addition, curves for pairs including a peak-to-minimum midpoint are asymmetric, reflecting greater connectivity with neurons having stimulus preference that is closer to the peak. This, too, enables propagation of activity towards the attractors.

Strong attractor patterns (i.e., those resistive to noise) require lower and upper bounds on **f**. We denote by $\Delta\mu_M$ the inter-peak-interval normalized to the full range width, where M is the number of distribution peaks. When **f** is narrower than $\Delta\mu_M$ (e.g., $\mathbf{f} = \Delta\mu_5$ with a three-peak distribution; see Fig. 5*C*) long-range connections are rare and limited in scope, leading to weak local attractors with broad boundaries. Up to a certain level, the broader the initial tuning, the greater the long-range connectivity especially around the minima, leading to stronger attractors. On the other hand, when **f** is much broader than $\Delta\mu_M$, short-range connectivity around the minimum also becomes massive, which weakens the attractors (e.g., $\mathbf{f} = \Delta\mu_3$ with a four-peak distribution). Broad **f** also leads to overall high connectivity around the midpoint neurons with less asymmetry in their connectivity with neurons that have a stimulus perference near the peak vs. near the minimum .

With $\mathbf{f} = \Delta\mu_4$, the graphs reveal similar short-range connectivities in the three- and four-peak distributions, reflected in similar P_{LTP} levels around the peaks of the curves (Fig. 5*C*, middle row). The midpoint curves for the two distributions are also similarly asymmetric. Still, these two distribution cases clearly differ in their long-range connectivities ($>1.5\sigma$) around the minima, reflected in higher P_{LTP} here for the four-peak distribution. This difference is not seen around the distribution peaks. In light of the above considerations, this connectivity difference would lead to stronger and more rapidly evolving attractors in the four-peak case, contrary to the behavioral finding. We therefore reject $\mathbf{f} = \Delta\mu_4$.

When $\mathbf{f} = \Delta\mu_3$, the broader tuning leads to a broader range of densely potentiated connections (Fig. 5*C*, bottom row). Still, in the three-peak distribution case (left column), there is a pronounced difference in short-range connectivity near the peak and the minimum, in favor of the peak. A much smaller difference is seen in the four-peak case (right column). This smaller difference results in a weaker attractor. Actually, the peak vs. minimum difference at short ranges is so small that an attractor pattern is possible only because of dense long-range connections around the minimum. Long-range connections are slower to develop because the probability of evoking two distant neurons is smaller, implying slow formation of attractor patterns (if at all). In addition, there is less asymmetry in the midpoint curve in this case relative to the three-peak case. Taken together, a clear advantage is seen for $\mathbf{f} = \Delta\mu_3$ for the three-peak relative to the four-peak case in the strength and formation rate of attractor patterns. This is similar to the stronger and more rapidly developing distribution effects found behaviorally in the three-peak case. These considerations imply the surprising prediction of a broad tuning width in neurons that are involved in internal class representation.