Appendix A

The purpose of the following analysis is to derive equation (2.1.1.2), representing the equilibrium between the force generated in the fiber and the shear resisting force of the endomysium.

First, the shear force exerted on the fiber by the endomysium, F_{end}, was calculated as a function of fiber stretch λ.

Assuming that shear strain is linearly distributed along the length of the endomysium layer we can write:

$$\beta_{\text{end}}(\lambda, z) = \frac{\delta(\lambda, z)}{h(\lambda)}, \tag{A.1}$$

where $\beta_{\text{end}}(\lambda, z)$ is the shear strain in the endomysium and

$$\delta(\lambda, z) = \left(\frac{1-\lambda}{\lambda}\right)z \tag{A.2}$$

is the displacement of a point on the periphery of the fiber at distance z from the fixed end of the fiber (Fig. 1B). $h(\lambda)$ is the thickness of the endomysium (Fig. 1B) which is a function of fiber stretch λ.

The shear force exerted on the fiber by the endomysium is obtained by integrating the shear stress over the surface of the fiber, where we have assumed that shear strain is constant through the thickness of the endomysium.

With the use of equation (2.1.1.1) and (A.1-2) we can write:

$$\begin{aligned} F_{\text{end}} &= \int_0^{\lambda L_0} 2G_{\text{end}}\beta_{\text{end}}(\lambda, z)\big(2\pi r(\lambda)\big)dz \\ &= 2\pi G_{\text{end}} L_0^2 \frac{r(\lambda)}{h(\lambda)}(1-\lambda)\lambda, \end{aligned} \tag{A.3}$$

where $r(\lambda)$ is the radius of the fiber at length $l = \lambda L_0$. Assuming incompressibility of the fiber gives $r(\lambda) = r_0 / \sqrt{\lambda}$, where r_0 is the initial fiber diameter (Fig. 1A).

We calculated $h(\lambda)$ using the assumption of incompressibility for the endomysium. The initial volume of the endomysium layer is: $V_0^{(\mathrm{end})} = L_0\pi \left\{ (r_0 + h_0)^2 - r_0^2 \right\} = \pi L_0 \left(2r_0 h_0 + h_0^2 \right)$, where h_0 is the initial thickness of the endomysium (Fig. 1A). The endomysium volume at fiber length l is given by $V_{\mathrm{end}} = \pi L_0 \left(\frac{\lambda+1}{2} \right) \left(2r(\lambda) h(\lambda) + h^2(\lambda) \right)$. Assuming incompressibility of the endomysium $V_{\mathrm{end}} = V_0^{(\mathrm{end})}$:

$$\pi L_0 \left(\frac{\lambda+1}{2} \right) \left(2r(\lambda) h(\lambda) + h^2(\lambda) \right) = V_0^{(\mathrm{end})},$$

$$h^2 + 2rh - \frac{V_0^{(\mathrm{end})}}{\pi L_0 \left(\frac{\lambda+1}{2} \right)} = 0. \tag{A.4}$$

Equation (A.4) has one positive solution:

$$h(\lambda) = -r + \sqrt{r^2 + \frac{V_0^{(\mathrm{end})}}{\pi L_0 \left(\frac{\lambda+1}{2} \right)}}. \tag{A.5}$$

The expression for $r(\lambda)/h(\lambda)$ was further simplified as:

$$\frac{r(\lambda)}{h(\lambda)} = \frac{1}{-1 + \sqrt{1 + 2\kappa(2+\kappa)\left(\frac{\lambda}{\lambda+1} \right)}}, \tag{A.6}$$

where $\kappa = h_0 / r_0$.

Equation (A.6) was substituted into (A.3) to give:

$$F_{\text{end}} = 2\pi G_{\text{end}} L_0^2 \lambda \left(\frac{1-\lambda}{-1+\sqrt{1+2\kappa(2+\kappa)\left(\frac{\lambda}{\lambda+1}\right)}} \right). \tag{A.7}$$

Similarly the force generated in the fiber was calculated using the stress-strain behavior given in equation (2.1.1):

$$F_{\text{fiber=}} \left(\frac{A_0}{\lambda} \right) p_1 \sigma_{\text{iso}} \left(e^{6.6(\lambda-1)} - 1 \right) + A_0 \alpha \sigma_{\text{iso}} \begin{cases} 9(\lambda-0.4)^2 & \lambda \le 0.6 \\ 1-4(1-\lambda)^2 & 0.6 < \lambda < 1.4 \\ 9(\lambda-1.6)^2 & \lambda \ge 1.4 \end{cases}, \tag{A.8}$$

where A_0 is the initial fiber cross-sectional area, and the following substitution was made for fiber area A, based on the assumption that muscle fibers are incompressible: $A = A_0/\lambda$.

Setting the right hand sides of equations (A.7) and (A.8) equal to each other produces the equilibrium equation (2.1.1.2).

As stated above, this analysis relies on the assumption that shear strain is constant through the thickness of the endomysium. This assumption results in an underestimation of shear force and thereby an underestimation of active fiber force particularly for fibers with lower optimal length and lower fiber volume fractions. This underestimation results from the fact that the shear strains would not be constant through the thickness, but larger at the fiber and smaller at the outside surface of the endomysium. We tested the effects of this assumption by comparison with FE models (Fig. 8).

Appendix B

The following is the derivation of the equilibrium equation for force transmission through tension in the endomysium (Fig. 2).

When the fiber is activated it shortens until its active force is balanced by the passive tension within the endomysium. We have used the following equation to describe the tensile behavior of endomysium:

$$\sigma_{\text{end}} = C^{\text{end}} \left(e^{6.6\left(\lambda^{\text{end}} - 1\right)} - 1 \right), \tag{B.1}$$

where C^{end} is the tensile modulus of endomysium. $\lambda^{\text{end}} = l^{\text{endomysium}} / L_0^{\text{endomysium}}$ and can be written as:

$$\lambda^{\text{end}} = \frac{1 - \lambda L_0 / L_0^{\text{fascicle}}}{1 - L_0 / L_0^{\text{fascicle}}}, \tag{B.2}$$

where L_0 is as before the resting length of the fiber and L_0^{fascicle} is the length of the fascicle assumed to remain constant throughout the deformation.

The equation of equilibrium was obtained from $\sigma_{\text{end}} = \sigma_{\text{fiber}}$:

$$\begin{aligned} & C^{\text{end}} \left(e^{6.6(1-\lambda)\left(\frac{L_0/L_0^{\text{fascicle}}}{1-L_0/L_0^{\text{fascicle}}}\right)} - 1 \right) \\ & = p_1 \sigma_{\text{iso}} \left(e^{6.6(\lambda-1)} - 1 \right) + \lambda \alpha \sigma_{\text{iso}} \begin{cases} 9\left(\lambda - 0.4\right)^2 & \lambda \le 0.6 \\ 1 - 4\left(1-\lambda\right)^2 & 0.6 < \lambda < 1.4 \end{cases} \end{aligned}, \tag{B.3}$$

where (B.2) has been substituted into (B.1).

Appendix C

In this appendix we derive the equilibrium equation for force transmission from an intrafascicularly terminating fiber within a muscle that undergoes length change. The force generated in and transmitted from the fiber was calculated once static equilibrium is reached. The fiber in its deformed configuration can still be described using the schematic in Fig. 1B. However, L_0, the outer length of the endomysium layer, is no longer equal to L_{rest}, the resting length of the fiber. L_0, is determined by the overall stretch of the surrounding muscle tissue, defined as λ_0, so that:

$$\frac{L_0}{L_{\text{rest}}} = \lambda_0 . \tag{C.1}$$

The final length of the fiber was obtained by determining the equilibrium between the active force generated in the fiber and the shear force within the endomysium that resists the shortening of the fiber, $F_{\text{end}} = F_{\text{fiber}}$. F_{end} can be calculated following the same logic as Equations (A.1-7) and is equal to:

$$F_{\text{end}} = 2\pi G_{\text{end}} L_{\text{rest}}^2 \lambda \left(\frac{\lambda_0 - \lambda}{-1 + \sqrt{1 - \left(\frac{\lambda}{\lambda_0 + \lambda} \right) 2\kappa (2+\kappa)}} \right). \tag{C.2}$$

The equation of equilibrium becomes:

$$\frac{2\left(\frac{G_{\text{end}}}{\sigma_{\text{iso}}}\right)\left(\frac{L_{\text{rest}}}{r_{\text{rest}}}\right)^2 \lambda(\lambda_0 - \lambda)}{-1 + \sqrt{1 - \left(\frac{\lambda}{\lambda_0 + \lambda} \right) 2\kappa (2+\kappa)}} = p_1 \left(\frac{e^{6.6(\lambda-1)} - 1}{\lambda} \right) + \alpha \begin{cases} 9(\lambda - 0.4)^2 & \lambda \le 0.6 \\ 1 - 4(1-\lambda)^2 & 0.6 < \lambda < 1.4 \\ 9(\lambda - 1.6)^2 & \lambda \ge 1.4 \end{cases}, \tag{C.3}$$

where r_{rest} is the fiber radius at resting length and $\kappa = h_{\text{rest}} / r_{\text{rest}}$ is the endomysium thickness to fiber radius at resting length.

We solved equation (C.3) for a fiber with an endomysium shear modulus of 5 Pa for three example cases of $\lambda_0 = 0.6$, $\lambda_0 = 1.2$ and $\lambda_0 = 1.6$ in order to compare with the force transmitted from the same fiber for $\lambda_0 = 1$ (where equation (C.3) becomes identical to (2.1.1.2)). We plotted the ratio of force transmitted from the terminating fiber to the force that the fiber would generate if it were held isometrically at a fixed length of $L_0 = \lambda_0 L_{\text{rest}}$, $F_{\text{iso}}\left(\lambda_0\right)$.

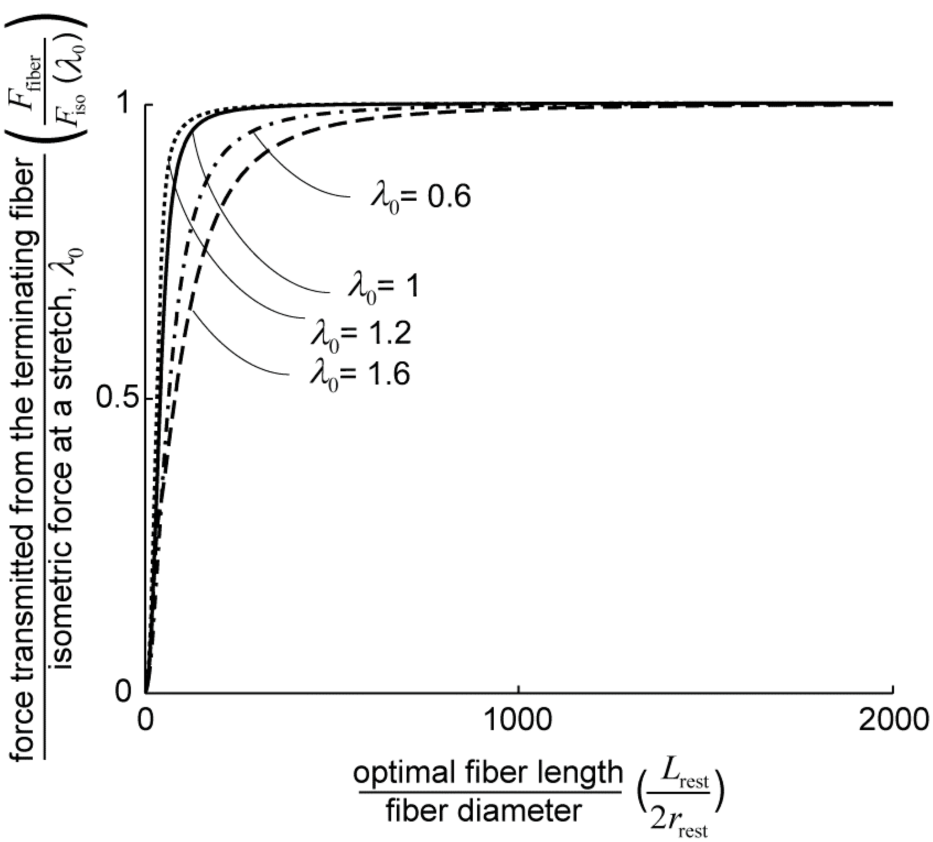

Figure C.1. The ratio of force transmitted from the terminating fiber to the isometric force that can be transmitted from a fiber of the same cross-sectional area, if it were held at a fixed length, L_0, plotted against the ratio of fiber resting length to fiber diameter at resting length. Fiber volume fraction, V_{f}, is 90% and endomysium shear modulus, G_{end}, is $5Pa$.