# Rate-Independent Constructs for Chemical Computation

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## Appendix: Logarithm Reactions

We present chemical reactions that implement the pseudo-code presented in the text.

**System Initialization** As with our other modules, we have a small set of reactions to control the overall timing of our system.

$$2\,x + x'_{\text{ab}} + y'_{\text{ab}} \xrightarrow{\text{slow}} 2\,x + g^P \tag{1}$$
$$g^P + x' \xrightarrow{\text{fast}} x' \tag{2}$$
$$g^P + y' \xrightarrow{\text{fast}} y' \tag{3}$$
$$g^P \xrightarrow{\text{slow}} g^1 + g^2 \tag{4}$$

**Halving** We use a slight variation of our decrement module to implement the operation `x = x / 2`.

$$x + g^1 \xrightarrow{\text{slow}} x' + g^1 \tag{5}$$
$$g^1 + x_{\text{ab}} \xrightarrow{\text{slow}} \varnothing \tag{6}$$
$$2\,x' + g'^1_{\text{ab}} \xrightarrow{\text{fast}} x + x^{\text{rx}} \tag{7}$$
$$x^{\text{rx}} \xrightarrow{\text{slow}} \varnothing \tag{8}$$
$$x' + x^{\text{rx}}_{\text{ab}} + g'^1_{\text{ab}} \xrightarrow{\text{slow}} \varnothing \tag{9}$$
$$\tag{10}$$

**Increment** We use our increment module to implement the line `y = y + 1`. We set $y$ to be 1 initially.

$$y + g^2 \xrightarrow{\text{slow}} y' + g^2 \tag{11}$$
$$g^2 + y_{\text{ab}} \xrightarrow{\text{slow}} \varnothing \tag{12}$$
$$2\,y' + g'^2_{\text{ab}} \xrightarrow{\text{fast}} y' + y + y^{\text{rx}} \tag{13}$$
$$y^{\text{rx}} \xrightarrow{\text{slow}} \varnothing \tag{14}$$
$$y' + y^{\text{rx}}_{\text{ab}} + g'^2_{\text{ab}} \xrightarrow{\text{slow}} 2\,y \tag{15}$$
$$\tag{16}$$

**Cleanup** Once the module has completed, we decrement $y$ by one, storing the result in $y_f$.

$$x'_{\mathrm{ab}} + y'_{\mathrm{ab}} \xrightarrow{\text{slow}} done^{\mathrm{P}} \tag{17}$$

$$done^{\mathrm{P}} + 2\,x \xrightarrow{\text{fast}} 2\,x \tag{18}$$

$$done^{\mathrm{P}} + x' \xrightarrow{\text{fast}} x' \tag{19}$$

$$done^{\mathrm{P}} + y' \xrightarrow{\text{fast}} y' \tag{20}$$

$$done^{\mathrm{P}} \xrightarrow{\text{slow}} done \tag{21}$$

$$2\,done \xrightarrow{\text{fast}} done \tag{22}$$

$$done + 2\,y \xrightarrow{\text{slow}} y + y_f \tag{23}$$

**Absence Indicators** Two special absence indicators are used by the halving and increment modules above; a total of 13 are needed for the system to function properly. They are of the same form as all other absence indicators, described in the paper.