

A Mathematical Model for the Flow of Carbon Through the Aspartate, Threonine and the Branched Chain Amino Acid Pathways of *Escherichia coli* K12.

```
(* call Cellerator and kMech into Mathematica Kernel *)
```

```
<< cellerator.m;
```

```
<< kMech.m;
```

```
Cellerator™ 1.4.6 (30-July-2004) loaded 27-Aug-2005 17:01  
using Mathematica Version 5.0 for Microsoft Windows (June 11, 2003)
```

```
kMech is loaded
```

```
(* kMech Input of Enzyme Mechanisms for the Mathematical Modeling  
of Threonine (Thr), L-isoleucine (ILE), L-valine (VAL),  
and L-leucine (LEU) Biosynthesis *)
```

```
Asp$Thr$Synthesis = Union[
```

```
  { { {AKI}  $\xRightarrow{\text{ASA}}$  {HDHI},  
      {} , {} , {} } ,
```

```
    GMWC[cGMWC → cASA, LGMWC → LASA, nGMWC → nASA, KGMWC → {KmHDHIASA},  
      kcatGMWC → kcat$AKI$HDHI] } } ,
```

```
  { { {HDHI}  $\xRightarrow{\text{Asp}}$  {AKI},  
      {} , {} , {} } ,
```

```
    GMWC[cGMWC → cAsp, LGMWC → LAsp, nGMWC → nAsp, KGMWC → {KmAKIASp},  
      kcatGMWC → kcat$HDHI$AKI] } } ,
```

```
  { { {Asp}  $\xRightarrow{\text{AKI}}$  {AspP},  
      {} , {Thr} , {} } ,
```

```
    GMWC[cGMWC → cAKI, LGMWC → LAKI, nGMWC → nAKI, KGMWC → {KmAKIASp, KiAKIASpThr},
```

```

    kcatGMWC → kcat$AKI$Asp] }},
  Enz [ {Asp, ATP} ⇌ {AspP, ADP}, BiBi [kfAKIIAsp, krAKIIAsp, kcat$AKII$Asp] ],
  { { {Asp} ⇒ {AspP},
      { {}, {Lys}, {} }
    },
    GMWC [cGMWC → cAKIII, LGMWC → LAKIII, nGMWC → nAKIII,
      KGMWC → {KmAKIIIAsp, KiAKIIIAspLys}, kcatGMWC → kcat$AKIII$Asp] }},
  Enz [ {AspP, NADPH} ⇌ {ASA, NADP, Phosphate}, BiTri [kfASDAspP, krASDAspP, kcat$ASD$ASA],
  Enz [ {ASA, NADP, Phosphate} ⇌ {AspP, NADPH}, TriBi [kfASDASA, krASDASA, kcat$ASD$ASA],
  { { {ASA} ⇒ {Hse},
      { {}, {Thr}, {} }
    },
    GMWC [cGMWC → cHDHI, LGMWC → LHDHI, nGMWC → nHDHI, KGMWC → {KmHDHIASA, KiHDHIASA},
      kcatGMWC → kcat$HDHI$ASA] }},
  { {ASA → ∅, kfASA} },
  { {Hse → ∅, kfHse} },
  Enz [ {ASA, NADPH} ⇌ {Hse, NADP}, BiBi [kfHDHIIASA, krHDHIIASA, kcat$HDHII$ASA] ],
  Enz [ {Hse, ATP} ⇌ {HseP, ADP}, BiBi [kfHSKHse, krHSKHse, kcat$HSK$Hse],
    CI [Thr, kfiHSKHseThr, kriHSKHseThr] ],
  { { {HseP} ⇌ Thr, kftTSHseP, krTSHseP, kcat$TS$HseP} }
];

```

```

ILESynthesis = Union [

```

```

  { { {Thr} ⇒ {aKB, NH3},
      { {Val}, {Ile} }
    },

```

```

    GMWC [nGMWC → nTDA, cGMWC → cTDA, LGMWC → L0TDA, kcatGMWC → kcat$TDA,
      KGMWC → {KmThr, KaVal, KiIle} ] }},

```

```

{ {
    KDC
    aKB ⇌ propionylCoA, kfKDCaKB, krKDCaKB, kcat$KDC$aKB } },

Enz [ { AHASI,AHASICH3CO
Pyr, aKB } ⇌ { CO2, aAHB }, PingPong [kfAHASIPyr, krAHASIPyr, kcat$AHASI$Pyr,
    kfAHASIIaKB, krAHASIIaKB, kcat$AHASII$aKB],
    NCI [Val, kfIAHASIPyrVal, kriAHASIPyrVal, kfIAHASIIaKBVal, kriAHASIIaKBVal,
    residualRateAHASIIValaKB] ],

Enz [ { AHASII,AHASIICH3CO
Pyr, aKB } ⇌ { CO2, aAHB }, PingPong [kfAHASIIIPyr, krAHASIIIPyr, kcat$AHASII$Pyr,
    kfAHASIIIaKB, krAHASIIIaKB, kcat$AHASIII$aKB] ],

Enz [ { AHASIII,AHASIIIICH3CO
Pyr, aKB } ⇌ { CO2, aAHB }, PingPong [kfAHASIIIPyr, krAHASIIIPyr, kcat$AHASIII$Py
    kfAHASIIIIaKB, krAHASIIIIaKB, kcat$AHASIIII$aKB],
    NCI [Val, kfIAHASIIIPyrVal, kriAHASIIIPyrVal, kfIAHASIIIIaKBVal, kriAHASIIIIaKBVal
    residualRateAHASIIIIValaKB] ],

Enz [ { aAHB, NADPH } IR ⇌ { aDMV, NADP }, BiBi [kfIRaAHB, krIRaAHB, kcat$IR$aAHB] ],

{ { DAD
aDMV ⇌ aKMV, kfDADaDMV, krDADaDMV, kcat$DAD$aDMV } },

Enz [ { TB,TBNH2
Glu, aKMV } ⇌ { aKG, Ile }, PingPong [kfTBGlu, fkrTBGlu, fkcat$TB$Glu, fkfTBaKM
    fkrTBaKMV, fkcat$TB$aKMV] ],

Enz [ { TB,TBNH2
Ile, aKG } ⇌ { aKMV, Glu }, PingPong [rkfTBile, rkrTBile, rkcat$TB$Ile, rkfTBaKG,
    rkrTBaKG, rkcat$TB$aKG] ],

```

```

    { {LIVIexIle ⇌ Ile, kfLIVIIexIle, krLIVIIexIle, kcat$LIVII$exIle} },
    { {LIVIIexIle ⇌ Ile, kfLIVIIIexIle, krLIVIIIexIle, kcat$LIVIII$exIle} },

    { {aKMV → acetylCoA, kcat$aKMV} },

    { {Ile → protein, kcat$Ile} } ];

VALSynthesis = Union[
  Enz[AHASI,AHASIICH3CO{Pyr, Pyr} ⇌ {CO2, aAL}, PingPong[kfAHASIPyr, krAHASIPyr, kcat$AHASI$Pyr,
    kfAHASIPyr2, krAHASIPyr2, kcat$AHASI$Pyr2],
    NCI[Val, kfiAHASIPyrVal, kriAHASIPyrVal, kfiAHASIPyr2Val, kriAHASIPyr2Val,
    residualRateAHASIValPyr2] ],
  Enz[AHASII,AHASIICH3CO{Pyr, Pyr} ⇌ {CO2, aAL}, PingPong[kfAHASIIIPyr, krAHASIIIPyr, kcat$AHASII$Pyr,
    kfAHASIIIPyr2, krAHASIIIPyr2, kcat$AHASII$Pyr2] ],
  Enz[AHASIII,AHASIICH3CO{Pyr, Pyr} ⇌ {CO2, aAL}, PingPong[kfAHASIIIIPyr, krAHASIIIIPyr, kcat$AHASIII$Pyr,
    kfAHASIIIIPyr2, krAHASIIIIPyr2, kcat$AHASIII$Pyr2],
    NCI[Val, kfiAHASIIIIPyrVal, kriAHASIIIIPyrVal, kfiAHASIIIIPyr2Val, kriAHASIIIIPyr2V
    residualRateAHASIIIIValPyr2] ],
  Enz[IR{aAL, NADPH} ⇌ {ADHIV, NADP}, BiBi[kfIRaAL, krIRaAL, kcat$IR$aAL] ],
  { {DADadHIV ⇌ aKIV, kfdADaDHIV, krdADaDHIV, kcat$DAD$aDHIV} },

```

```

Enz [ {Ala, aKIV}  $\rightleftharpoons$  {Pyr, Val}, PingPong [fkfTCAla, fkrTCAla, fkcat$TC$Ala, fkfTCaKIV,
  fkrTCaKIV, fkcat$TC$aKIV] ],
Enz [ {Val, Pyr}  $\rightleftharpoons$  {aKIV, Ala}, PingPong [rkfTCVal, rkrTCVal, rkcat$TC$Val, rkfTCPyr,
  rkrTCPyr, rkcat$TC$Pyr] ],

Enz [ {Glu, aKIV}  $\rightleftharpoons$  {aKG, Val}, PingPong [fkfTBGlu, fkrTBGlu, fkcat$TB$Glu, fkfTBaKIV,
  fkrTBaKIV, fkcat$TB$aKIV] ],
Enz [ {Val, aKG}  $\rightleftharpoons$  {aKIV, Glu}, PingPong [rkfTBVal, rkrTBVal, rkcat$TB$Val, rkfTBaKG,
  rkrTBaKG, rkcat$TB$aKG] ],

{ {exVal  $\rightleftharpoons$  Val, kfLIVIexVal, krLIVIexVal, kcat$LIVI$exVal} },
{ {exVal  $\rightleftharpoons$  Val, kfLIVIIexVal, krLIVIIexVal, kcat$LIVII$exVal} },

{ {aKIV  $\rightarrow$  pantothenate, kcat$aKIV} },

{ {Val  $\rightarrow$  protein, kcat$Val} } ];

LEUSynthesis = Union [
  Enz [ {acetylCoA, aKIV}  $\rightleftharpoons$  {CoA, aIPM},
    PingPong [kfIPMSacetylCoA, krIPMSacetylCoA, kcat$IPMS$acetylCoA, kfIPMSaKIV,
      krIPMSaKIV, kcat$IPMS$aKIV], CI [Leu, kfiIPMSLeu, kriIPMSLeu],
    NCI [Leu, kfiIPMSacetylLeu, kriIPMSacetylLeu] ],

```

```

      IPMI
      { { aIPM ⇌ bIPM, kfIPMIaIPM, krIPMIaIPM, kcat$IPMI$aIPM } },
      { { bIPM ⇌ aIPM, kfIPMIbIPM, krIPMIbIPM, kcat$IPMI$bIPM } },

      IPMDH
      Enz [ { bIPM, NAD } ⇌ { aKIC, NADH }, BiBi [ kfIPMDHbIPM, krIPMDHbIPM, kcat$IPMDH$bIPM ] ],

      TB, TBNH2
      Enz [ { Glu, aKIC } ⇌ { aKG, Leu }, PingPong [ kfTBGlu, fkrTBGlu, fkcat$TB$Glu, fkfTBaKIC,
        fkrTBaKIC, fkcat$TB$aKIC ] ],

      TB, TBNH2
      Enz [ { Leu, aKG } ⇌ { aKIC, Glu }, PingPong [ rkfTBLeu, rkrTBLeu, rkcat$TB$Leu, rkfTBaKG,
        rkrTBaKG, rkcat$TB$aKG ] ],

      { { aKIC → glutarylCoA, kcat$aKIC } },

      LIVI
      { { exLeu ⇌ Leu, kfLIVIexLeu, krLIVIexLeu, kcat$LIVI$exLeu } },
      LIVII
      { { exLeu ⇌ Leu, kfLIVIIexLeu, krLIVIIexLeu, kcat$LIVII$exLeu } },
      LS
      { { exLeu ⇌ Leu, kfLSexLeu, krLSexLeu, kcat$LS$exLeu } },

      { { Leu → protein, kcat$Leu } } ];

THR$ILE$VAL$LEU$Synthesis = Union [ Asp$Thr$Synthesis, ILE$Synthesis, VAL$Synthesis,
  LEU$Synthesis ];

```

(* Cellerator-interpreted Ordinary Differential Equations (ODEs) and Equations for the MWC Model and Substrate Generators . *)

```
interpret[THR$ILE$VAL$LEU$Synthesis];
```

(*

Modifications for Substrate Generators:

The generated ODEs require some modifications to include the constant substrate fluxes for the steady state simulation

1. Substrate Generators:

Set the first derivatives of the following substrates to 0 to simulate constant flux,

```
Asp'[t]==0, ATP'[t]==0, NADP'[t]==0, NADPH'[t]==0, Phosphate'[t]==0,
Pyr'[t]==0, Glu'[t]==0, Ala'[t]==0, exIle'[t]==0, exVal'[t]==0,
exLeu'[t]==0, acetylCoA'[t]==0, NAD'[t]==0,
```

*)

```
{myODEs, myVars} =
```

```
{ {aAHB'[t] == -kfIRaAHB aAHB[t] IR[t] NADPH[t] + kcat$AHASI$aKB $Complex$aKB$AHASICH3CO
kcat$AHASI$aKB residualRateAHASIValaKB $Complex$aKB$AHASICH3CO$Val[t] +
kcat$AHASII$aKB $Complex$aKB$AHASIIICH3CO[t] +
kcat$AHASIII$aKB $Complex$aKB$AHASIIIICH3CO[t] +
kcat$AHASIII$aKB residualRateAHASIIIValaKB $Complex$aKB$AHASIIIICH3CO$Val[t] +
krIRaAHB $Complex$IR$aAHB$NADPH[t] ,
aAL'[t] == -kfIRaAL aAL[t] IR[t] NADPH[t] + krIRaAL $Complex$IR$aAL$NADPH[t] +
kcat$AHASI$Pyr2 $Complex$Pyr$AHASICH3CO[t] +
kcat$AHASI$Pyr2 residualRateAHASIVaIPyr2 $Complex$Pyr$AHASICH3CO$Val[t] +
kcat$AHASII$Pyr2 $Complex$Pyr$AHASIIICH3CO[t] +
```


$$\begin{aligned}
& \text{kcat}\$AHASIII\$Pyr2 \$Complex\$Pyr\$AHASIIIICH3CO\$[t] + \\
& \text{kcat}\$AHASIII\$Pyr2 \text{residualRateAHASIIIValPyr2 } \$Complex\$Pyr\$AHASIIIICH3CO\$Val\$[t] \\
& \text{acetylCoA}'[t] == 0, \\
& \text{aDHIV}'[t] == -\text{kfDADaDHIV aDHIV}[t] \text{DAD}[t] + \text{krDADaDHIV } \$Complex\$aDHIV\$DAD\$[t] + \\
& \quad \text{kcat}\$IR\$aAL \$Complex\$IR\$aAL\$NADPH\$[t], \\
& \text{aDMV}'[t] == -\text{kfDADaDMV aDMV}[t] \text{DAD}[t] + \text{krDADaDMV } \$Complex\$aDMV\$DAD\$[t] + \\
& \quad \text{kcat}\$IR\$aAHB \$Complex\$IR\$aAHB\$NADPH\$[t], \\
& \text{ADP}'[t] == \text{kcat}\$AKII\$Asp \$Complex\$AKII\$Asp\$ATP\$[t] + \text{kcat}\$HSK\$Hse \$Complex\$HSK\$Hse\$ \\
& \text{AHASI}'[t] == -\text{kfAHASIPyr AHASI}[t] \text{Pyr}[t] - \text{kfiAHASIPyrVal AHASI}[t] \text{Val}[t] + \\
& \quad \text{kriAHASIPyrVal } \$Complex\$AHASI\$Val\$[t] + \text{kcat}\$AHASI\$aKB \$Complex\$aKB\$AHASICH3CO\$ \\
& \quad \text{kcat}\$AHASI\$Pyr2 \$Complex\$Pyr\$AHASICH3CO\$[t] + \text{krAHASIPyr } \$Complex\$Pyr\$AHASI\$[t] \\
& \text{AHASICH3CO}'[t] == -\text{kfAHASIIaKB AHASICH3CO}[t] \text{aKB}[t] - \text{kfAHASIPyr2 AHASICH3CO}[t] \text{Pyr}[t] \\
& \quad \text{kfiAHASIIaKBVal AHASICH3CO}[t] \text{Val}[t] + \text{kriAHASIIaKBVal } \$Complex\$AHASICH3CO\$Val\$[t] \\
& \quad \text{krAHASIIaKB } \$Complex\$aKB\$AHASICH3CO\$[t] + \text{krAHASIPyr2 } \$Complex\$Pyr\$AHASICH3CO\$[t] \\
& \quad \text{kcat}\$AHASI\$Pyr \$Complex\$Pyr\$AHASI\$[t], \\
& \text{AHASII}'[t] == -\text{kfAHASIIIPyr AHASII}[t] \text{Pyr}[t] + \text{kcat}\$AHASII\$aKB \$Complex\$aKB\$AHASIIICH \\
& \quad \text{kcat}\$AHASII\$Pyr2 \$Complex\$Pyr\$AHASIIICH3CO\$[t] + \text{krAHASIIIPyr } \$Complex\$Pyr\$AHASII \\
& \text{AHASIIICH3CO}'[t] == -\text{kfAHASIIaKB AHASIIICH3CO}[t] \text{aKB}[t] - \text{kfAHASIIIPyr2 AHASIIICH3CO}[t] \\
& \quad \text{krAHASIIaKB } \$Complex\$aKB\$AHASIIICH3CO\$[t] + \text{krAHASIIIPyr2 } \$Complex\$Pyr\$AHASIIICH3C \\
& \quad \text{kcat}\$AHASII\$Pyr \$Complex\$Pyr\$AHASII\$[t], \\
& \text{AHASIII}'[t] == -\text{kfAHASIIIIPyr AHASIII}[t] \text{Pyr}[t] - \text{kfiAHASIIIIPyrVal AHASIII}[t] \text{Val}[t] \\
& \quad \text{kriAHASIIIIPyrVal } \$Complex\$AHASIII\$Val\$[t] + \\
& \quad \text{kcat}\$AHASIII\$aKB \$Complex\$aKB\$AHASIIIICH3CO\$[t] + \\
& \quad \text{kcat}\$AHASIII\$Pyr2 \$Complex\$Pyr\$AHASIIIICH3CO\$[t] + \text{krAHASIIIIPyr } \$Complex\$Pyr\$AH \\
& \text{AHASIIIICH3CO}'[t] == -\text{kfAHASIIIaKB AHASIIIICH3CO}[t] \text{aKB}[t] - \\
& \quad \text{kfAHASIIIIPyr2 AHASIIIICH3CO}[t] \text{Pyr}[t] - \text{kfiAHASIIIaKBVal AHASIIIICH3CO}[t] \text{Val}[t] + \\
& \quad \text{kriAHASIIIaKBVal } \$Complex\$AHASIIIICH3CO\$Val\$[t] + \\
& \quad \text{krAHASIIIaKB } \$Complex\$aKB\$AHASIIIICH3CO\$[t] + \\
& \quad \text{krAHASIIIIPyr2 } \$Complex\$Pyr\$AHASIIIICH3CO\$[t] + \text{kcat}\$AHASIII\$Pyr \$Complex\$Pyr\$AH
\end{aligned}$$

$$\begin{aligned}
& a_{IPM}'[t] = -k_{fIPMI} a_{IPM} a_{IPM}[t] IPMI[t] + k_{rIPMI} a_{IPM} \$Complex\$a_{IPM}\$IPMI\$[t] + \\
& \quad k_{cat}\$IPMS\$a_{KIV} \$Complex\$a_{KIV}\$IPMSacetyl\$[t] + k_{cat}\$IPMI\$b_{IPM} \$Complex\$b_{IPM}\$IPMI \\
& a_{KB}'[t] = -k_{fAHASII} a_{KB} AHASICH3CO[t] a_{KB}[t] - k_{fAHASIII} a_{KB} AHASIIICH3CO[t] a_{KB}[t] - \\
& \quad k_{fAHASIIII} a_{KB} AHASIIICH3CO[t] a_{KB}[t] - k_{fKDCa} a_{KB} a_{KB}[t] KDC[t] + \\
& \quad \left(k_{cat}\$TDA TDA[t] \left(\frac{c_{TDA} L_{OTDA} \left(1 + \frac{Ile[t]}{K_{iIle}}\right)^{n_{TDA}} Thr[t] \left(1 + \frac{c_{TDA} Thr[t]}{K_{mThr}}\right)^{-1+n_{TDA}}}{K_{mThr}} + \right. \right. \\
& \quad \left. \left. \frac{Thr[t] \left(1 + \frac{Thr[t]}{K_{mThr}}\right)^{-1+n_{TDA}} \left(1 + \frac{Val[t]}{K_{aVal}}\right)^{n_{TDA}}}{K_{mThr}} \right) \right) / \\
& \quad \left(L_{OTDA} \left(1 + \frac{Ile[t]}{K_{iIle}}\right)^{n_{TDA}} \left(1 + \frac{c_{TDA} Thr[t]}{K_{mThr}}\right)^{n_{TDA}} + \left(1 + \frac{Thr[t]}{K_{mThr}}\right)^{n_{TDA}} \left(1 + \frac{Val[t]}{K_{aVal}}\right)^{n_{TDA}} \right) \cdot \\
& \quad k_{fAHASII} a_{KB} a_{KB}[t] \$Complex\$AHASICH3CO\$Val\$[t] - \\
& \quad k_{fAHASIIII} a_{KB} a_{KB}[t] \$Complex\$AHASIIICH3CO\$Val\$[t] + \\
& \quad k_{rAHASII} a_{KB} \$Complex\$a_{KB}\$AHASICH3CO\$[t] + k_{rAHASIII} a_{KB} \$Complex\$a_{KB}\$AHASICH3CO\$Val \\
& \quad k_{rAHASIIII} a_{KB} \$Complex\$a_{KB}\$AHASIIICH3CO\$[t] + k_{rAHASIIII} a_{KB} \$Complex\$a_{KB}\$AHASIIICH: \\
& \quad k_{rAHASIIII} a_{KB} \$Complex\$a_{KB}\$AHASIIICH3CO\$Val\$[t] + k_{rKDCa} a_{KB} \$Complex\$a_{KB}\$KDC\$[t] , \\
& a_{KG}'[t] = -r_{kFTBa} a_{KG} a_{KG}[t] TBNH2[t] + r_{krTBa} a_{KG} \$Complex\$a_{KG}\$TBNH2\$[t] + \\
& \quad f_{kcat}\$TB\$Glu \$Complex\$Glu\$TB\$[t] , \\
& a_{KI}'[t] = \\
& \quad - \left(k_{cat}\$AKI\$HDHI \left(\frac{AKI[t] \left(1 + \frac{AKI[t]}{K_{mHDHIIASA}}\right)^{-1+n_{ASA}}}{K_{mHDHIIASA}} + \frac{c_{ASA} L_{ASA} AKI[t] \left(1 + \frac{c_{ASA} AKI[t]}{K_{mHDHIIASA}}\right)^{-1+n_{ASA}}}{K_{mHDHIIASA}} \right. \right. \\
& \quad \left. \left. ASA[t] \right) / \left(\left(1 + \frac{AKI[t]}{K_{mHDHIIASA}}\right)^{n_{ASA}} + L_{ASA} \left(1 + \frac{c_{ASA} AKI[t]}{K_{mHDHIIASA}}\right)^{n_{ASA}} \right) +
\end{aligned}$$

$$\left(\begin{array}{l} \text{kcat\$HDHI\$AKI Asp}[t] \\ \left(\frac{\text{HDHI}[t] \left(1 + \frac{\text{HDHI}[t]}{\text{KmAKIAsp}}\right)^{-1+n\text{Asp}}}{\text{KmAKIAsp}} + \frac{\text{cAsp LAsp HDHI}[t] \left(1 + \frac{\text{cAsp HDHI}[t]}{\text{KmAKIAsp}}\right)^{-1+n\text{Asp}}}{\text{KmAKIAsp}} \right) \end{array} \right) /$$

$$\left(\left(1 + \frac{\text{HDHI}[t]}{\text{KmAKIAsp}}\right)^{n\text{Asp}} + \text{LAsp} \left(1 + \frac{\text{cAsp HDHI}[t]}{\text{KmAKIAsp}}\right)^{n\text{Asp}} \right),$$

$$\begin{aligned} \text{aKIC}'[t] = & -\text{kcat\$aKIC aKIC}[t] - \text{fkfTBaKIC aKIC}[t] \text{ TBNH2}[t] + \\ & \text{fkrTBaKIC \$Complex\$aKIC\$TBNH2\$}[t] + \text{kcat\$IPMDH\$bIPM \$Complex\$IPMDH\$bIPM\$NAD\$}[t] \\ & \text{rkcat\$TB\$Leu \$Complex\$Leu\$TB\$}[t], \end{aligned}$$

$$\begin{aligned} \text{AKII}'[t] = & -\text{kfAKIIAsp AKII}[t] \text{ Asp}[t] \text{ ATP}[t] + \text{kcat\$AKII\$Asp \$Complex\$AKII\$Asp\$ATP\$} \\ & \text{krAKIIAsp \$Complex\$AKII\$Asp\$ATP\$}[t], \text{ AKIII}'[t] = 0, \end{aligned}$$

$$\begin{aligned} \text{aKIV}'[t] = & -\text{kcat\$aKIV aKIV}[t] - \text{kfIPMSaKIV aKIV}[t] \text{ IPMSacetyl}[t] - \\ & \text{fkfTBaKIV aKIV}[t] \text{ TBNH2}[t] - \text{fkfTCaKIV aKIV}[t] \text{ TCNH2}[t] + \\ & \text{kcat\$DAD\$aDHIV \$Complex\$aDHIV\$DAD\$}[t] + \text{krIPMSaKIV \$Complex\$aKIV\$IPMSacetyl\$}[t] \\ & \text{krIPMSaKIV \$Complex\$aKIV\$IPMSacetyl\$Leu\$}[t] + \text{fkrTBaKIV \$Complex\$aKIV\$TBNH2\$}[t] \\ & \text{fkrTCaKIV \$Complex\$aKIV\$TCNH2\$}[t] - \text{kfIPMSaKIV aKIV}[t] \text{ \$Complex\$IPMSacetyl\$Leu\$} \\ & \text{rkcat\$TB\$Val \$Complex\$Val\$TB\$}[t] + \text{rkcat\$TC\$Val \$Complex\$Val\$TC\$}[t], \end{aligned}$$

$$\begin{aligned} \text{aKMV}'[t] = & -\text{kcat\$aKMV aKMV}[t] - \text{fkfTBaKMV aKMV}[t] \text{ TBNH2}[t] + \\ & \text{kcat\$DAD\$aDMV \$Complex\$aDMV\$DAD\$}[t] + \text{fkrTBaKMV \$Complex\$aKMV\$TBNH2\$}[t] + \\ & \text{rkcat\$TB\$Ile \$Complex\$Ile\$TB\$}[t], \end{aligned}$$

$$\text{Ala}'[t] = 0,$$

$$\begin{aligned} \text{ASA}'[t] = & -\text{kfASA ASA}[t] - \text{kfHDHIIASA ASA}[t] \text{ HDHII}[t] \text{ NADPH}[t] - \\ & \text{kfASDASA ASA}[t] \text{ ASD}[t] \text{ NADP}[t] \text{ Phosphate}[t] - \end{aligned}$$

$$\left(\begin{array}{l} \text{kcat\$HDHI\$ASA HDHI}[t] \end{array} \right)$$

$$\left(\frac{\text{ASA}[t] \left(1 + \frac{\text{ASA}[t]}{\text{KmHDHIIASA}}\right)^{-1+n\text{HDHI}}}{\text{KmHDHIIASA}} + \frac{\text{cHDHI LHDHI ASA}[t] \left(1 + \frac{\text{cHDHI ASA}[t]}{\text{KmHDHIIASA}}\right)^{-1+n\text{HDHI}} \left(1 + \frac{\text{Thr}[t]}{\text{KiHDHIIASAThr}}\right)^{n\text{HDHI}}}{\text{KmHDHIIASA}} \right) /$$

$$\left(\left(1 + \frac{\text{ASA}[t]}{\text{KmHDHIIASA}}\right)^{n\text{HDHI}} + \text{LHDHI} \left(1 + \frac{\text{cHDHI ASA}[t]}{\text{KmHDHIIASA}}\right)^{n\text{HDHI}} \left(1 + \frac{\text{Thr}[t]}{\text{KiHDHIIASAThr}}\right)^{n\text{HDHI}} \right) +$$

$$\text{krASDASA \$Complex\$ASD\$ASA\$NADP\$Phosphate\$[t]} +$$

$$\text{kcat\$ASD\$AspP \$Complex\$ASD\$AspP\$NADPH\$[t]} + \text{krHDHIIASA \$Complex\$HDHII\$ASA\$NADP\$[t]}$$

$$\text{ASD}'[t] = -\text{kfASDAspP ASD}[t] \text{AspP}[t] \text{NADPH}[t] - \text{kfASDASA ASA}[t] \text{ASD}[t] \text{NADP}[t] \text{Phosph}$$

$$\text{kcat\$ASD\$ASA \$Complex\$ASD\$ASA\$NADP\$Phosphate\$[t]} +$$

$$\text{krASDASA \$Complex\$ASD\$ASA\$NADP\$Phosphate\$[t]} +$$

$$\text{kcat\$ASD\$AspP \$Complex\$ASD\$AspP\$NADPH\$[t]} + \text{krASDAspP \$Complex\$ASD\$AspP\$NADPH\$[t]}$$

$$\text{Asp}'[t] = 0,$$

$$\text{AspP}'[t] =$$

$$\left(\text{kcat\$AKIII\$Asp AKIII}[t] \right)$$

$$\left(\frac{\text{Asp}[t] \left(1 + \frac{\text{Asp}[t]}{\text{KmAKIIIAsp}}\right)^{-1+n\text{AKIII}}}{\text{KmAKIIIAsp}} + \frac{1}{\text{KmAKIIIAsp}} \right)$$

$$\left(\text{cAKIII LAKIII Asp}[t] \left(1 + \frac{\text{cAKIII Asp}[t]}{\text{KmAKIIIAsp}}\right)^{-1+n\text{AKIII}} \left(1 + \frac{\text{Lys}[t]}{\text{KiAKIIIAspLys}}\right)^{n\text{AKIII}} \right)$$

$$\left(\left(1 + \frac{\text{Asp}[t]}{\text{KmAKIIIAsp}}\right)^{n\text{AKIII}} + \text{LAKIII} \left(1 + \frac{\text{cAKIII Asp}[t]}{\text{KmAKIIIAsp}}\right)^{n\text{AKIII}} \left(1 + \frac{\text{Lys}[t]}{\text{KiAKIIIAspLys}}\right)^{n\text{AKIII}} \right)$$

$$\begin{aligned}
& kfASDAspP \text{ ASD}[t] \text{ AspP}[t] \text{ NADPH}[t] + \\
& \left(\begin{aligned} & kcat\$AKI\$Asp \text{ AKI}[t] \\ & \left(\frac{\text{Asp}[t] \left(1 + \frac{\text{Asp}[t]}{KmAKIAsp}\right)^{-1+nAKI}}{KmAKIAsp} + \right. \\ & \left. \frac{cAKI \text{ LAKI Asp}[t] \left(1 + \frac{cAKI \text{ Asp}[t]}{KmAKIAsp}\right)^{-1+nAKI} \left(1 + \frac{\text{Thr}[t]}{KiAKIAspThr}\right)^{nAKI}}{KmAKIAsp} \right) \right) / \\ & \left(\left(1 + \frac{\text{Asp}[t]}{KmAKIAsp}\right)^{nAKI} + \text{LAKI} \left(1 + \frac{cAKI \text{ Asp}[t]}{KmAKIAsp}\right)^{nAKI} \left(1 + \frac{\text{Thr}[t]}{KiAKIAspThr}\right)^{nAKI} \right) + \\ & kcat\$AKII\$Asp \$Complex\$AKII\$Asp\$ATP\$[t] + \\ & kcat\$ASD\$ASA \$Complex\$ASD\$ASA\$NADP\$Phosphate\$[t] + \\ & krASDAspP \$Complex\$ASD\$AspP\$NADPH\$[t], \\ \text{ATP}'[t] & = 0, \\ \text{bIPM}'[t] & = -kfIPMIbIPM \text{ bIPM}[t] \text{ IPMI}[t] - kfIPMDHbIPM \text{ bIPM}[t] \text{ IPMDH}[t] \text{ NAD}[t] + \\ & kcat\$IPMI\$aIPM \$Complex\$aIPM\$IPMI\$[t] + krIPMIbIPM \$Complex\$bIPM\$IPMI\$[t] + \\ & krIPMDHbIPM \$Complex\$IPMDH\$bIPM\$NAD\$[t], \\ \text{CO2}'[t] & = kcat\$AHASIII\$Pyr \$Complex\$Pyr\$AHASIII\$[t] + \\ & kcat\$AHASIII\$Pyr \text{ residualRateAHASIIIValaKB} \$Complex\$Pyr\$AHASIII\$Val\$[t] + \\ & kcat\$AHASII\$Pyr \$Complex\$Pyr\$AHASII\$[t] + kcat\$AHASI\$Pyr \$Complex\$Pyr\$AHASI\$[t] + \\ & kcat\$AHASI\$Pyr \text{ residualRateAHASIValaKB} \$Complex\$Pyr\$AHASI\$Val\$[t], \\ \text{CoA}'[t] & = kcat\$IPMS\$acetylCoA \$Complex\$acetylCoA\$IPMS\$[t], \\ \text{DAD}'[t] & = -kFDADaDHIV \text{ aDHIV}[t] \text{ DAD}[t] - kFDADaDMV \text{ aDMV}[t] \text{ DAD}[t] + \\ & kcat\$DAD\$aDHIV \$Complex\$aDHIV\$DAD\$[t] + krDADaDHIV \$Complex\$aDHIV\$DAD\$[t] + \\ & kcat\$DAD\$aDMV \$Complex\$aDMV\$DAD\$[t] + krDADaDMV \$Complex\$aDMV\$DAD\$[t], \\ \text{exIle}'[t] & = 0,
\end{aligned}
\end{aligned}$$

$$\text{exLeu}'[t] == 0,$$

$$\text{exVal}'[t] == 0,$$

$$\text{Glu}'[t] == 0,$$

$$\text{glutarylCoA}'[t] == \text{kcat}\$aKIC \text{aKIC}[t],$$

$$\text{HDHI}'[t] ==$$

$$\left(\text{kcat}\$AKI\$HDHI \left(\frac{\text{AKI}[t] \left(1 + \frac{\text{AKI}[t]}{\text{KmHDHIASA}} \right)^{-1+nASA}}{\text{KmHDHIASA}} + \frac{\text{cASA LASA AKI}[t] \left(1 + \frac{\text{cASA AKI}[t]}{\text{KmHDHIASA}} \right)^{-1+nASA}}{\text{KmHDHIASA}} \right. \right. \\ \left. \left. \text{ASA}[t] \right) / \left(\left(1 + \frac{\text{AKI}[t]}{\text{KmHDHIASA}} \right)^{nASA} + \text{LASA} \left(1 + \frac{\text{cASA AKI}[t]}{\text{KmHDHIASA}} \right)^{nASA} \right) -$$

$$\left(\text{kcat}\$HDHI\$AKI \text{Asp}[t]$$

$$\left(\frac{\text{HDHI}[t] \left(1 + \frac{\text{HDHI}[t]}{\text{KmAKIAsp}} \right)^{-1+nAsp}}{\text{KmAKIAsp}} + \frac{\text{cAsp LAsp HDHI}[t] \left(1 + \frac{\text{cAsp HDHI}[t]}{\text{KmAKIAsp}} \right)^{-1+nAsp}}{\text{KmAKIAsp}} \right) /$$

$$\left(\left(1 + \frac{\text{HDHI}[t]}{\text{KmAKIAsp}} \right)^{nAsp} + \text{LAsp} \left(1 + \frac{\text{cAsp HDHI}[t]}{\text{KmAKIAsp}} \right)^{nAsp} \right),$$

$$\text{HDHII}'[t] == -\text{kfHDHIIASA ASA}[t] \text{HDHII}[t] \text{NADPH}[t] +$$

$$\text{kcat}\$HDHII\$ASA \$Complex\$HDHII\$ASA\$NADPH\$[t] + \text{krHDHIIASA \$Complex\$HDHII\$ASA\$NAI}$$

$$\text{Hse}'[t] == -\text{kfHse Hse}[t] - \text{kfHSKHse ATP}[t] \text{Hse}[t] \text{HSK}[t] +$$

$$\left(\text{kcat}\$HDHI\$ASA \text{HDHI}[t]$$

$$\left(\frac{\text{ASA}[t] \left(1 + \frac{\text{ASA}[t]}{\text{KmHDHIIASA}}\right)^{-1+n\text{HDHI}}}{\text{KmHDHIIASA}} + \frac{\text{cHDHI LHDHI ASA}[t] \left(1 + \frac{\text{cHDHI ASA}[t]}{\text{KmHDHIIASA}}\right)^{-1+n\text{HDHI}} \left(1 + \frac{\text{Thr}[t]}{\text{KiHDHIIASAThr}}\right)^{n\text{HDHI}}}{\text{KmHDHIIASA}} \right) /$$

$$\left(\left(1 + \frac{\text{ASA}[t]}{\text{KmHDHIIASA}}\right)^{n\text{HDHI}} + \text{LHDHI} \left(1 + \frac{\text{cHDHI ASA}[t]}{\text{KmHDHIIASA}}\right)^{n\text{HDHI}} \left(1 + \frac{\text{Thr}[t]}{\text{KiHDHIIASAThr}}\right)^{n\text{HDHI}} \right) +$$

$$\text{kcat}\$HDHII\$ASA \$Complex\$HDHII\$ASA\$NADPH\$[t] + \text{kr}\$HSKHse \$Complex\$HSK\$Hse\$ATP\$[t]$$

$$\text{HseP}'[t] == -\text{kf}\$TSHseP \$Complex\$HseP\$TS\$[t] + \text{kr}\$TSHseP \$Complex\$HseP\$TS\$[t] +$$

$$\text{kcat}\$HSK\$Hse \$Complex\$HSK\$Hse\$ATP\$[t],$$

$$\text{HSK}'[t] == -\text{kf}\$HSKHse \$Complex\$HSK\$Hse\$ATP\$[t] + \text{kr}\$HSKHse \$Complex\$HSK\$Hse\$ATP\$[t] +$$

$$\text{kri}\$HSKHseThr \$Complex\$HSK\$Thr\$[t],$$

$$\text{Ile}'[t] == -\text{kcat}\$Ile \$Complex\$Ile\$TB\$[t] - \text{rkf}\$TB\$aKMV \$Complex\$aKMV\$TBN$$

$$\text{kcat}\$LIVII\$exIle \$Complex\$exIle\$LIVII\$[t] + \text{kcat}\$LIVI\$exIle \$Complex\$exIle\$LIVI$$

$$\text{rkr}\$TB\$Ile \$Complex\$Ile\$TB\$[t],$$

$$\text{IPMDH}'[t] == -\text{kf}\$IPMDH\$bIPM \$Complex\$IPMDH\$bIPM\$NAD\$[t] + \text{kr}\$IPMDH\$bIPM \$Complex\$IPMDH\$bIPM\$I$$

$$\text{IPMI}'[t] == -\text{kf}\$IPMI\$aIPM \$Complex\$aIPM\$IPMI\$[t] + \text{kr}\$IPMI\$aIPM \$Complex\$aIPM\$IPMI\$[t] +$$

$$\text{kcat}\$IPMI\$bIPM \$Complex\$bIPM\$IPMI\$[t] + \text{kr}\$IPMI\$bIPM \$Complex\$bIPM\$IPMI\$[t],$$

$$\text{IPMS}'[t] == -\text{kf}\$IPMS\$acetylCoA \$Complex\$acetylCoA\$IPMS\$[t] - \text{kfi}\$IPMS\$Leu \$Complex\$IPMS\$Leu \$[t] +$$

$$\text{kr}\$IPMS\$acetylCoA \$Complex\$acetylCoA\$IPMS\$[t] +$$

$$\text{kcat}\$IPMS\$aKIV \$Complex\$aKIV\$IPMS\$acetyl\$[t] + \text{kri}\$IPMS\$Leu \$Complex\$IPMS\$Leu \$[t],$$

$$\text{IPMSacetyl}'[t] == -\text{kf}\$IPMS\$aKIV \$Complex\$aKIV\$IPMS\$acetyl\$[t] - \text{kfi}\$IPMS\$acetyl\$Leu \$Complex\$IPMS\$acetyl\$[t]$$

$$\text{kcat}\$IPMS\$acetylCoA \$Complex\$acetylCoA\$IPMS\$[t] +$$

$$\text{kr}\$IPMS\$aKIV \$Complex\$aKIV\$IPMS\$acetyl\$[t] + \text{kri}\$IPMS\$acetyl\$Leu \$Complex\$IPMS\$acetyl\$[t]$$

$$\text{IR}'[t] == -\text{kf}\$IRaAHB \$Complex\$aAHB\$IR\$[t] + \text{kr}\$IRaAHB \$Complex\$aAHB\$IR\$[t] - \text{kf}\$IRaAL \$Complex\$aAL\$IR\$[t] + \text{kr}\$IRaAL \$Complex\$aAL\$IR\$[t]$$

$$\begin{aligned}
& kcat\$IR\$aAHB \$Complex\$IR\$aAHB\$NADPH\$[t] + krIRaAHB \$Complex\$IR\$aAHB\$NADPH\$[t] + \\
& kcat\$IR\$aAL \$Complex\$IR\$aAL\$NADPH\$[t] + krIRaAL \$Complex\$IR\$aAL\$NADPH\$[t] , \\
KDC'[t] = & -kfKDCaKB aKB[t] KDC[t] + kcat\KDCaKB \$Complex$aKBKDC[t] + \\
& krKDCaKB \$Complex$aKBKDC[t] , \\
Leu'[t] = & -kcat$Leu Leu[t] - kfiIPMSLeu IPMS[t] Leu[t] - \\
& kfiIPMSacetylLeu IPMSacetyl[t] Leu[t] - rkfTBLeu Leu[t] TB[t] + \\
& fkcatTBaKIC \$Complex$aKIC$TBNH2$[t] - \\
& kfiIPMSacetylLeu Leu[t] $Complex$aKIV$IPMSacetyl$[t] + \\
& kriIPMSacetylLeu $Complex$aKIV$IPMSacetyl$Leu$[t] + \\
& kcat$LIVII$exLeu $Complex$exLeu$LIVII$[t] + kcat$LIVI$exLeu $Complex$exLeu$LIVI$[t] + \\
& kcatLSexLeu $Complex$exLeuLS[t] + kriIPMSacetylLeu $Complex$IPMSacetylLeu[t] + \\
& kriIPMSLeu $Complex$IPMSLeu[t] + rkrTBLeu $Complex$LeuTB[t] , \\
LIVI'[t] = & -kfLIVIexIle exIle[t] LIVI[t] - kfLIVIexLeu exLeu[t] LIVI[t] - \\
& kflLIVIexVal exVal[t] LIVI[t] + kcat$LIVI$exIle $Complex$exIle$LIVI$[t] + \\
& krLIVIexIle $Complex$exIle$LIVI$[t] + kcat$LIVI$exLeu $Complex$exLeu$LIVI$[t] + \\
& krLIVIexLeu $Complex$exLeu$LIVI$[t] + kcat$LIVI$exVal $Complex$exVal$LIVI$[t] + \\
& krLIVIexVal $Complex$exVal$LIVI$[t] , \\
LIVII'[t] = & -kfLIVIIexIle exIle[t] LIVII[t] - kfLIVIIexLeu exLeu[t] LIVII[t] - \\
& kflLIVIIexVal exVal[t] LIVII[t] + kcat$LIVII$exIle $Complex$exIle$LIVII$[t] + \\
& krLIVIIexIle $Complex$exIle$LIVII$[t] + kcat$LIVII$exLeu $Complex$exLeu$LIVII$[t] + \\
& krLIVIIexLeu $Complex$exLeu$LIVII$[t] + kcat$LIVII$exVal $Complex$exVal$LIVII$[t] + \\
& krLIVIIexVal $Complex$exVal$LIVII$[t] , \\
LS'[t] = & -kfLSexLeu exLeu[t] LS[t] + kcatLSexLeu $Complex$exLeuLS[t] + \\
& krLSexLeu $Complex$exLeuLS[t] , \\
Lys'[t] = & 0 , \\
NAD'[t] = & 0 , \\
NADH'[t] = & kcat$IPMDH$bIPM $Complex$IPMDH$bIPM$NAD$[t] , \\
NADP'[t] = & 0 , \\
NADPH'[t] = & 0 ,
\end{aligned}$$

NH3' [t] ==

$$\left(\text{kcat}\$TDA\ TDA [t] \left(\frac{\text{cTDA}\ LOTDA \left(1 + \frac{\text{Ile}[t]}{\text{KiIle}} \right)^{nTDA} \text{Thr}[t] \left(1 + \frac{\text{cTDA}\ \text{Thr}[t]}{\text{KmThr}} \right)^{-1+nTDA}}{\text{KmThr}} + \frac{\text{Thr}[t] \left(1 + \frac{\text{Thr}[t]}{\text{KmThr}} \right)^{-1+nTDA} \left(1 + \frac{\text{Val}[t]}{\text{KaVal}} \right)^{nTDA}}{\text{KmThr}} \right) \right) /$$

$$\left(LOTDA \left(1 + \frac{\text{Ile}[t]}{\text{KiIle}} \right)^{nTDA} \left(1 + \frac{\text{cTDA}\ \text{Thr}[t]}{\text{KmThr}} \right)^{nTDA} + \left(1 + \frac{\text{Thr}[t]}{\text{KmThr}} \right)^{nTDA} \left(1 + \frac{\text{Val}[t]}{\text{KaVal}} \right)^{nTDA} \right),$$

pantothenate' [t] == kcat\$aKIV aKIV[t] ,

Phosphate' [t] == 0 ,

propionylCoA' [t] == kcat\$KDC\$aKB \$Complex\$aKB\$KDC\$[t] ,

protein' [t] == kcat\$Ile Ile[t] + kcat\$Leu Leu[t] + kcat\$Val Val[t] ,

Pyr' [t] == 0 ,

TB' [t] == -fkfTBGlu Glu[t] TB[t] - rkfTBile Ile[t] TB[t] - rkfTBLeu Leu[t] TB[t] -
 rkfTBVal TB[t] Val[t] + rkcat\$TB\$aKG \$Complex\$aKG\$TBNH2\$[t] +
 fkcat\$TB\$aKIC \$Complex\$aKIC\$TBNH2\$[t] + fkcat\$TB\$aKIV \$Complex\$aKIV\$TBNH2\$[t] +
 fkcat\$TB\$aKMV \$Complex\$aKMV\$TBNH2\$[t] + fkrTBGlu \$Complex\$Glu\$TB\$[t] +
 rkrTBile \$Complex\$Ile\$TB\$[t] + rkrTBLeu \$Complex\$Leu\$TB\$[t] +
 rkrTBVal \$Complex\$Val\$TB\$[t] ,

TBNH2' [t] == -rkfTBaKG aKG[t] TBNH2[t] - fkfTBaKIC aKIC[t] TBNH2[t] -
 fkfTBaKIV aKIV[t] TBNH2[t] - fkfTBaKMV aKMV[t] TBNH2[t] + rkrTBaKG \$Complex\$aKG\$T
 fkrTBaKIC \$Complex\$aKIC\$TBNH2\$[t] + fkrTBaKIV \$Complex\$aKIV\$TBNH2\$[t] +
 fkrTBaKMV \$Complex\$aKMV\$TBNH2\$[t] + fkcat\$TB\$Glu \$Complex\$Glu\$TB\$[t] +
 rkcat\$TB\$Ile \$Complex\$Ile\$TB\$[t] + rkcat\$TB\$Leu \$Complex\$Leu\$TB\$[t] +
 rkcat\$TB\$Val \$Complex\$Val\$TB\$[t] ,

TC' [t] == -fkfTCAla Ala[t] TC[t] - rkfTCVal TC[t] Val[t] +
 fkcat\$TC\$aKIV \$Complex\$aKIV\$TCNH2\$[t] + fkrTCAla \$Complex\$Ala\$TC\$[t] +
 rkcat\$TC\$Pyr \$Complex\$Pyr\$TCNH2\$[t] + rkrTCVal \$Complex\$Val\$TC\$[t] ,

$$\begin{aligned}
\text{TCNH2}'[t] &= -\text{fkfTCaKIV aKIV}[t] \text{TCNH2}[t] - \text{rkfTCPyr Pyr}[t] \text{TCNH2}[t] + \\
&\quad \text{fkrTCaKIV \$Complex$aKIV\$TCNH2\$}[t] + \text{fkcat\$TC\$Ala \$Complex\$Ala\$TC\$}[t] + \\
&\quad \text{rkrTCPyr \$Complex\$Pyr\$TCNH2\$}[t] + \text{rkcat\$TC\$Val \$Complex\$Val\$TC\$}[t], \text{TDA}'[t] = 0, \\
\text{Thr}'[t] &= -\text{kfiHSKHseThr HSK}[t] \text{Thr}[t] - \\
&\quad \left(\text{kcat\$TDA TDA}[t] \left(\frac{\text{cTDA LOTDA} \left(1 + \frac{\text{Ile}[t]}{\text{KiIle}}\right)^{\text{nTDA}} \text{Thr}[t] \left(1 + \frac{\text{cTDA Thr}[t]}{\text{KmThr}}\right)^{-1+\text{nTDA}}}{\text{KmThr}} + \right. \right. \\
&\quad \left. \left. \frac{\text{Thr}[t] \left(1 + \frac{\text{Thr}[t]}{\text{KmThr}}\right)^{-1+\text{nTDA}} \left(1 + \frac{\text{Val}[t]}{\text{KaVal}}\right)^{\text{nTDA}}}{\text{KmThr}} \right) \right) / \\
&\quad \left(\text{LOTDA} \left(1 + \frac{\text{Ile}[t]}{\text{KiIle}}\right)^{\text{nTDA}} \left(1 + \frac{\text{cTDA Thr}[t]}{\text{KmThr}}\right)^{\text{nTDA}} + \left(1 + \frac{\text{Thr}[t]}{\text{KmThr}}\right)^{\text{nTDA}} \left(1 + \frac{\text{Val}[t]}{\text{KaVal}}\right)^{\text{nTDA}} \right) \cdot \\
&\quad \text{kcat\$TS\$HseP \$Complex\$HseP\$TS\$}[t] + \text{kriHSKHseThr \$Complex\$HSK\$Thr\$}[t], \\
\text{TS}'[t] &= -\text{kfTSHseP HseP}[t] \text{TS}[t] + \text{kcat\$TS\$HseP \$Complex\$HseP\$TS\$}[t] + \\
&\quad \text{krTSHseP \$Complex\$HseP\$TS\$}[t], \\
\text{Val}'[t] &= -\text{kcat\$Val Val}[t] - \text{kfiAHASIPyrVal AHASI}[t] \text{Val}[t] - \\
&\quad \text{kfiAHASIAKBVal AHASICH3CO}[t] \text{Val}[t] - \text{kfiAHASIIIPyrVal AHASIII}[t] \text{Val}[t] - \\
&\quad \text{kfiAHASIIIaKBVal AHASIIICH3CO}[t] \text{Val}[t] - \text{rkfTBVal TB}[t] \text{Val}[t] - \text{rkfTCVal TC}[t] \text{V} \\
&\quad \text{kriAHASIAKBVal \$Complex\$AHASICH3CO\$Val\$}[t] + \\
&\quad \text{kriAHASIIIaKBVal \$Complex\$AHASIIICH3CO\$Val\$}[t] + \\
&\quad \text{kriAHASIIIPyrVal \$Complex\$AHASIII\$Val\$}[t] + \text{kriAHASIPyrVal \$Complex\$AHASI\$Val\$}[t] \\
&\quad \text{kfiAHASIAKBVal Val}[t] \$Complex$aKB\$AHASICH3CO\$}[t] + \\
&\quad \text{kriAHASIAKBVal \$Complex$aKB\$AHASICH3CO\$Val\$}[t] - \\
&\quad \text{kfiAHASIIIaKBVal Val}[t] \$Complex$aKB\$AHASIIICH3CO\$}[t] + \\
&\quad \text{kriAHASIIIaKBVal \$Complex$aKB\$AHASIIICH3CO\$Val\$}[t] + \\
&\quad \text{fkcat\TBaKIV \$Complex$aKIV\$TBNH2\$}[t] + \text{fkcat\TCaKIV \$Complex$aKIV\$TCNH2\$}[t] + \\
&\quad \text{kcat\$LIVII$exVal \$Complex$exVal\$LIVII\$}[t] + \text{kcat\$LIVI$exVal \$Complex$exVal\$LIVI\$}[t] \\
&\quad \text{kfiAHASIPyr2Val Val}[t] \$Complex\$Pyr\$AHASICH3CO\$}[t] + \\
&\quad \text{kriAHASIPyr2Val \$Complex\$Pyr\$AHASICH3CO\$Val\$}[t] -
\end{aligned}$$

$$\begin{aligned}
& \text{kfiAHASIIIPyr2Val Val [t] } \$\text{Complex}\$Pyr\$AHASIIICH3CO\$[t] + \\
& \text{kriAHASIIIPyr2Val } \$\text{Complex}\$Pyr\$AHASIIICH3CO\$Val\$[t] - \\
& \text{kfiAHASIIIPyrVal Val [t] } \$\text{Complex}\$Pyr\$AHASIII\$[t] + \\
& \text{kriAHASIIIPyrVal } \$\text{Complex}\$Pyr\$AHASIII\$Val\$[t] - \\
& \text{kfiAHASIPyrVal Val [t] } \$\text{Complex}\$Pyr\$AHASI\$[t] + \text{kriAHASIPyrVal } \$\text{Complex}\$Pyr\$AHAS \\
& \text{rkrTBVal } \$\text{Complex}\$Val\$TB\$[t] + \text{rkrTCVal } \$\text{Complex}\$Val\$TC\$[t] , \\
& \$\text{Complex}\$acetylCoA\$IPMS\$' [t] == \text{kfiIPMSacetylCoA acetylCoA [t] IPMS [t] -} \\
& \text{kcat\$IPMS\$acetylCoA } \$\text{Complex}\$acetylCoA\$IPMS\$[t] - \\
& \text{krIPMSacetylCoA } \$\text{Complex}\$acetylCoA\$IPMS\$[t] , \\
& \$\text{Complex}\$aDHIV\$DAD\$' [t] == \text{kfDADaDHIV aDHIV [t] DAD [t] -} \\
& \text{kcat\$DAD\$aDHIV } \$\text{Complex}\$aDHIV\$DAD\$[t] - \text{krDADaDHIV } \$\text{Complex}\$aDHIV\$DAD\$[t] , \\
& \$\text{Complex}\$aDMV\$DAD\$' [t] == \text{kfDADaDMV aDMV [t] DAD [t] - kcat\$DAD\$aDMV } \$\text{Complex}\$aDMV\$DA \\
& \text{krDADaDMV } \$\text{Complex}\$aDMV\$DAD\$[t] , \\
& \$\text{Complex}\$AHASICH3CO\$Val\$' [t] == \text{kfiAHASiAKBVal AHASICH3CO [t] Val [t] -} \\
& \text{kriAHASiAKBVal } \$\text{Complex}\$AHASICH3CO\$Val\$[t] - \\
& \text{kfAHASiAKB aKB [t] } \$\text{Complex}\$AHASICH3CO\$Val\$[t] - \\
& \text{kfAHASIPyr2 Pyr [t] } \$\text{Complex}\$AHASICH3CO\$Val\$[t] + \\
& \text{krAHASiAKB } \$\text{Complex}\$aKB\$AHASICH3CO\$Val\$[t] + \\
& \text{kcat\$AHASI\$aKB residualRateAHASiValaKB } \$\text{Complex}\$aKB\$AHASICH3CO\$Val\$[t] + \\
& \text{krAHASIPyr2 } \$\text{Complex}\$Pyr\$AHASICH3CO\$Val\$[t] + \\
& \text{kcat\$AHASI\$Pyr2 residualRateAHASiValPyr2 } \$\text{Complex}\$Pyr\$AHASICH3CO\$Val\$[t] , \\
& \$\text{Complex}\$AHASIIICH3CO\$Val\$' [t] == \\
& \text{kfiAHASIIIaKBVal AHASIIICH3CO [t] Val [t] - kriAHASIIIaKBVal } \$\text{Complex}\$AHASIIICH3CO \\
& \text{kfAHASIIIaKB aKB [t] } \$\text{Complex}\$AHASIIICH3CO\$Val\$[t] - \\
& \text{kfAHASIIIPyr2 Pyr [t] } \$\text{Complex}\$AHASIIICH3CO\$Val\$[t] + \\
& \text{krAHASIIIaKB } \$\text{Complex}\$aKB\$AHASIIICH3CO\$Val\$[t] + \\
& \text{kcat\$AHASIII\$aKB residualRateAHASIIIValaKB } \$\text{Complex}\$aKB\$AHASIIICH3CO\$Val\$[t] + \\
& \text{krAHASIIIPyr2 } \$\text{Complex}\$Pyr\$AHASIIICH3CO\$Val\$[t] + \\
& \text{kcat\$AHASIII\$Pyr2 residualRateAHASIIIValPyr2 } \$\text{Complex}\$Pyr\$AHASIIICH3CO\$Val\$[t] .
\end{aligned}$$

$$\begin{aligned}
& \$Complex\$AHASIII\$Val\$' [t] == kfiAHASIIIPyrVal AHASIII[t] Val[t] - \\
& \quad kriAHASIIIPyrVal \$Complex\$AHASIII\$Val\$[t] - kFAHASIIIPyr Pyr[t] \$Complex\$AHASII \\
& \quad krAHASIIIPyr \$Complex\$Pyr\$AHASIII\$Val\$[t] + \\
& \quad kcat\$AHASIII\$Pyr residualRateAHASIIIValaKB \$Complex\$Pyr\$AHASIII\$Val\$[t], \\
& \$Complex\$AHASI\$Val\$' [t] == kfiAHASIPyrVal AHASI[t] Val[t] - \\
& \quad kriAHASIPyrVal \$Complex\$AHASI\$Val\$[t] - kFAHASIPyr Pyr[t] \$Complex\$AHASI\$Val\$[t] \\
& \quad krAHASIPyr \$Complex\$Pyr\$AHASI\$Val\$[t] + \\
& \quad kcat\$AHASI\$Pyr residualRateAHASIValaKB \$Complex\$Pyr\$AHASI\$Val\$[t], \\
& \$Complex\$aIPM\$IPMI\$' [t] == kfiPMIaIPM aIPM[t] IPMI[t] - \\
& \quad kcat\$IPMI\$aIPM \$Complex\$aIPM\$IPMI\$[t] - krIPMIaIPM \$Complex\$aIPM\$IPMI\$[t], \\
& \$Complex\$aKB\$AHASICH3CO\$' [t] == kFAHASIaKB AHASICH3CO[t] aKB[t] - \\
& \quad kcat\$AHASI\$aKB \$Complex\$aKB\$AHASICH3CO\$[t] - krAHASIaKB \$Complex\$aKB\$AHASICH3CO \\
& \quad kfiAHASIaKBVal Val[t] \$Complex\$aKB\$AHASICH3CO\$[t] + \\
& \quad kriAHASIaKBVal \$Complex\$aKB\$AHASICH3CO\$Val\$[t], \\
& \$Complex\$aKB\$AHASICH3CO\$Val\$' [t] == \\
& \quad kFAHASIaKB aKB[t] \$Complex\$AHASICH3CO\$Val\$[t] + \\
& \quad kfiAHASIaKBVal Val[t] \$Complex\$aKB\$AHASICH3CO\$[t] - \\
& \quad krAHASIaKB \$Complex\$aKB\$AHASICH3CO\$Val\$[t] - \\
& \quad kriAHASIaKBVal \$Complex\$aKB\$AHASICH3CO\$Val\$[t] - \\
& \quad kcat\$AHASI\$aKB residualRateAHASIValaKB \$Complex\$aKB\$AHASICH3CO\$Val\$[t], \\
& \$Complex\$aKB\$AHASIIICH3CO\$' [t] == \\
& \quad kFAHASIIaKB AHASIIICH3CO[t] aKB[t] - kcat\$AHASII\$aKB \$Complex\$aKB\$AHASIIICH3CO\$[t] \\
& \quad krAHASIIaKB \$Complex\$aKB\$AHASIIICH3CO\$[t], \\
& \$Complex\$aKB\$AHASIIIICH3CO\$' [t] == \\
& \quad kFAHASIIIaKB AHASIIIICH3CO[t] aKB[t] - kcat\$AHASIII\$aKB \$Complex\$aKB\$AHASIIIICH3CO \\
& \quad krAHASIIIaKB \$Complex\$aKB\$AHASIIIICH3CO\$[t] - \\
& \quad kfiAHASIIIaKBVal Val[t] \$Complex\$aKB\$AHASIIIICH3CO\$[t] + \\
& \quad kriAHASIIIaKBVal \$Complex\$aKB\$AHASIIIICH3CO\$Val\$[t], \\
& \$Complex\$aKB\$AHASIIICH3CO\$Val\$' [t] ==
\end{aligned}$$

$$\begin{aligned}
& \text{kfAHASIIIaKB aKB}[t] \text{\$Complex\$AHASIIIICH3CO\$Val\$}[t] + \\
& \text{kfiAHASIIIaKBVal Val}[t] \text{\$Complex\$aKB\$AHASIIIICH3CO\$}[t] - \\
& \text{krAHASIIIaKB \$Complex\$aKB\$AHASIIIICH3CO\$Val\$}[t] - \\
& \text{kriAHASIIIaKBVal \$Complex\$aKB\$AHASIIIICH3CO\$Val\$}[t] - \\
& \text{kcat\$AHASIII\$aKB residualRateAHASIIIValaKB \$Complex\$aKB\$AHASIIIICH3CO\$Val\$}[t], \\
& \text{\$Complex\$aKB\$KDC\$}'[t] == \text{kfKDCaKB aKB}[t] \text{KDC}[t] - \text{kcat\$KDC\$aKB \$Complex\$aKB\$KDC\$}[t] \\
& \text{krKDCaKB \$Complex\$aKB\$KDC\$}[t], \\
& \text{\$Complex\$aKG\$TBNH2\$}'[t] == \text{rkftBaKG aKG}[t] \text{TBNH2}[t] - \text{rkcat\$TB\$aKG \$Complex\$aKG\$TBNH2\$}[t] \\
& \text{rkrTBaKG \$Complex\$aKG\$TBNH2\$}[t], \\
& \text{\$Complex\$aKIC\$TBNH2\$}'[t] == \text{fkftBaKIC aKIC}[t] \text{TBNH2}[t] - \\
& \text{fkcat\$TB\$aKIC \$Complex\$aKIC\$TBNH2\$}[t] - \text{fkrTBaKIC \$Complex\$aKIC\$TBNH2\$}[t], \\
& \text{\$Complex\$AKII\$Asp\$ATP\$}'[t] == \text{kfAKIIAsp AKII}[t] \text{Asp}[t] \text{ATP}[t] - \\
& \text{kcat\$AKII\$Asp \$Complex\$AKII\$Asp\$ATP\$}[t] - \text{krAKIIAsp \$Complex\$AKII\$Asp\$ATP\$}[t], \\
& \text{\$Complex\$aKIV\$IPMSacetyl\$}'[t] == \\
& \text{kfIPMSaKIV aKIV}[t] \text{IPMSacetyl}[t] - \text{kcat\$IPMS\$aKIV \$Complex\$aKIV\$IPMSacetyl\$}[t] - \\
& \text{krIPMSaKIV \$Complex\$aKIV\$IPMSacetyl\$}[t] - \\
& \text{kfiIPMSacetylLeu Leu}[t] \text{\$Complex\$aKIV\$IPMSacetyl\$}[t] + \\
& \text{kriIPMSacetylLeu \$Complex\$aKIV\$IPMSacetyl\$Leu\$}[t], \\
& \text{\$Complex\$aKIV\$IPMSacetyl\$Leu\$}'[t] == \\
& \text{kfiIPMSacetylLeu Leu}[t] \text{\$Complex\$aKIV\$IPMSacetyl\$}[t] - \\
& \text{kriIPMSacetylLeu \$Complex\$aKIV\$IPMSacetyl\$Leu\$}[t] - \\
& \text{krIPMSaKIV \$Complex\$aKIV\$IPMSacetyl\$Leu\$}[t] + \\
& \text{kfIPMSaKIV aKIV}[t] \text{\$Complex\$IPMSacetyl\$Leu\$}[t], \\
& \text{\$Complex\$aKIV\$TBNH2\$}'[t] == \text{fkftBaKIV aKIV}[t] \text{TBNH2}[t] - \\
& \text{fkcat\$TB\$aKIV \$Complex\$aKIV\$TBNH2\$}[t] - \text{fkrTBaKIV \$Complex\$aKIV\$TBNH2\$}[t], \\
& \text{\$Complex\$aKIV\$TCNH2\$}'[t] == \text{fkftCaKIV aKIV}[t] \text{TCNH2}[t] - \\
& \text{fkcat\$TC\$aKIV \$Complex\$aKIV\$TCNH2\$}[t] - \text{fkrTCaKIV \$Complex\$aKIV\$TCNH2\$}[t], \\
& \text{\$Complex\$aKMV\$TBNH2\$}'[t] == \text{fkftBaKMV aKMV}[t] \text{TBNH2}[t] - \\
& \text{fkcat\$TB\$aKMV \$Complex\$aKMV\$TBNH2\$}[t] - \text{fkrTBaKMV \$Complex\$aKMV\$TBNH2\$}[t],
\end{aligned}$$

$$\begin{aligned}
& \$Complex\$Ala\$TC\$' [t] == fkftCAla Ala [t] TC [t] - fkcat\$TC\$Ala \$Complex\$Ala\$TC\$ [t] - \\
& \quad fkrTCAla \$Complex\$Ala\$TC\$ [t] , \\
& \$Complex\$ASD\$ASA\$NADP\$Phosphate\$' [t] == \\
& \quad kfASDASA ASA [t] ASD [t] NADP [t] Phosphate [t] - \\
& \quad kcat\$ASD\$ASA \$Complex\$ASD\$ASA\$NADP\$Phosphate\$ [t] - \\
& \quad krASDASA \$Complex\$ASD\$ASA\$NADP\$Phosphate\$ [t] , \\
& \$Complex\$ASD\$AspP\$NADPH\$' [t] == kfASDAspP ASD [t] AspP [t] NADPH [t] - \\
& \quad kcat\$ASD\$AspP \$Complex\$ASD\$AspP\$NADPH\$ [t] - krASDAspP \$Complex\$ASD\$AspP\$NADPH\$ [t] , \\
& \$Complex\$bIPM\$IPMI\$' [t] == kfIPMIbIPM bIPM [t] IPMI [t] - \\
& \quad kcat\$IPMI\$bIPM \$Complex\$bIPM\$IPMI\$ [t] - krIPMIbIPM \$Complex\$bIPM\$IPMI\$ [t] , \\
& \$Complex\$exIle\$LIVII\$' [t] == kfLIVIIexIle exIle [t] LIVII [t] - \\
& \quad kcat\$LIVII\$exIle \$Complex\$exIle\$LIVII\$ [t] - krLIVIIexIle \$Complex\$exIle\$LIVII\$ [t] , \\
& \$Complex\$exIle\$LIVI\$' [t] == kfLIVIIexIle exIle [t] LIVI [t] - \\
& \quad kcat\$LIVII\$exIle \$Complex\$exIle\$LIVI\$ [t] - krLIVIIexIle \$Complex\$exIle\$LIVI\$ [t] , \\
& \$Complex\$exLeu\$LIVII\$' [t] == kfLIVIIexLeu exLeu [t] LIVII [t] - \\
& \quad kcat\$LIVII\$exLeu \$Complex\$exLeu\$LIVII\$ [t] - krLIVIIexLeu \$Complex\$exLeu\$LIVII\$ [t] , \\
& \$Complex\$exLeu\$LIVI\$' [t] == kfLIVIIexLeu exLeu [t] LIVI [t] - \\
& \quad kcat\$LIVII\$exLeu \$Complex\$exLeu\$LIVI\$ [t] - krLIVIIexLeu \$Complex\$exLeu\$LIVI\$ [t] , \\
& \$Complex\$exLeu\$LS\$' [t] == kfLSexLeu exLeu [t] LS [t] - kcat\$LS\$exLeu \$Complex\$exLeu\$LS\$ [t] - \\
& \quad krLSexLeu \$Complex\$exLeu\$LS\$ [t] , \\
& \$Complex\$exVal\$LIVII\$' [t] == kfLIVIIexVal exVal [t] LIVII [t] - \\
& \quad kcat\$LIVII\$exVal \$Complex\$exVal\$LIVII\$ [t] - krLIVIIexVal \$Complex\$exVal\$LIVII\$ [t] , \\
& \$Complex\$exVal\$LIVI\$' [t] == kfLIVIIexVal exVal [t] LIVI [t] - \\
& \quad kcat\$LIVII\$exVal \$Complex\$exVal\$LIVI\$ [t] - krLIVIIexVal \$Complex\$exVal\$LIVI\$ [t] , \\
& \$Complex\$Glu\$TB\$' [t] == fkftBGlu Glu [t] TB [t] - fkcat\$TB\$Glu \$Complex\$Glu\$TB\$ [t] - \\
& \quad fkrTBGlu \$Complex\$Glu\$TB\$ [t] , \\
& \$Complex\$HDHII\$ASA\$NADPH\$' [t] == kfHDHIIASA ASA [t] HDHII [t] NADPH [t] - \\
& \quad kcat\$HDHII\$ASA \$Complex\$HDHII\$ASA\$NADPH\$ [t] - krHDHIIASA \$Complex\$HDHII\$ASA\$NADPH\$ [t] , \\
& \$Complex\$HseP\$TS\$' [t] == kftSHseP HseP [t] TS [t] - kcat\$TS\$HseP \$Complex\$HseP\$TS\$ [t] - \\
& \quad krSHseP \$Complex\$HseP\$TS\$ [t] ,
\end{aligned}$$

```

krTSHseP $Complex$HseP$TS$[t] ,
$Complex$HSK$Hse$ATP$[t] == kfHSKHse ATP[t] Hse[t] HSK[t] -
kcat$HSK$Hse $Complex$HSK$Hse$ATP$[t] - krHSKHse $Complex$HSK$Hse$ATP$[t] ,
$Complex$HSK$Thr$[t] == kfiHSKHseThr HSK[t] Thr[t] - kriHSKHseThr $Complex$HSK$Thr
$Complex$Ile$TB$[t] == rkfTBile Ile[t] TB[t] - rkcat$TB$Ile $Complex$Ile$TB$[t] -
rkrTBile $Complex$Ile$TB$[t] ,
$Complex$IPMDH$bIPM$NAD$[t] == kfIPMDHbIPM bIPM[t] IPMDH[t] NAD[t] -
kcat$IPMDH$bIPM $Complex$IPMDH$bIPM$NAD$[t] - krIPMDHbIPM $Complex$IPMDH$bIPM$
$Complex$IPMSacetyl$Leu$[t] == kfiIPMSacetylLeu IPMSacetyl[t] Leu[t] +
krIPMSaKIV $Complex$aKIV$IPMSacetyl$Leu$[t] -
kriIPMSacetylLeu $Complex$IPMSacetyl$Leu$[t] -
kfIPMSaKIV aKIV[t] $Complex$IPMSacetyl$Leu$[t] ,
$Complex$IPMS$Leu$[t] == kfiIPMSLeu IPMS[t] Leu[t] - kriIPMSLeu $Complex$IPMS$Leu$
$Complex$IR$aAHB$NADPH$[t] == kfIRaAHB aAHB[t] IR[t] NADPH[t] -
kcat$IR$aAHB $Complex$IR$aAHB$NADPH$[t] - krIRaAHB $Complex$IR$aAHB$NADPH$[t] ,
$Complex$IR$aAL$NADPH$[t] == kfIRaAL aAL[t] IR[t] NADPH[t] -
kcat$IR$aAL $Complex$IR$aAL$NADPH$[t] - krIRaAL $Complex$IR$aAL$NADPH$[t] ,
$Complex$Leu$TB$[t] == rkfTBLeu Leu[t] TB[t] - rkcat$TB$Leu $Complex$Leu$TB$[t] -
rkrTBLeu $Complex$Leu$TB$[t] ,
$Complex$Pyr$AHASICH3CO$[t] == kFAHASIPyr2 AHASICH3CO[t] Pyr[t] -
kcat$AHASI$Pyr2 $Complex$Pyr$AHASICH3CO$[t] - krAHASIPyr2 $Complex$Pyr$AHASICH3
kfiAHASIPyr2Val Val[t] $Complex$Pyr$AHASICH3CO$[t] +
kriAHASIPyr2Val $Complex$Pyr$AHASICH3CO$Val$[t] ,
$Complex$Pyr$AHASICH3CO$Val$[t] ==
kFAHASIPyr2 Pyr[t] $Complex$AHASICH3CO$Val$[t] +
kfiAHASIPyr2Val Val[t] $Complex$Pyr$AHASICH3CO$[t] -
krAHASIPyr2 $Complex$Pyr$AHASICH3CO$Val$[t] -
kriAHASIPyr2Val $Complex$Pyr$AHASICH3CO$Val$[t] -
kcat$AHASI$Pyr2 residualRateAHASIValPyr2 $Complex$Pyr$AHASICH3CO$Val$[t] ,

```

$$\begin{aligned}
& \$Complex\$Pyr\$AHASIIICH3CO\$' [t] == kFAHASIIPyr2 AHASIIICH3CO[t] Pyr [t] - \\
& \quad kcat\$AHASII\$Pyr2 \$Complex\$Pyr\$AHASIIICH3CO\$[t] - \\
& \quad krAHASIIPyr2 \$Complex\$Pyr\$AHASIIICH3CO\$[t] , \\
& \$Complex\$Pyr\$AHASIIIICH3CO\$' [t] == \\
& \quad kFAHASIIIPyr2 AHASIIIICH3CO[t] Pyr [t] - kcat\$AHASIII\$Pyr2 \$Complex\$Pyr\$AHASIIIICH3 \\
& \quad \quad krAHASIIIPyr2 \$Complex\$Pyr\$AHASIIIICH3CO\$[t] - \\
& \quad \quad kfIAHASIIIPyr2Val Val [t] \$Complex\$Pyr\$AHASIIIICH3CO\$[t] + \\
& \quad \quad kriAHASIIIPyr2Val \$Complex\$Pyr\$AHASIIIICH3CO\$Val\$[t] , \\
& \$Complex\$Pyr\$AHASIIIICH3CO\$Val\$' [t] == \\
& \quad kFAHASIIIPyr2 Pyr [t] \$Complex\$AHASIIIICH3CO\$Val\$[t] + \\
& \quad kfIAHASIIIPyr2Val Val [t] \$Complex\$Pyr\$AHASIIIICH3CO\$[t] - \\
& \quad krAHASIIIPyr2 \$Complex\$Pyr\$AHASIIIICH3CO\$Val\$[t] - \\
& \quad kriAHASIIIPyr2Val \$Complex\$Pyr\$AHASIIIICH3CO\$Val\$[t] - \\
& \quad kcat\$AHASIII\$Pyr2 residualRateAHASIIIVAlPyr2 \$Complex\$Pyr\$AHASIIIICH3CO\$Val\$[t] , \\
& \$Complex\$Pyr\$AHASIII\$' [t] == kFAHASIIIPyr AHASIII[t] Pyr [t] - \\
& \quad kcat\$AHASIII\$Pyr \$Complex\$Pyr\$AHASIII\$[t] - krAHASIIIPyr \$Complex\$Pyr\$AHASIII\$[\\
& \quad kfIAHASIIIPyrVal Val [t] \$Complex\$Pyr\$AHASIII\$[t] + \\
& \quad kriAHASIIIPyrVal \$Complex\$Pyr\$AHASIII\$Val\$[t] , \\
& \$Complex\$Pyr\$AHASIII\$Val\$' [t] == kFAHASIIIPyr Pyr [t] \$Complex\$AHASIII\$Val\$[t] + \\
& \quad kfIAHASIIIPyrVal Val [t] \$Complex\$Pyr\$AHASIII\$[t] - \\
& \quad krAHASIIIPyr \$Complex\$Pyr\$AHASIII\$Val\$[t] - \\
& \quad kriAHASIIIPyrVal \$Complex\$Pyr\$AHASIII\$Val\$[t] - \\
& \quad kcat\$AHASIII\$Pyr residualRateAHASIIIVAlaKB \$Complex\$Pyr\$AHASIII\$Val\$[t] , \\
& \$Complex\$Pyr\$AHASII\$' [t] == kFAHASIIPyr AHASII[t] Pyr [t] - \\
& \quad kcat\$AHASII\$Pyr \$Complex\$Pyr\$AHASII\$[t] - krAHASIIPyr \$Complex\$Pyr\$AHASII\$[t] , \\
& \$Complex\$Pyr\$AHASI\$' [t] == kFAHASIPyr AHASI[t] Pyr [t] - \\
& \quad kcat\$AHASI\$Pyr \$Complex\$Pyr\$AHASI\$[t] - krAHASIPyr \$Complex\$Pyr\$AHASI\$[t] - \\
& \quad kfIAHASIPyrVal Val [t] \$Complex\$Pyr\$AHASI\$[t] + kriAHASIPyrVal \$Complex\$Pyr\$AHAS \\
& \$Complex\$Pyr\$AHASI\$Val\$' [t] == kFAHASIPyr Pyr [t] \$Complex\$AHASI\$Val\$[t] +
\end{aligned}$$


```

kfiAHASIPyrVal Val[t] $Complex$Pyr$AHASI$[t] - kraHASIPyr $Complex$Pyr$AHASI$Va
kriAHASIPyrVal $Complex$Pyr$AHASI$Val$[t] -
kcat$AHASI$Pyr residualRateAHASIValaKB $Complex$Pyr$AHASI$Val$[t] ,
$Complex$Pyr$TCNH2$'[t] == rkftCYPyr Pyr[t] TCNH2[t] - rkcat$TC$Pyr $Complex$Pyr$TCN
rkrTCYPyr $Complex$Pyr$TCNH2$[t] ,
$Complex$Val$TB$'[t] == rkftBVal TB[t] Val[t] - rkcat$TB$Val $Complex$Val$TB$[t] -
rkrTBVal $Complex$Val$TB$[t] ,
$Complex$Val$TC$'[t] == rkftCVal TC[t] Val[t] - rkcat$TC$Val $Complex$Val$TC$[t] -
rkrTCVal $Complex$Val$TC$[t] } ,
{aAHB, aAL, acetylCoA, aDHIV, aDMV, ADP, AHASI, AHASICH3CO, AHASII, AHASIICH3CO,
AHASIII, AHASIIIICH3CO, aIPM, aKB, aKG, AKI, aKIC, AKII, AKIII, aKIV, aKMV, Ala,
ASA, ASD, Asp, AspP, ATP, bIPM, CO2, CoA, DAD, exIle, exLeu, exVal, Glu, glutarylCo
HDHI, HDHII, Hse, HseP, HSK, Ile, IPMDH, IPMI, IPMS, IPMSacetyl, IR, KDC, Leu,
LIVI, LIVII, LS, Lys, NAD, NADH, NADP, NADPH, NH3, pantothenate, Phosphate,
propionylCoA, protein, Pyr, TB, TBNH2, TC, TCNH2, TDA, Thr, TS, Val,
$Complex$aCetylCoA$IPMS$, $Complex$aDHIV$DAD$, $Complex$aDMV$DAD$,
$Complex$AHASICH3CO$Val$, $Complex$AHASIIICH3CO$Val$, $Complex$AHASIII$Val$,
$Complex$AHASI$Val$, $Complex$aIPM$IPMI$, $Complex$aKB$AHASICH3CO$,
$Complex$aKB$AHASICH3CO$Val$, $Complex$aKB$AHASIIICH3CO$, $Complex$aKB$AHASIIICH
$Complex$aKB$AHASIIICH3CO$Val$, $Complex$aKB$KDC$, $Complex$aKG$TBNH2$,
$Complex$aKIC$TBNH2$, $Complex$AKII$Asp$ATP$, $Complex$aKIV$IPMSacetyl$,
$Complex$aKIV$IPMSacetyl$Leu$, $Complex$aKIV$TBNH2$, $Complex$aKIV$TCNH2$,
$Complex$aKMV$TBNH2$, $Complex$Ala$TC$, $Complex$ASD$ASA$NADP$Phosphate$,
$Complex$ASD$AspP$NADPH$, $Complex$bIPM$IPMI$, $Complex$exIle$LIVIII$,
$Complex$exIle$LIVI$, $Complex$exLeu$LIVII$, $Complex$exLeu$LIVI$, $Complex$exLe
$Complex$exVal$LIVII$, $Complex$exVal$LIVI$, $Complex$Glu$TB$, $Complex$HDHII$A
$Complex$HseP$TS$, $Complex$HSK$Hse$ATP$, $Complex$HSK$Thr$, $Complex$Ile$TB$,
$Complex$IPMDH$bIPM$NAD$, $Complex$IPMSacetyl$Leu$, $Complex$IPMS$Leu$,
$Complex$IR$aAHB$NADPH$, $Complex$IR$aAL$NADPH$, $Complex$Leu$TB$,

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```

$Complex$Pyr$AHASICH3CO$, $Complex$Pyr$AHASICH3CO$Val$, $Complex$Pyr$AHASIIICH3C
$Complex$Pyr$AHASIIICH3CO$, $Complex$Pyr$AHASIIICH3CO$Val$, $Complex$Pyr$AHASII
$Complex$Pyr$AHASIII$Val$, $Complex$Pyr$AHASII$, $Complex$Pyr$AHASI$,
$Complex$Pyr$AHASI$Val$, $Complex$Pyr$TCNH2$, $Complex$Val$TB$, $Complex$Val$TC:

```

```

(*
Inputs for Values of  $K_m$  ,  $K_i$  and  $k_{cat}$  for each Enzyme.
Lambda Approximation Functions: Kf[], Kf2S[] for  $k_f$ ,
                                Kr[] for  $k_r$ 
Omega Approximation Functions: Kfi[] for  $k_{fi}$ ,
                                Kri[] for  $k_{ri}$ .
*)

```

```
Lambda = 100;
```

```
Omega = 1;
```

```
myKs = {
```

```
  cASA = 1;
```

```
  LASA = 1;
```

```
  nASA = 4;
```

```
  KmHDHIASA = 80;
```

```
  kcat$AKI$HDHI = 10000;
```

```
  cAsp = 1;
```

```
  LAsp = 1;
```

```
  nAsp = 4;
```

```
  KmAKIASp = 220;
```

```
  kcat$HDHI$AKI = 10000;
```

```
cAKI = 0.0001;  
LAKI = 10000;  
nAKI = 4;  
KmAKIAsp = 220;  
KiAKIAspThr = 135;  
kcat$AKI$Asp = 100;
```

```
cAKIII = 0.00005;  
LAKIII = 1000;  
nAKIII = 2;  
KmAKIIIAsp = 30;  
KiAKIIIAspLys = 15;  
kcat$AKIII$Asp = 20;
```

```
cHDHI = 0.008;  
LHDHI = 3000;  
nHDHI = 4;  
KmHDHIASA = 80;  
KiHDHIASAThr = 120;  
kcat$HDHI$ASA = 800;
```

```
kfHDHIAKI = 1000;  
kfAKIHDHI = 1000;
```

```
KmAKIIAsp = 100; kcat$AKII$Asp = 100;  
kfAKIIAsp → Kf [KmAKIIAsp, kcat$AKII$Asp, Lambda] ,  
krAKIIAsp → Kr [kcat$AKII$Asp, Lambda] ,
```

```
kmASDAspP = 22; kcat$ASD$AspP = 40;
```

```
kfASDAspP → Kf [kmASDAspP, kcat$ASD$AspP, Lambda] ,
krASDAspP → Kr [kcat$ASD$AspP, Lambda] ,

KmASDASA = 110; KmASDNADP = 140; KmASDPhosphate = 10200; kcat$ASD$ASA = 80;
kfASDASA → Kf3S [KmASDASA, KmASDNADP, KmASDPhosphate, kcat$ASD$ASA, Lambda] ,
krASDASA → Kr [kcat$ASD$ASA, Lambda] ,

KmHDHIIASA = 100; kcat$HDHII$ASA = 100;
kfHDHIIASA → Kf [KmHDHIIASA, kcat$HDHII$ASA, Lambda] ,
krHDHIIASA → Kr [kcat$HDHII$ASA, Lambda] ,

KmHSKHse = 110; kcat$HSK$Hse = 60;
kfHSKHse → Kf [KmHSKHse, kcat$HSK$Hse, Lambda] ,
krHSKHse → Kr [kcat$HSK$Hse, Lambda] ,

KiHSKHseThr = 1000;
kfiHSKHseThr → Kfi [KmHSKHse, kcat$HSK$Hse, Lambda, Omega] ,
kriHSKHseThr → Kri [KmHSKHse, kcat$HSK$Hse, Lambda, Omega, KiHSKHseThr] ,

kfASA = 0.05;
kfHse = 0.05;

KmTSHseP = 310; kcat$TS$HseP = 60;
kftSHseP → Kf [KmTSHseP, kcat$TS$HseP, Lambda] ,
krTSHseP → Kr [kcat$TS$HseP, Lambda] ,

nTDA → 4,
cTDA → 0.013,
LOTDA → 1.05,
```

```
kcat$TDA → 6000,  
KmThr → 2700,  
KaVal → 550,  
KiIle → 15,                (* TDA feedback resistant mutant, KiIle=100000 *)  
  
KmKDCaKB = 1000; kcat$KDC$aKB = 3000;  
kfKDCaKB → Kf [KmKDCaKB, kcat$KDC$aKB, Lambda] ,  
krKDCaKB → Kr [kcat$KDC$aKB, Lambda] ,  
  
KmAHASIPyr = 10; kcat$AHASI$Pyr = 7000;  
kfAHASIPyr → Kf [KmAHASIPyr, kcat$AHASI$Pyr, Lambda] ,  
krAHASIPyr → Kr [kcat$AHASI$Pyr, Lambda] ,  
  
KmAHASIIaKB = 5000; kcat$AHASII$aKB = 7000;  
kfAHASIIaKB → Kf [KmAHASIIaKB, kcat$AHASII$aKB, Lambda] ,  
krAHASIIaKB → Kr [kcat$AHASII$aKB, Lambda] ,  
residualRateAHASIIaKB → 0,  
  
KiaHASIVal = 200;  
kfiAHASIPyrVal → Kfi [KmAHASIPyr, kcat$AHASI$Pyr, Lambda, Omega] ,  
kriAHASIPyrVal → Kri [KmAHASIPyr, kcat$AHASI$Pyr, Lambda, Omega, KiaHASIVal] ,  
kfiAHASIIaKBVal → Kfi [KmAHASIIaKB, kcat$AHASII$aKB, Lambda, Omega] ,  
kriAHASIIaKBVal → Kri [KmAHASIIaKB, kcat$AHASII$aKB, Lambda, Omega, KiaHASIVal] ,  
  
KmAHASIIIPyr = 10; kcat$AHASIIIPyr = 7000;  
kfAHASIIIPyr → Kf [KmAHASIIIPyr, kcat$AHASIIIPyr, Lambda] ,  
krAHASIIIPyr → Kr [kcat$AHASIIIPyr, Lambda] ,  
  
KmAHASIIaKB = 150; kcat$AHASII$aKB = 7000;
```

```
kfAHASIIaKB → Kf [KmAHASIIaKB, kcat$AHASII$aKB, Lambda] ,  
krAHASIIaKB → Kr [kcat$AHASII$aKB, Lambda] ,
```

```
KmAHASIIIIPyr = 10; kcat$AHASIII$Pyr = 7000;  
kfAHASIIIIPyr → Kf [KmAHASIIIIPyr, kcat$AHASIII$Pyr, Lambda] ,  
krAHASIIIIPyr → Kr [kcat$AHASIII$Pyr, Lambda] ,
```

```
KmAHASIIIaKB = 150; kcat$AHASIII$aKB = 7000;  
kfAHASIIIaKB → Kf [KmAHASIIIaKB, kcat$AHASIII$aKB, Lambda] ,  
krAHASIIIaKB → Kr [kcat$AHASIII$aKB, Lambda] ,  
residualRateAHASIIIValaKB → 0.15,
```

```
KiAHASIIIVal = 20;  
kfiAHASIIIIPyrVal → Kfi [KmAHASIIIIPyr, kcat$AHASIII$Pyr, Lambda, Omega] ,  
kriAHASIIIIPyrVal → Kri [KmAHASIIIIPyr, kcat$AHASIII$Pyr, Lambda, Omega, KiAHASIIIVal] ,  
kfiAHASIIIaKBVal → Kfi [KmAHASIIIaKB, kcat$AHASIII$aKB, Lambda, Omega] ,  
kriAHASIIIaKBVal → Kri [KmAHASIIIaKB, kcat$AHASIII$aKB, Lambda, Omega, KiAHASIIIVal] ,
```

```
KmIRaAHB = 780; KmIRNADPH = 15; kcat$IR$aAHB = 4700;  
kfIRaAHB → Kf2S [KmIRaAHB, KmIRNADPH, kcat$IR$aAHB, Lambda] ,  
krIRaAHB → Kr [kcat$IR$aAHB, Lambda] ,
```

```
KmDADaDMV = 750; kcat$DAD$aDMV = 24000;  
kfdDADaDMV → Kf [KmDADaDMV, kcat$DAD$aDMV, Lambda] ,  
krDADaDMV → Kr [kcat$DAD$aDMV, Lambda] ,
```

```
KmTBGlu = 1000; fkcat$TB$Glu = 2000;  
fkfTBGlu → Kf [KmTBGlu, fkcat$TB$Glu, Lambda] ,  
fkrTBGlu → Kr [fkcat$TB$Glu, Lambda] ,
```

```
KmTBaKMV = 200; fkcat$TB$aKMV = 1500;  
fkfTBaKMV → Kf[KmTBaKMV, fkcat$TB$aKMV, Lambda],  
fkrTBaKMV → Kr[fkcat$TB$aKMV, Lambda],
```

```
KmTBile = 600; rkcat$TB$Ile = 3000;  
rkfTBile → Kf[KmTBile, rkcat$TB$Ile, Lambda],  
rkrTBile → Kr[rkcat$TB$Ile, Lambda],
```

```
KmTBaKG = 2500; rkcat$TB$aKG = 2100;  
rkfTBaKG → Kf[KmTBaKG, rkcat$TB$aKG, Lambda],  
rkrTBaKG → Kr[rkcat$TB$aKG, Lambda],
```

```
KmLIVIexIle = 7; kcat$LIVI$exIle = 200;  
kflLIVIexIle → Kf[KmLIVIexIle, kcat$LIVI$exIle, Lambda],  
krLIVIexIle → Kr[kcat$LIVI$exIle, Lambda],
```

```
KmLIVIIexIle = 1; kcat$LIVII$exIle = 1;  
kflLIVIIexIle → Kf[KmLIVIIexIle, kcat$LIVII$exIle, Lambda],  
krLIVIIexIle → Kr[kcat$LIVII$exIle, Lambda],
```

```
kcat$aKMV → 5,  
kcat$Ile → 0.2,
```

```
KmAHASIPyr = 10; kcat$AHASI$Pyr = 7000;  
kfaHASIPyr → Kf[KmAHASIPyr, kcat$AHASI$Pyr, Lambda],  
kraHASIPyr → Kr[kcat$AHASI$Pyr, Lambda],
```

```
KmAHASIPyr2 = 1000; kcat$AHASI$Pyr2 = 7000;
```

```
kfAHASIPyr2 → Kf [KMAHASIPyr2, kcat$AHASI$Pyr2, Lambda],
krAHASIPyr2 → Kr [kcat$AHASI$Pyr2, Lambda],
residualRateAHASIValPyr2 → 0,

KIAHASIVal = 200;
kfiAHASIPyrVal → Kfi [KMAHASIPyr, kcat$AHASI$Pyr, Lambda, Omega],
kriAHASIPyrVal → Kri [KMAHASIPyr, kcat$AHASI$Pyr, Lambda, Omega, KIAHASIVal],
kfiAHASIPyr2Val → Kfi [KMAHASIPyr2, kcat$AHASI$Pyr2, Lambda, Omega],
kriAHASIPyr2Val → Kri [KMAHASIPyr2, kcat$AHASI$Pyr2, Lambda, Omega, KIAHASIVal],

KMAHASIIPyr = 10; kcat$AHASII$Pyr = 7000;
kfAHASIIIPyr → Kf [KMAHASIIPyr, kcat$AHASII$Pyr, Lambda],
krAHASIIIPyr → Kr [kcat$AHASII$Pyr, Lambda],

KMAHASIIPyr2 = 10000; kcat$AHASII$Pyr2 = 7000;
kfAHASIIIPyr2 → Kf [KMAHASIIPyr2, kcat$AHASII$Pyr2, Lambda],
krAHASIIIPyr2 → Kr [kcat$AHASII$Pyr2, Lambda],

KMAHASIIIIPyr = 10; kcat$AHASIII$Pyr = 7000;
kfAHASIIIIPyr → Kf [KMAHASIIIIPyr, kcat$AHASIII$Pyr, Lambda],
krAHASIIIIPyr → Kr [kcat$AHASIII$Pyr, Lambda],

KMAHASIIIIPyr2 = 7000; kcat$AHASIII$Pyr2 = 7000;
kfAHASIIIIPyr2 → Kf [KMAHASIIIIPyr2, kcat$AHASIII$Pyr2, Lambda],
krAHASIIIIPyr2 → Kr [kcat$AHASIII$Pyr2, Lambda],
residualRateAHASIIIIValPyr2 → 0.15,

KIAHASIIIIVal = 20;
kfiAHASIIIIPyrVal → Kfi [KMAHASIIIIPyr, kcat$AHASIII$Pyr, Lambda, Omega],
```



```
kriAHASIIIPyrVal → Kri [KmAHASIIIPyr, kcat$AHASIII$Pyr, Lambda, Omega, KiaHASIIIVa  
kfiAHASIIIPyr2Val → Kfi [KmAHASIIIPyr2, kcat$AHASIII$Pyr2, Lambda, Omega],  
kriAHASIIIPyr2Val → Kri [KmAHASIIIPyr2, kcat$AHASIII$Pyr2, Lambda, Omega, KiaHASII
```

```
KmIRaAL = 290; KmIRNADPH = 15; kcat$IR$aAL = 1100;  
kfIRaAL → Kf2S [KmIRaAL, KmIRNADPH, kcat$IR$aAL, Lambda],  
krIRaAL → Kr [kcat$IR$aAL, Lambda],
```

```
KmDADaDHIV = 2800; kcat$DAD$aDHIV = 24000;  
kfdADaDHIV → Kf [KmDADaDHIV, kcat$DAD$aDHIV, Lambda],  
krDADaDHIV → Kr [kcat$DAD$aDHIV, Lambda],
```

```
KmTBGlu = 1000; fkcat$TB$Glu = 2000;  
fkfTBGlu → Kf [KmTBGlu, fkcat$TB$Glu, Lambda],  
fkrTBGlu → Kr [fkcat$TB$Glu, Lambda],
```

```
KmTBaKIV = 300; fkcat$TB$aKIV = 930;  
fkfTBaKIV → Kf [KmTBaKIV, fkcat$TB$aKIV, Lambda],  
fkrTBaKIV → Kr [fkcat$TB$aKIV, Lambda],
```

```
KmTBVal = 2700; rkcat$TB$Val = 2000;  
rkfTBVal → Kf [KmTBVal, rkcat$TB$Val, Lambda],  
rkrTBVal → Kr [rkcat$TB$Val, Lambda],
```

```
KmTBaKG = 2500; rkcat$TB$aKG = 2100;  
rkfTBaKG → Kf [KmTBaKG, rkcat$TB$aKG, Lambda],  
rkrTBaKG → Kr [rkcat$TB$aKG, Lambda],
```

```
KmTCAla = 100; fkcat$TC$Ala = 2000;
```

```
fkfTCAla → Kf [KmTCAla, fkcat$TC$Ala, Lambda],
fkrTCAla → Kr [fkcat$TC$Ala, Lambda],

KmTCaKIV = 100; fkcat$TC$aKIV = 1500;
fkfTCaKIV → Kf [KmTCaKIV, fkcat$TC$aKIV, Lambda],
fkrTCaKIV → Kr [fkcat$TC$aKIV, Lambda],

KmTCVal = 3000; rkcat$TC$Val = 3000;
rkfTCVal → Kf [KmTCVal, rkcat$TC$Val, Lambda],
rkrTCVal → Kr [rkcat$TC$Val, Lambda],

KmTCPyr = 2000; rkcat$TC$Pyr = 3000;
rkfTCPyr → Kf [KmTCPyr, rkcat$TC$Pyr, Lambda],
rkrTCPyr → Kr [rkcat$TC$Pyr, Lambda],

KmLIVIexVal = 2; kcat$LIVI$exVal = 500;
kflLIVIexVal → Kf [KmLIVIexVal, kcat$LIVI$exVal, Lambda],
krLIVIexVal → Kr [kcat$LIVI$exVal, Lambda],

KmLIVIIexVal = 1; kcat$LIVII$exVal = 1;
kflLIVIIexVal → Kf [KmLIVIIexVal, kcat$LIVII$exVal, Lambda],
krLIVIIexVal → Kr [kcat$LIVII$exVal, Lambda],

kcat$aKIV → 70,
kcat$Val → 0.2,

KmIPMSacetylCoA = 200 ; kcat$IPMS$acetylCoA = 1000;
kflIPMSacetylCoA → Kf [KmIPMSacetylCoA, kcat$IPMS$acetylCoA, Lambda],
krIPMSacetylCoA → Kr [kcat$IPMS$acetylCoA, Lambda],
```

```
KmIPMSaKIV = 60; kcat$IPMS$aKIV = 1000;
kfiIPMSaKIV → Kf [KmIPMSaKIV, kcat$IPMS$aKIV, Lambda],
kriIPMSaKIV → Kr [kcat$IPMS$aKIV, Lambda],

KiIPMSLeu = 200;
kfiIPMSLeu → Kfi [KmIPMSacetylCoA, kcat$IPMS$acetylCoA, Lambda, Omega],
kriIPMSLeu → Kri [KmIPMSacetylCoA, kcat$IPMS$acetylCoA, Lambda, Omega, KiIPMSLeu],

KiIPMSacetylLeu = 500;
kfiIPMSacetylLeu → Kfi [KmIPMSaKIV, kcat$IPMS$aKIV, Lambda, Omega],
kriIPMSacetylLeu → Kri [KmIPMSaKIV, kcat$IPMS$aKIV, Lambda, Omega, KiIPMSacetylLeu

KmIPMIaIPM = 100; kcat$IPMI$aIPM = 1000;
kfiIPMIaIPM → Kf [KmIPMIaIPM, kcat$IPMI$aIPM, Lambda],
kriIPMIaIPM → Kr [kcat$IPMI$aIPM, Lambda],

KmIPMIbIPM = 100; kcat$IPMI$bIPM = 1000;
kfiIPMIbIPM → Kf [KmIPMIbIPM, kcat$IPMI$bIPM, Lambda],
kriIPMIbIPM → Kr [kcat$IPMI$bIPM, Lambda],

KmIPMDHbIPM = 105; KmIPMDHNAD = 320; kcat$IPMDH$bIPM = 4000;
kfiIPMDHbIPM → Kf2S [KmIPMDHbIPM, KmIPMDHNAD, kcat$IPMDH$bIPM, Lambda],
kriIPMDHbIPM → Kr [kcat$IPMDH$bIPM, Lambda],

KmTBGlu = 1000; fkcat$TB$Glu = 2000;
fkfTBGlu → Kf [KmTBGlu, fkcat$TB$Glu, Lambda],
fkrTBGlu → Kr [fkcat$TB$Glu, Lambda],
```

```
KmTBaKIC = 200; fkcat$TB$aKIC = 3600;
fkfTBaKIC → Kf[KmTBaKIC, fkcat$TB$aKIC, Lambda],
fkrTBaKIC → Kr[fkcat$TB$aKIC, Lambda],

KmTBLeu = 4400; rkcat$TB$Leu = 2800;
rkfTBLeu → Kf[KmTBLeu, rkcat$TB$Leu, Lambda],
rkrTBLeu → Kr[rkcat$TB$Leu, Lambda],

KmTBaKG = 2500; rkcat$TB$aKG = 2100;
rkfTBaKG → Kf[KmTBaKG, rkcat$TB$aKG, Lambda],
rkrTBaKG → Kr[rkcat$TB$aKG, Lambda],

KmLIVIexLeu = 4; kcat$LIVI$exLeu = 100;
kflLIVIexLeu → Kf[KmLIVIexLeu, kcat$LIVI$exLeu, Lambda],
krLIVIexLeu → Kr[kcat$LIVI$exLeu, Lambda],

KmLIVIIexLeu = 1; kcat$LIVII$exLeu = 1;
kflLIVIIexLeu → Kf[KmLIVIIexLeu, kcat$LIVII$exLeu, Lambda],
krLIVIIexLeu → Kr[kcat$LIVII$exLeu, Lambda],

KmLSexLeu = 0.5; kcat$LS$exLeu = 100;
kflSexLeu → Kf[KmLSexLeu, kcat$LS$exLeu, Lambda],
krLSexLeu → Kr[kcat$LS$exLeu, Lambda],

kcat$aKIC → 25,
kcat$Leu → 0.01
};
```

```
(*
  Inputs for Values of Substrate and Enzyme Concentrations ( $\mu\text{M}$ )
  { note: NADP[0]=0 in yang' s pathway}
*)

myICs = {
  AKI[0] == 8.1,
  AKII[0] == 0,
  AKIII[0] == 9.5,
  ASD[0] == 6.2,
  HDHI[0] == 8.1,
  HDHII[0] == 0,
  HSK[0] == 5.6,
  TS[0] == 8.3,

  Asp[0] == 3600,
  AspP[0] == 0,
  ASA[0] == 0,
  Hse[0] == 0,
  HseP[0] == 0,
  Lys[0] == 1000,
  Thr[0] == 0,

  ATP[0] == 1000,
  ADP[0] == 0,
  NADP[0] == 1000,
  NADPH[0] == 1000,
  Phosphate[0] == 1000,
```

```
$Complex$AKII$Asp$ATP$[0] == 0,  
$Complex$ASD$ASA$NADP$Phosphate$[0] == 0,  
$Complex$ASD$AspP$NADPH$[0] == 0,  
$Complex$HDHII$ASA$NADPH$[0] == 0,  
$Complex$HseP$TS$[0] == 0,  
$Complex$HSK$Hse$ATP$[0] == 0,  
$Complex$HSK$Thr$[0] == 0,
```

```
Pyr[0] == 1000,  
Leu[0] == 0,  
Ala[0] == 2000,  
Glu[0] == 2000,  
Ile[0] == 0,  
Val[0] == 0,  
aKB[0] == 0,  
NH3[0] == 0,  
aAHB[0] == 0,  
aDMV[0] == 0,  
aKMV[0] == 0,  
aKG[0] == 0,  
aAL[0] == 0,  
aDHIV[0] == 0,  
aKIV[0] == 0,  
aIPM[0] == 0,  
bIPM[0] == 0,  
aKIC[0] == 0,
```

```
propionylCoA[0] == 0,  
glutarylCoA[0] == 0,
```

```
pantothenate[0] == 0,  
acetylCoA[0] == 1000,  
CoA[0] == 0,  
NAD[0] == 1000,  
NADH[0] == 0,  
CO2[0] == 0,  
protein[0] == 0,  
  
TDA[0] == 3,  
KDC[0] == 2,  
  
AHASI[0] == 10,  
AHASICH3CO[0] == 0,  
AHASII[0] == 0, (* E. coli K12 has no active AHASII *)  
AHASIICH3CO[0] == 0,  
AHASIII[0] == 2,  
AHASIIICH3CO[0] == 0,  
  
IR[0] == 13.5,  
DAD[0] == 7,  
  
TB[0] == 2.5,  
TBNH2[0] == 0,  
TC[0] == 2,  
TCNH2[0] == 0,  
  
IPMS[0] == 5,  
IPMSacetyl[0] == 0,  
IPMI[0] == 6,
```

```
IPMDH[0] == 5,

LIVI[0] == 10,
LIVII[0] == 0,
LS[0] == 8,

                                (* extracellular amino acid treatment *)
exVal[0] == 0,                    (* 1000 for valine growth inhibition *)
exIle[0] == 0,                    (* 500 for isoleucine rescue *)
exLeu[0] == 0,

$Complex$aKB$aKDC$[0] == 0, $Complex$Pyr$AHASI$[0] == 0,
$Complex$AHASI$Val$[0] == 0, $Complex$Pyr$AHASI$Val$[0] == 0,
$Complex$AHASICH3CO$Val$[0] == 0, $Complex$aKB$AHASICH3CO$[0] == 0,
$Complex$aKB$AHASICH3CO$Val$[0] == 0, $Complex$Pyr$AHASICH3CO$[0] == 0,
$Complex$Pyr$AHASICH3CO$Val$[0] == 0, $Complex$Pyr$AHASII$[0] == 0,
$Complex$aKB$AHASIIICH3CO$[0] == 0, $Complex$Pyr$AHASIIICH3CO$[0] == 0,
$Complex$Pyr$AHASIII$[0] == 0, $Complex$AHASIII$Val$[0] == 0,
$Complex$Pyr$AHASIII$Val$[0] == 0, $Complex$AHASIIIICH3CO$Val$[0] == 0,
$Complex$aKB$AHASIIIICH3CO$[0] == 0, $Complex$aKB$AHASIIIICH3CO$Val$[0] == 0,
$Complex$Pyr$AHASIIIICH3CO$[0] == 0, $Complex$Pyr$AHASIIIICH3CO$Val$[0] == 0,
$Complex$IR$aAHB$aNADPH$[0] == 0, $Complex$IR$aAL$aNADPH$[0] == 0,
$Complex$aDHIV$aDAD$[0] == 0, $Complex$aDMV$aDAD$[0] == 0,
$Complex$aGlu$aTB$[0] == 0, $Complex$aKMV$aTBNH2$[0] == 0,
$Complex$aKIV$aTBNH2$[0] == 0, $Complex$aIle$aTB$[0] == 0,
$Complex$aVal$aTB$[0] == 0, $Complex$aKG$aTBNH2$[0] == 0,
$Complex$aAla$aTC$[0] == 0, $Complex$aKIV$aTCNH2$[0] == 0,
$Complex$aVal$aTC$[0] == 0, $Complex$Pyr$aTCNH2$[0] == 0,
$Complex$exIle$aLIVI$[0] == 0, $Complex$exIle$aLIVII$[0] == 0,
```



```

$Complex$exVal$LIVI$[0] == 0, $Complex$exVal$LIVII$[0] == 0,
$Complex$acetylCoA$IPMS$[0] == 0, $Complex$aKIV$IPMSacetyl$[0] == 0,
$Complex$IPMS$Leu$[0] == 0, $Complex$IPMSacetyl$Leu$[0] == 0,
$Complex$aKIV$IPMSacetyl$Leu$[0] == 0, $Complex$aIPM$IPMI$[0] == 0,
$Complex$bIPM$IPMI$[0] == 0, $Complex$IPMDH$bIPM$NAD$[0] == 0,
$Complex$aKIC$TBNH2$[0] == 0, $Complex$Leu$TB$[0] == 0,
$Complex$exLeu$LIVI$[0] == 0, $Complex$exLeu$LIVII$[0] == 0,
$Complex$exLeu$LS$[0] == 0};

```

```

(*
Call the Mathematica NDSolve Function to Solve the ODEs with Given
Values of Kinetic Parameters and Substrate and Enzyme Concentrations
Listed Above.

```

```

*)

```

```

tmax = 100; (* minutes *)
mySolution = NDSolve[Join[myODEs /. myKs, myICs], myVars, {t, 0, tmax}, AccuracyGoal ->
PrecisionGoal -> 3, MaxSteps -> 30000];

```

```

(* List of Substrates, Intermediates and Products for Graphic Outputs
Displayed Below *)

```

```

metabolites = {
Asp, AspP, ASA, Hse, HseP, HSK,
Thr, Pyr, aKB, aAHB, aAL, NADPH, aDMV, aDHIV, aKMV, aKIV, acetylCoA, aIPM, bIPM,
NAD, aKIC, Glu, Ala, aKG, Ile, Val, Leu
};

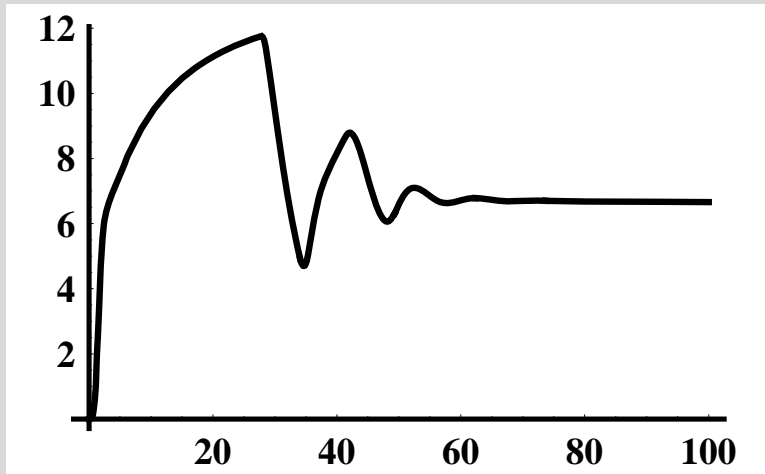
```

```
TBTCdistribution = {
  TB, TBNH2, $Complex$Glu$TB$, $Complex$aKMV$TBNH2$, $Complex$aKIV$TBNH2$,
  $Complex$aKIC$TBNH2$,
  $Complex$Ile$TB$, $Complex$Val$TB$, $Complex$Leu$TB$, $Complex$aKG$TBNH2$,
  TC, TCNH2, $Complex$Ala$TC$, $Complex$aKIV$TCNH2$,
  $Complex$Val$TC$, $Complex$Pyr$TCNH2$};
```

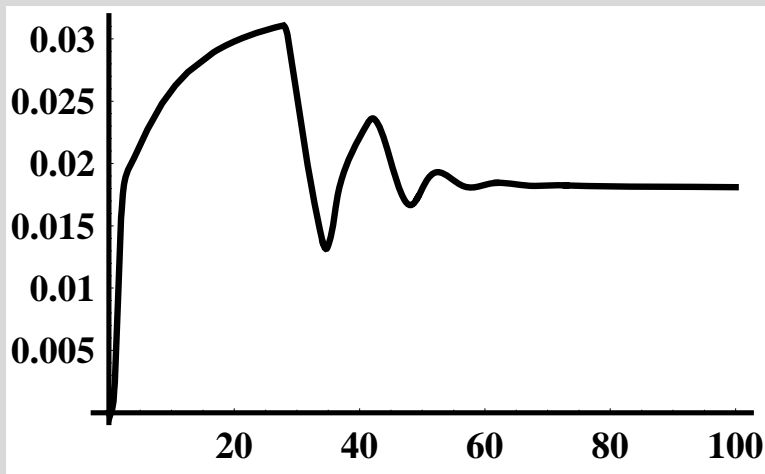
```
(* Define some plotting functions *)
FormalPlot[varName_, displayTime_, YplotRange_] :=
  Plot[varName[t] /. mySolution, {t, 0, displayTime}, PlotRange → YplotRange,
    Background → GrayLevel[1],
    TextStyle → {FontWeight → "Bold", FontSize → 14, FontFamily → "Times New Roman"},
    PlotStyle → {Thickness[0.01]}, AxesStyle → {Thickness[0.008]}};
DashPlot[varName_, displayTime_, YplotRange_] :=
  Plot[varName[t] /. mySolution, {t, 0, displayTime}, PlotRange → YplotRange,
    Background → GrayLevel[1],
    TextStyle → {FontWeight → "Bold", FontSize → 12, FontFamily → "Times New Roman"},
    PlotStyle → {Thickness[0.008], Dashing[{0.02, 0.025}]}, AxesStyle → {Thickness[0.
DotPlot[varName_, displayTime_, YplotRange_] :=
  Plot[varName[t] /. mySolution, {t, 0, displayTime}, PlotRange → YplotRange,
    Background → GrayLevel[1],
    TextStyle → {FontWeight → "Bold", FontSize → 12, FontFamily → "Times New Roman"},
    PlotStyle → {Thickness[0.008], Dashing[{0.001, 0.001, 0.001, 0.015}]},
    AxesStyle → {Thickness[0.0006]}};
```

```
(*  
  Display Results of Simulation of Branched Chain Amino Acid  
  Simulation: Rates of Production of Metabolic Intermediates and End-  
  products.  
  
    X axis is Time (min), and Y axis is Concentration ( $\mu\text{M}$ )  
*)  
  
displayTime = 100;  
Show[  
  GraphicsArray[  
    Partition[  
      Map[Plot[# [t] /. mySolution, {t, 0, displayTime}, PlotLabel -> #, PlotRange -> All,  
        DisplayFunction -> Identity] &, metabolites], 3]  
    ]  
  ];  
];
```

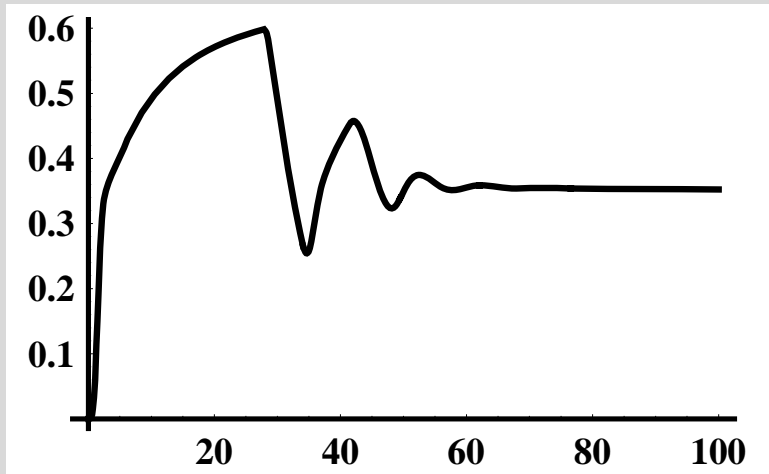
```
FormalPlot[aKB, 100, All];
```



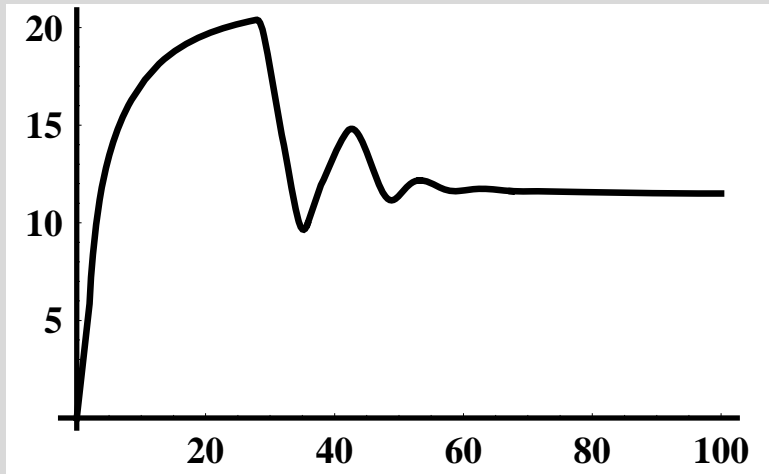
```
FormalPlot[aAHB, 100, All];
```



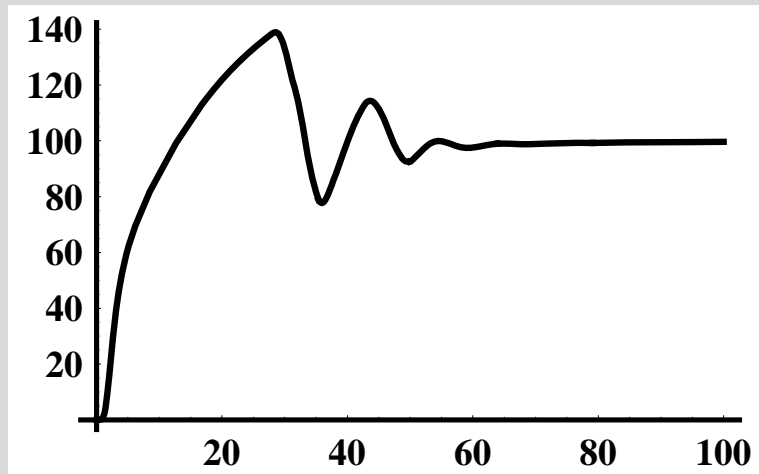
```
FormalPlot[aDMV, 100, All];
```



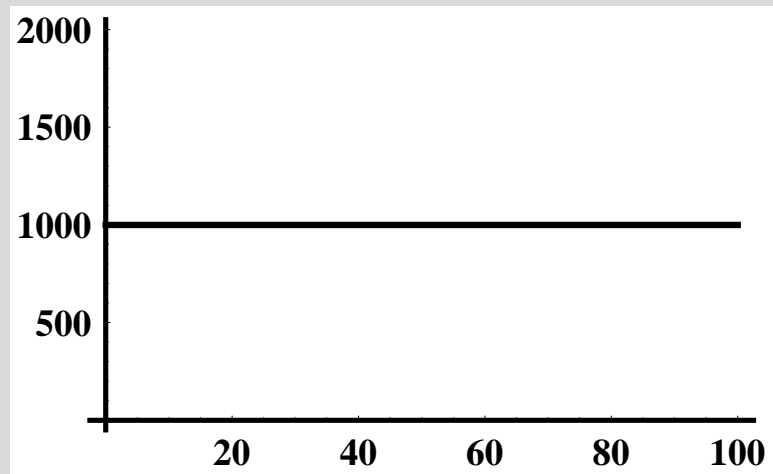
```
FormalPlot[aKMV, 100, All];
```



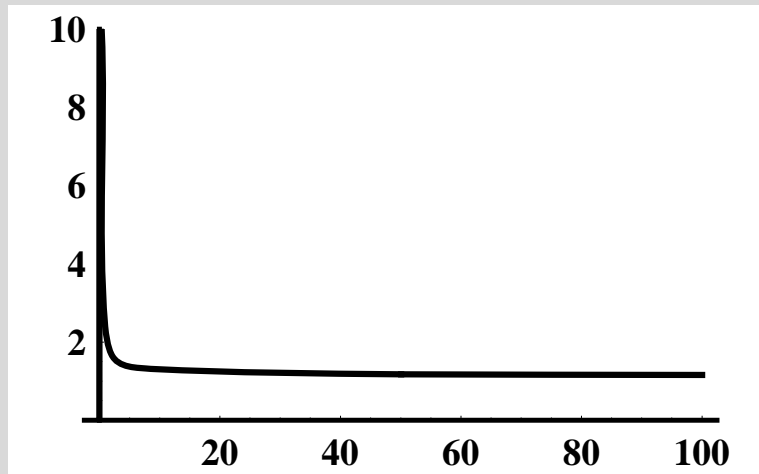
```
FormalPlot[Ile, 100, All];
```



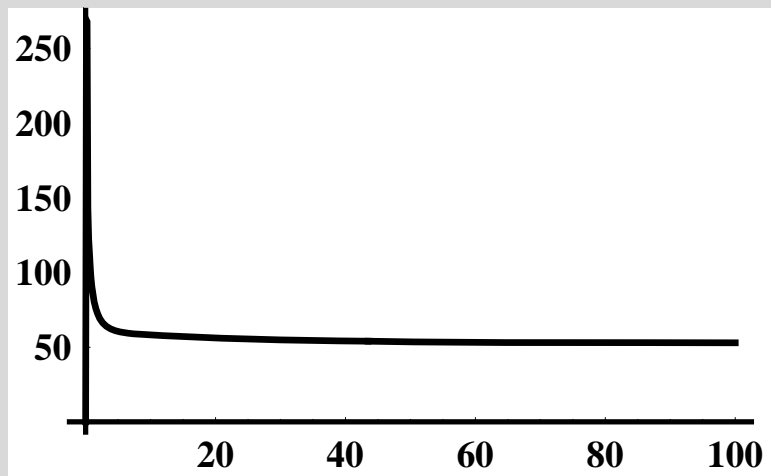
```
FormalPlot[Pyr, 100, All];
```



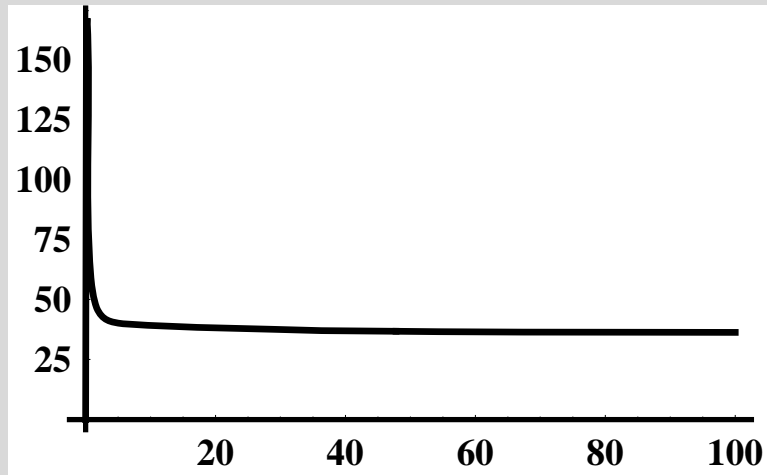
```
FormalPlot[aAL, 100, {0, 10}];
```



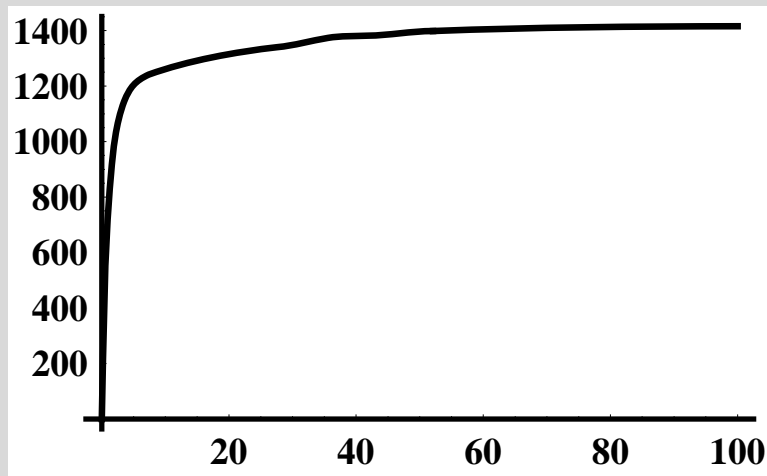
```
FormalPlot[aDHIV, 100, All];
```



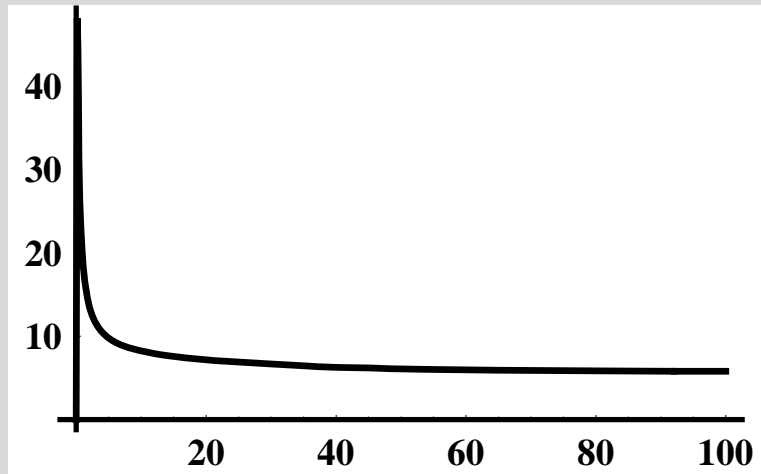
```
FormalPlot[aKIV, 100, All];
```



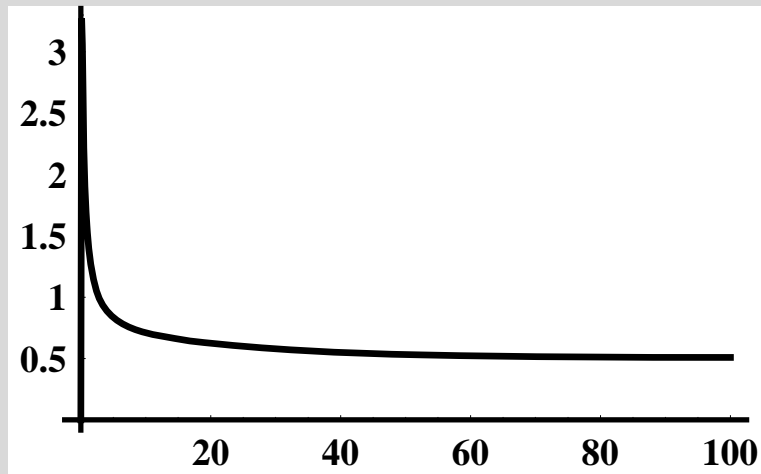
```
FormalPlot[Val, 100, All];
```



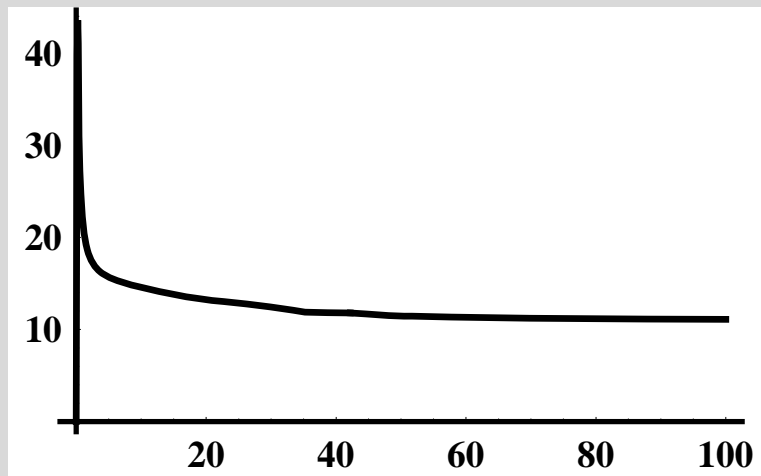

```
FormalPlot[aIPM, 100, All];
```



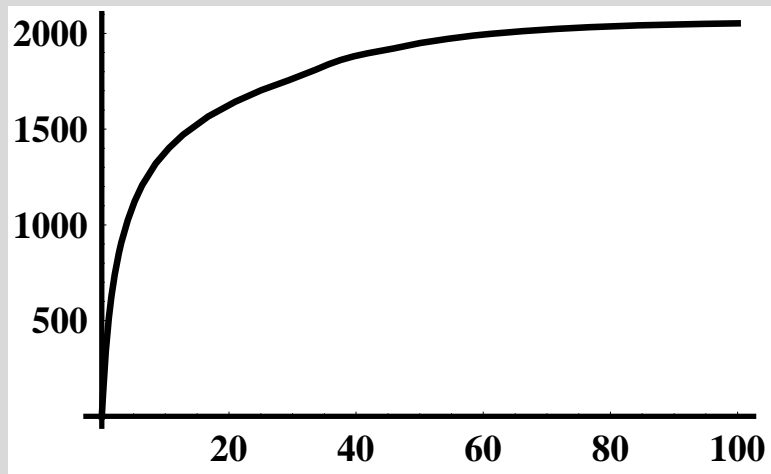
```
FormalPlot[bIPM, 100, All];
```



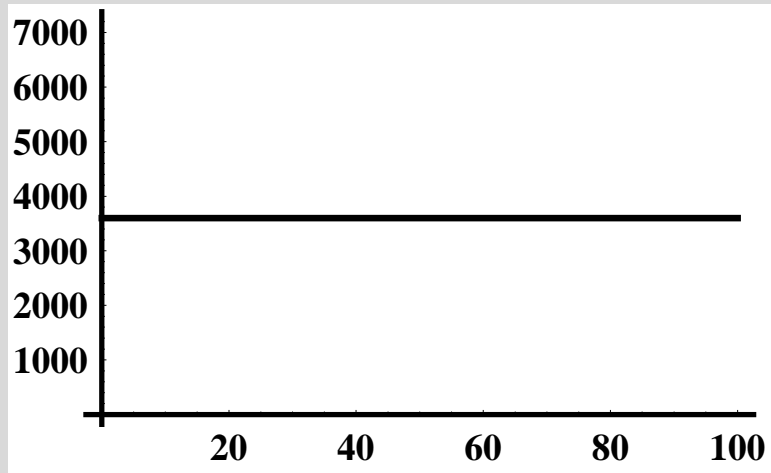
```
FormalPlot[aKIC, 100, All];
```



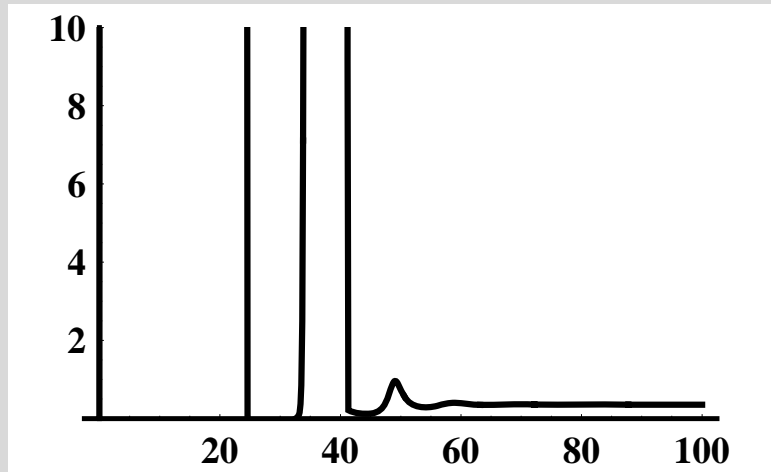
```
FormalPlot[Leu, 100, All];
```



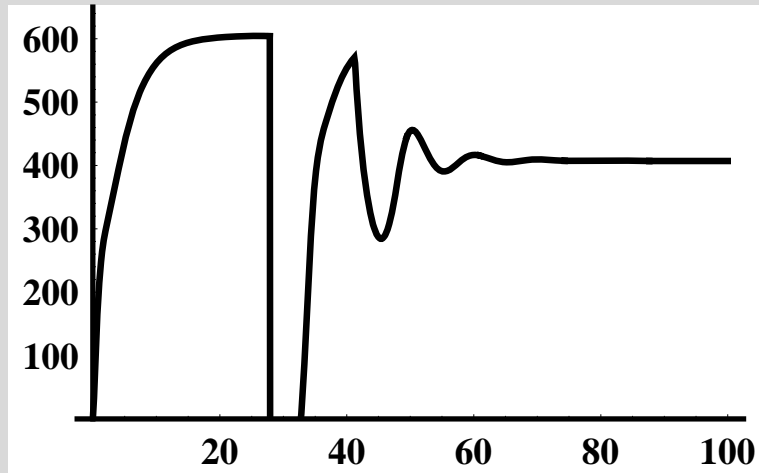
```
FormalPlot[Asp, 100, All];
```



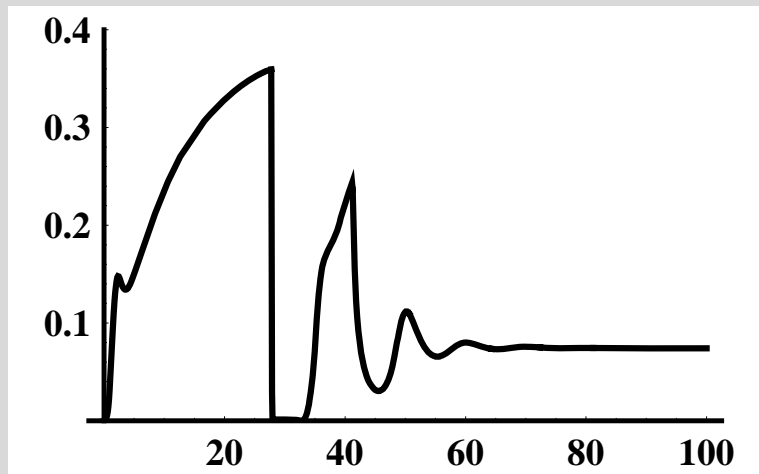
```
FormalPlot[AspP, 100, {0, 10}];
```



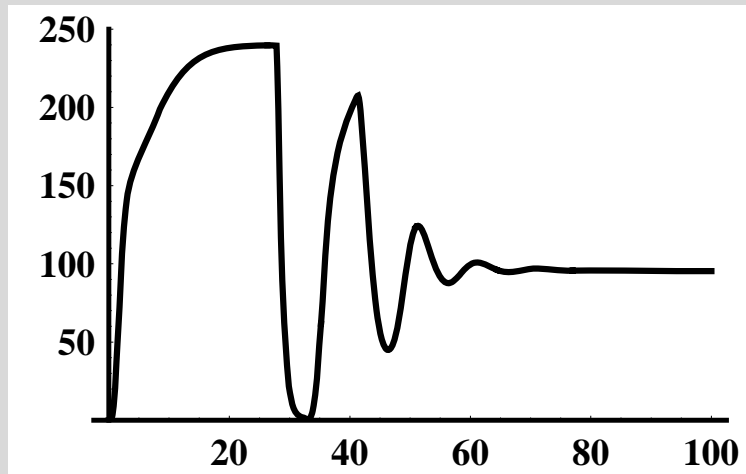
```
FormalPlot[ASA, 100, {0, 650}];
```



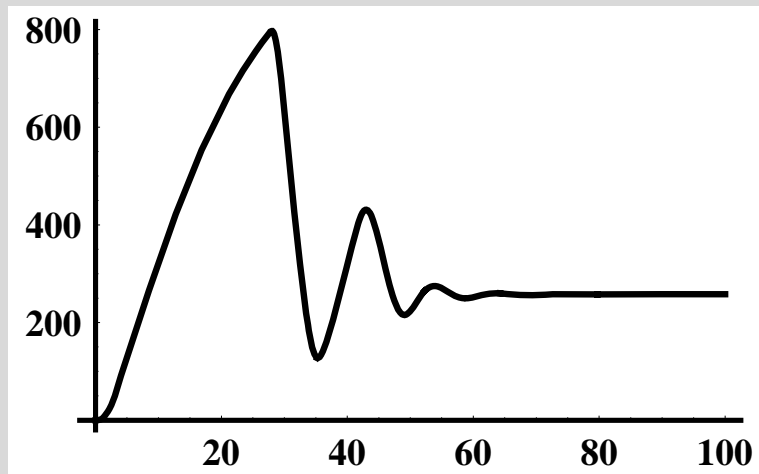
```
FormalPlot[Hse, 100, {0, 0.4}];
```



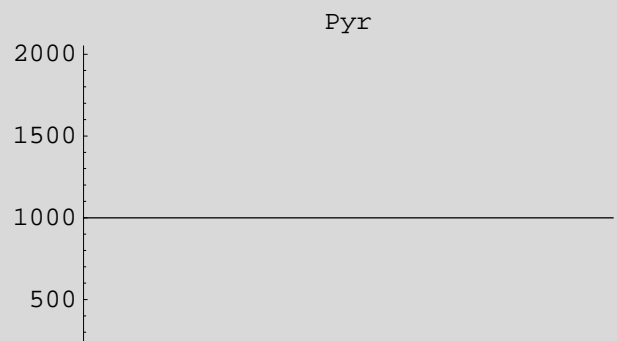
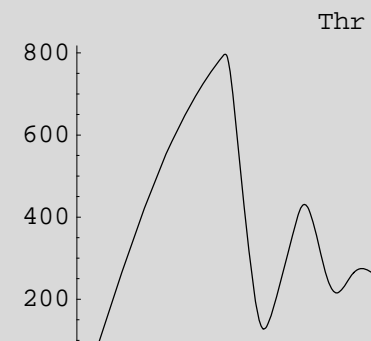
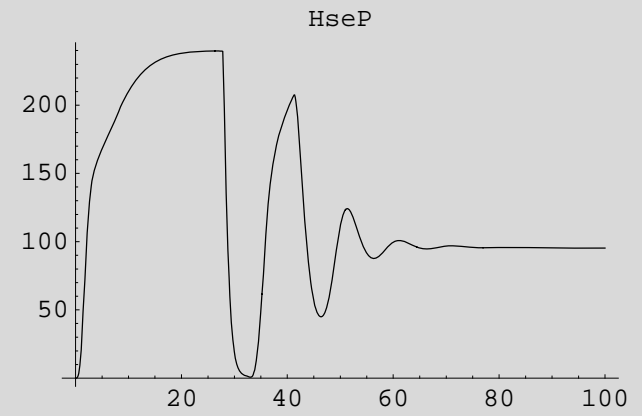
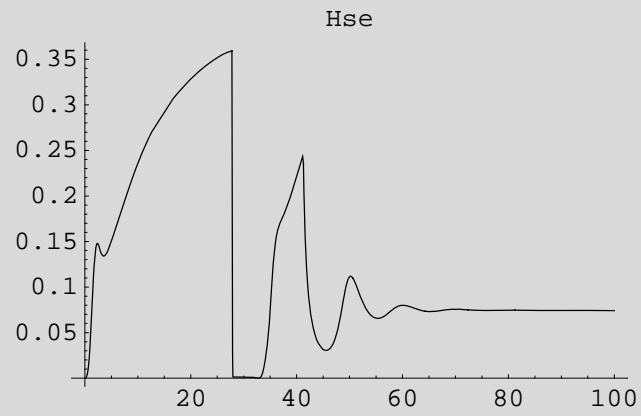
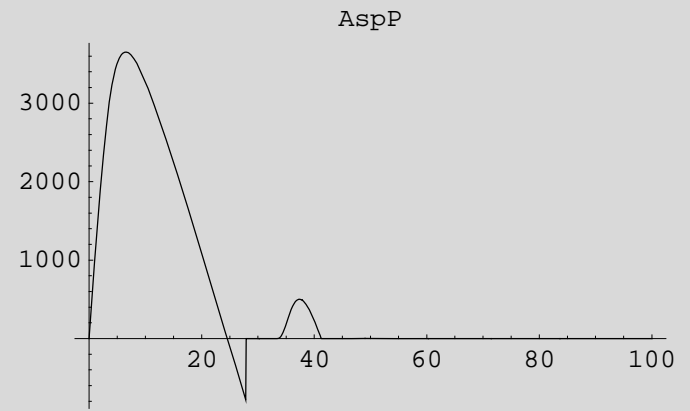
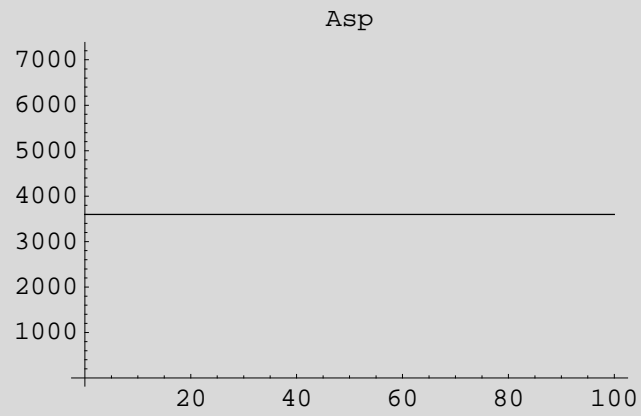
```
FormalPlot[HseP, 100, {0, 250}];
```

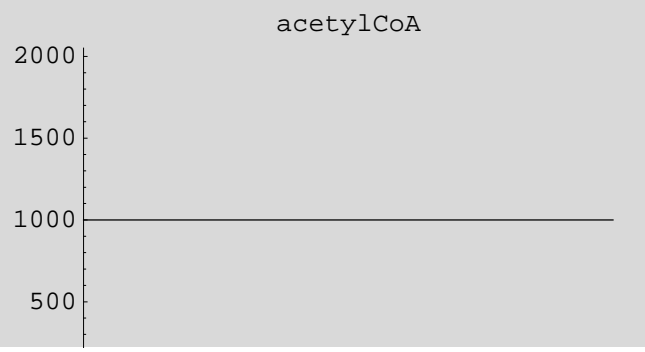
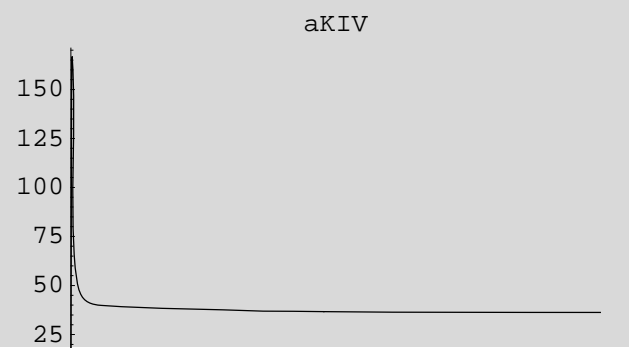
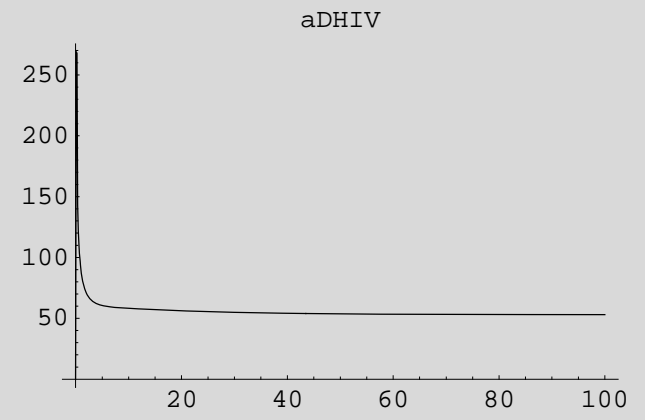
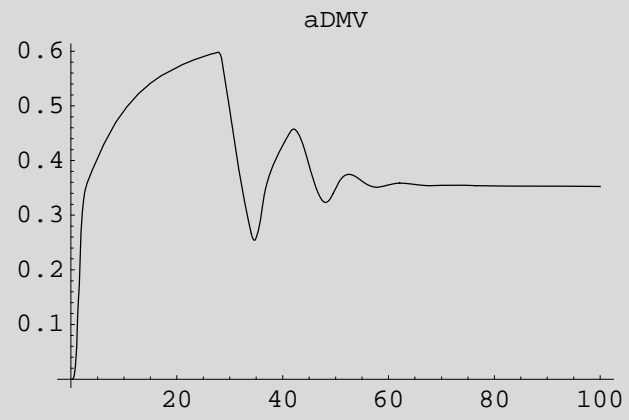
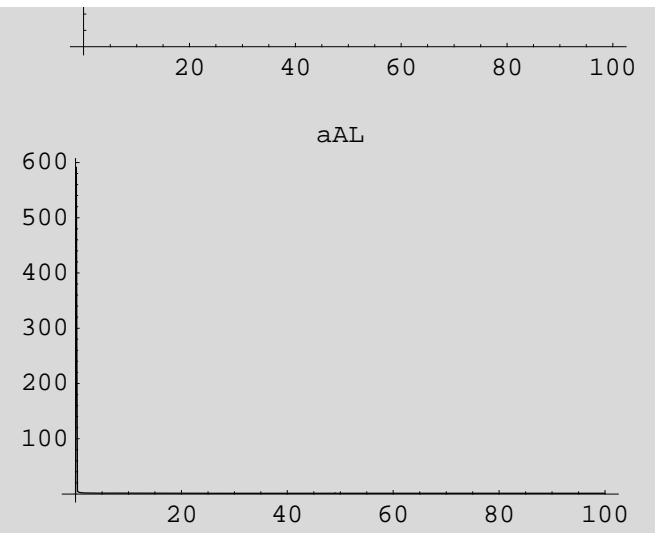
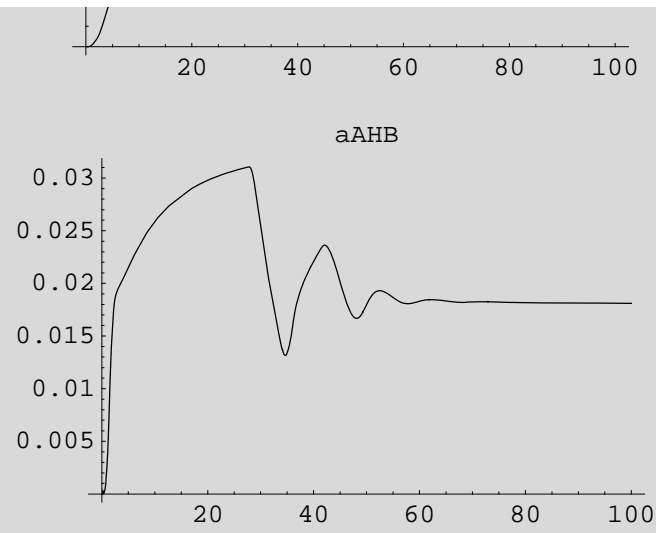


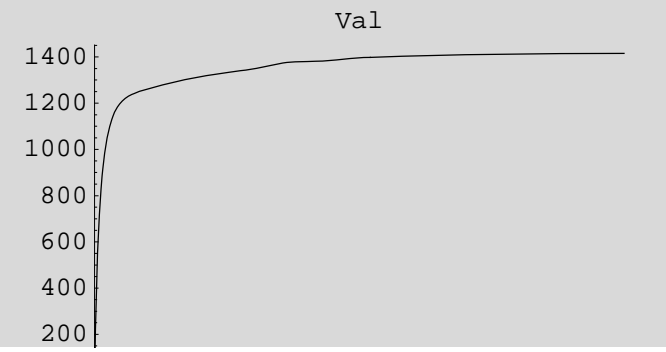
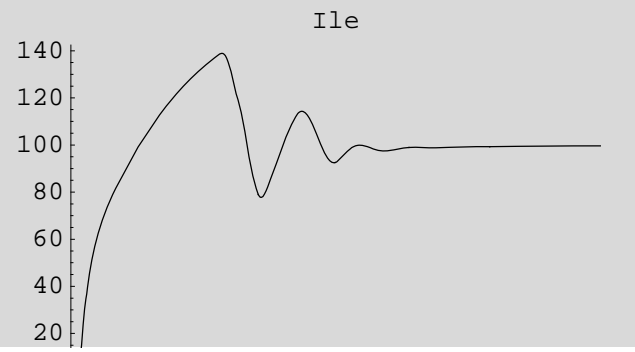
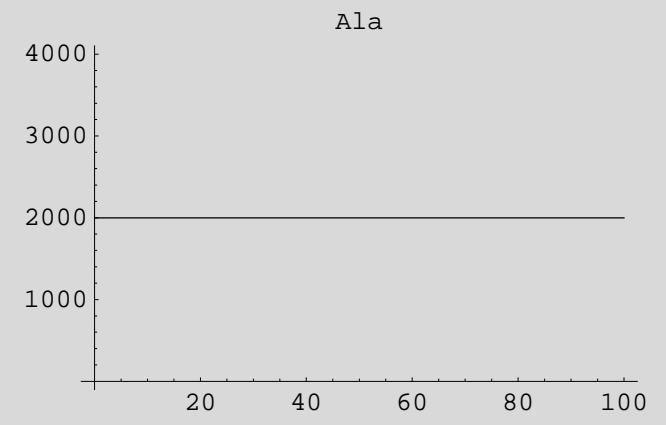
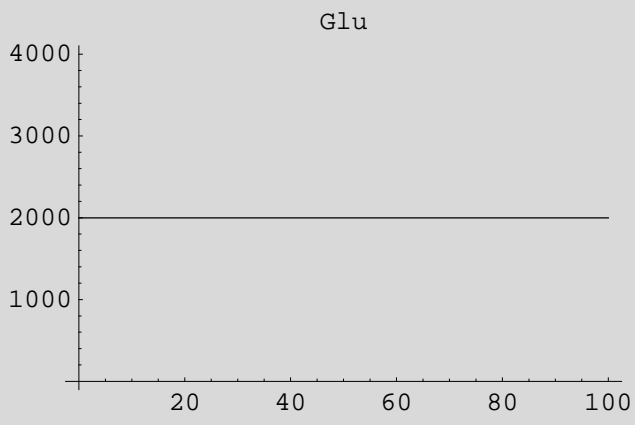
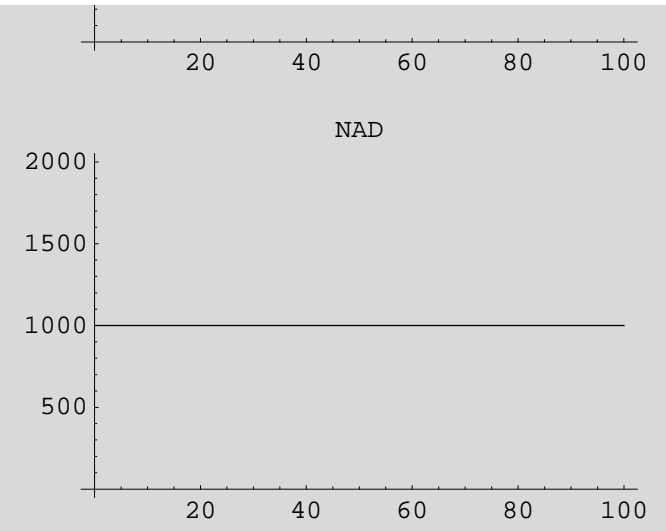
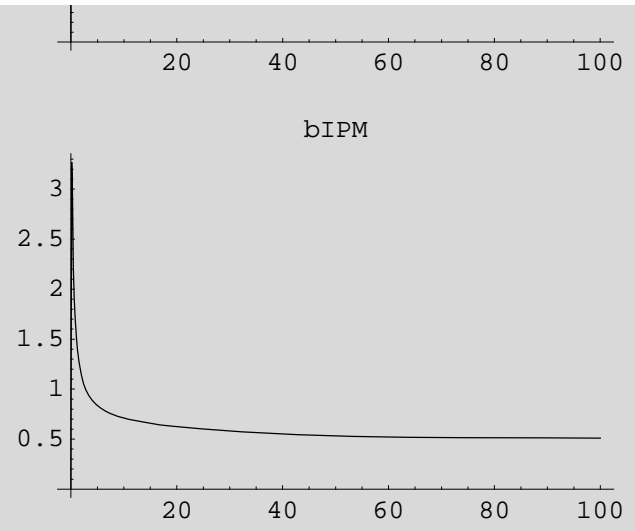
```
FormalPlot[Thr, 100, All];
```

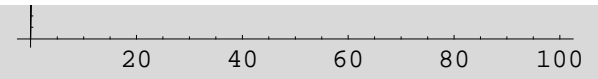


```
(* Pyr[0] = 1,000 *)
```

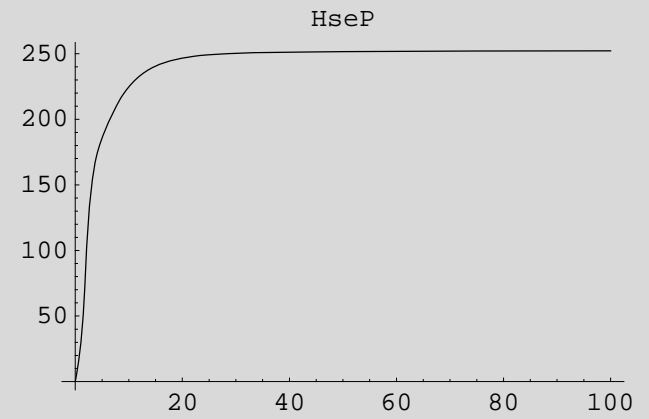
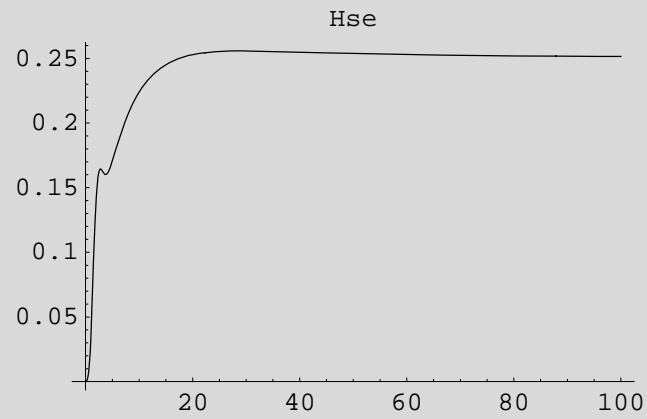
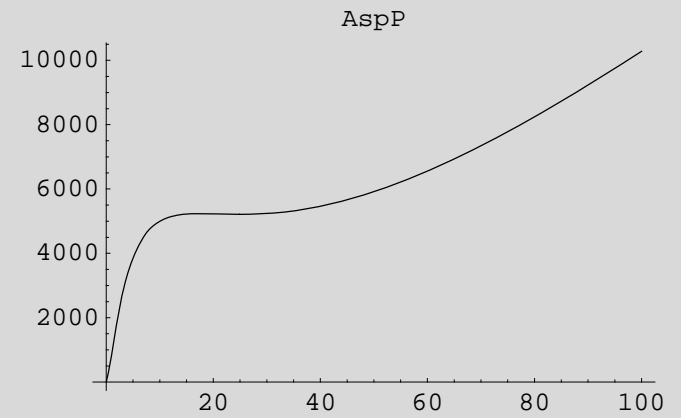
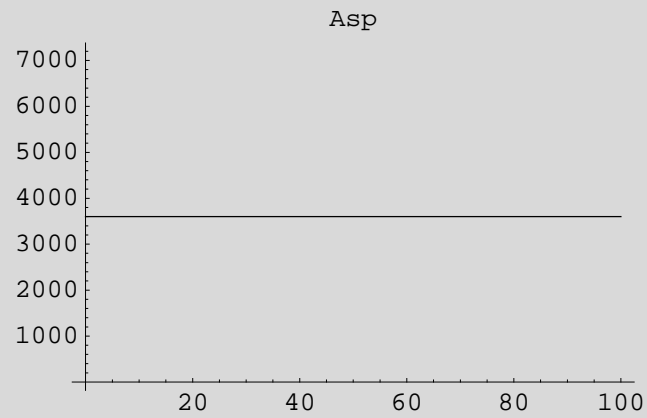


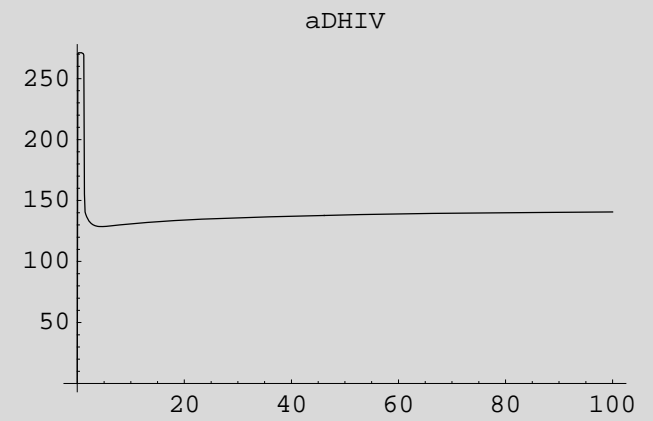
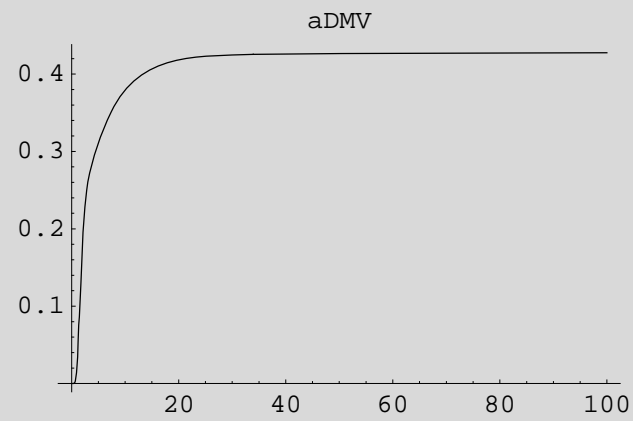
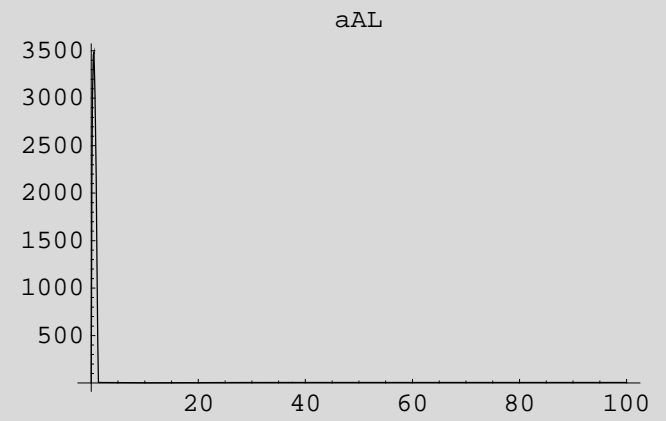
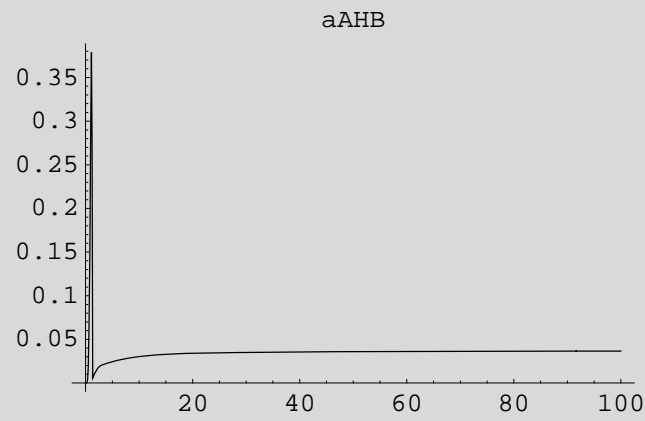
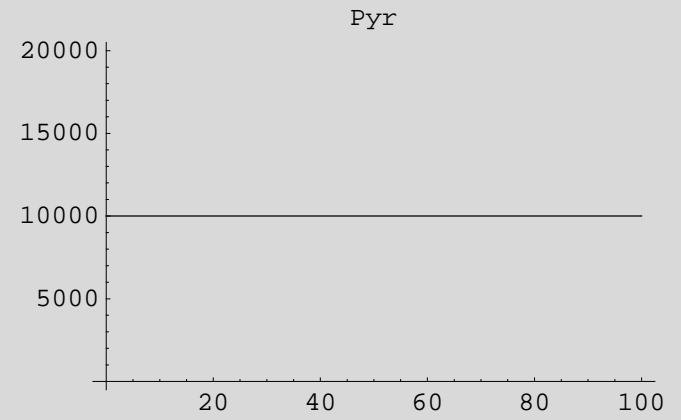
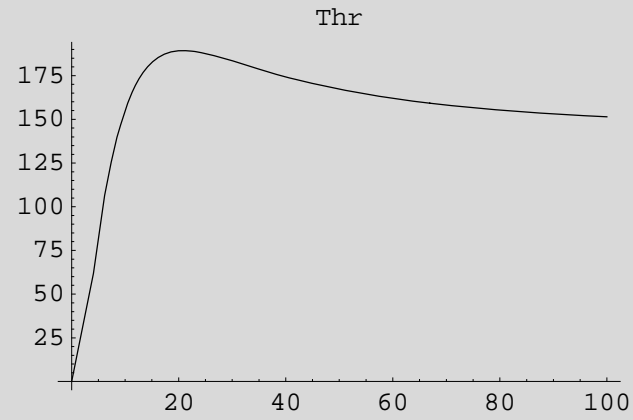


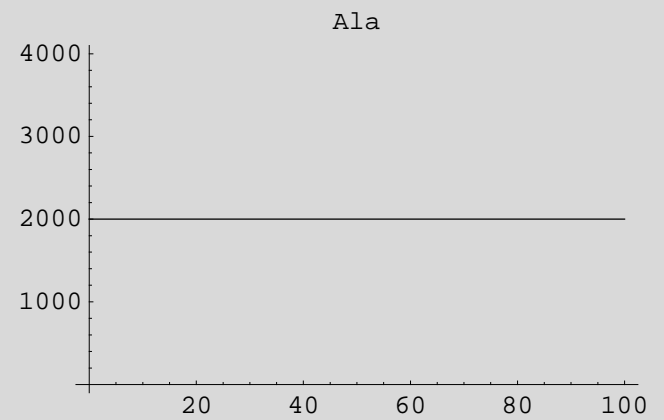
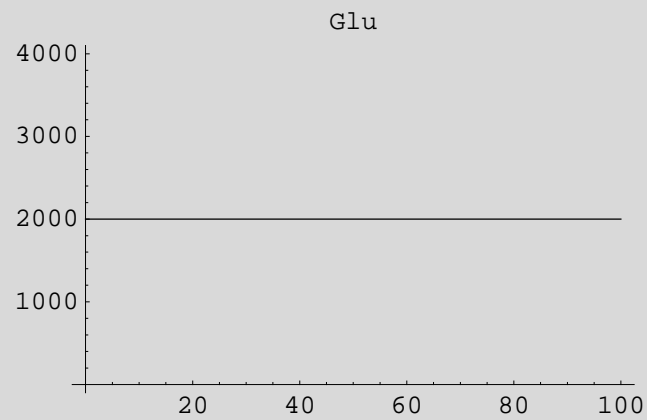
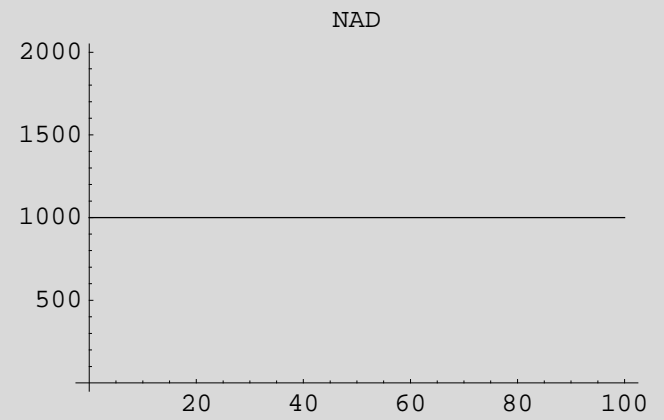
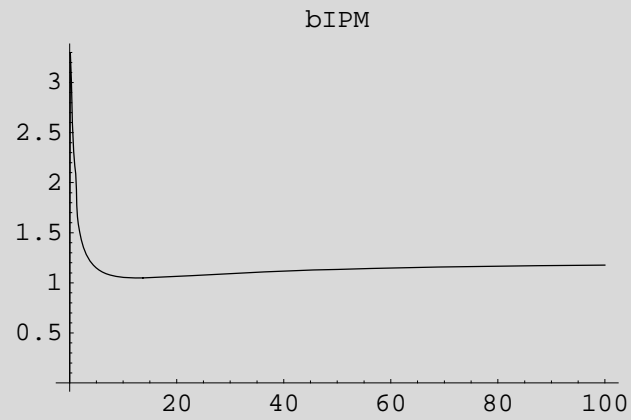
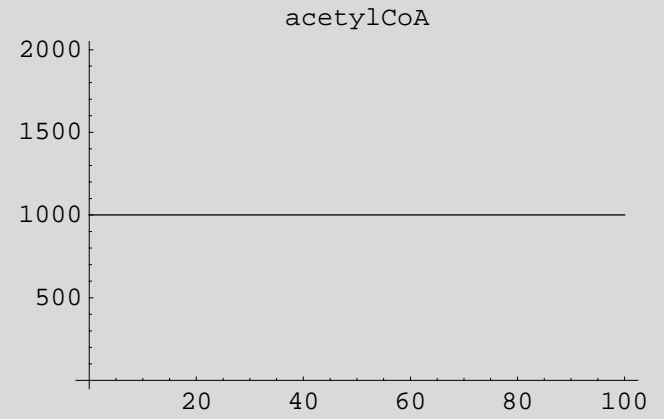
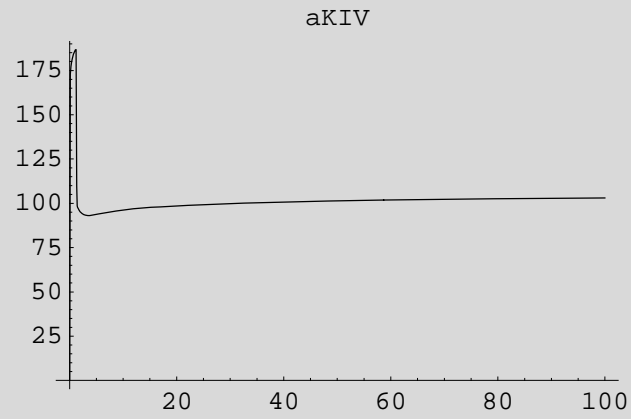


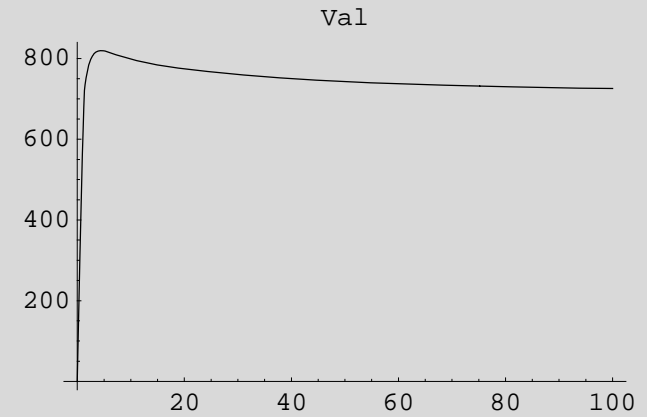
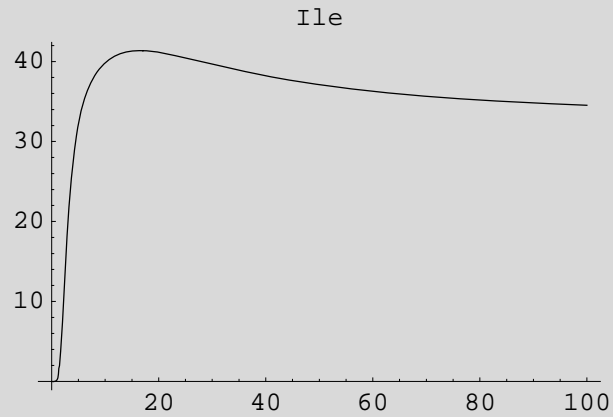


(* Pyr[0]= 10,000 *)



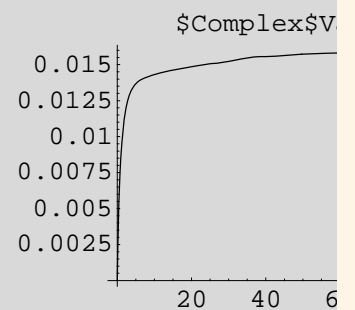
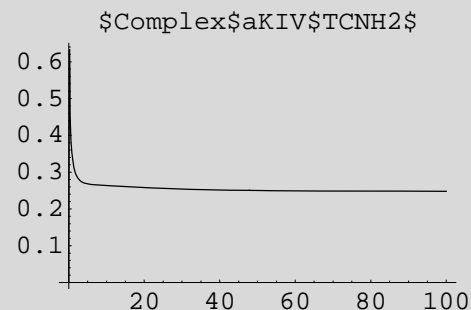
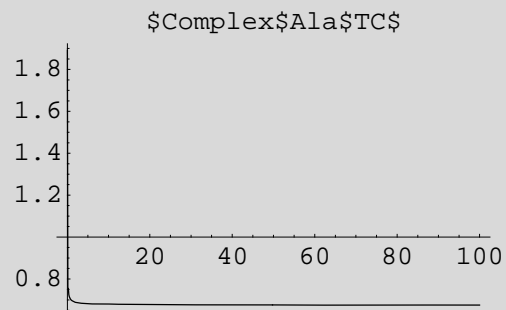
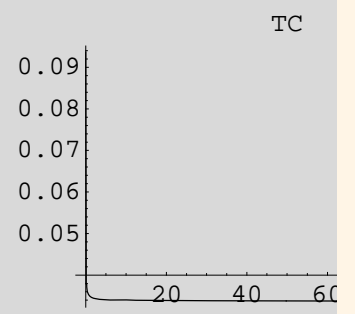
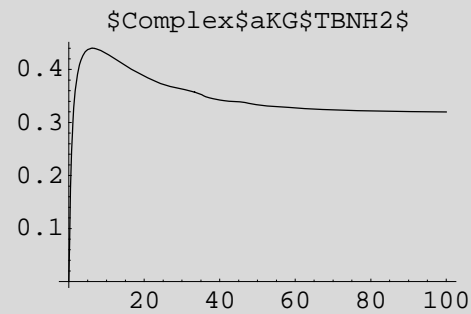
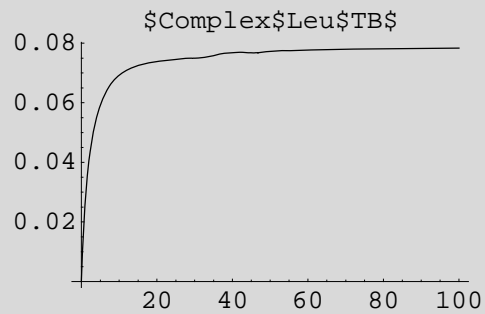
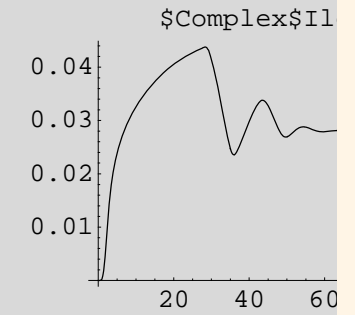
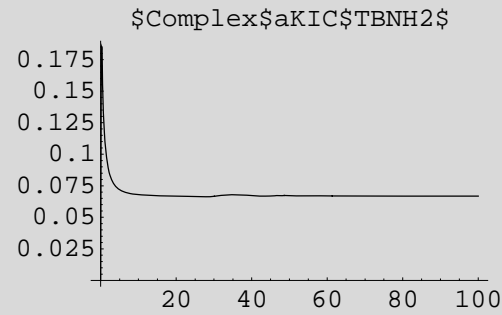
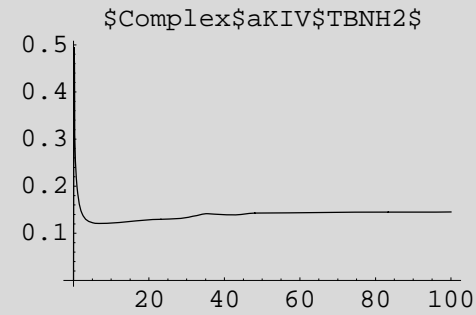
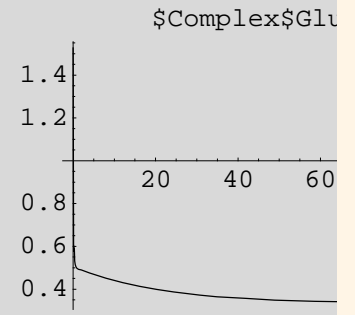
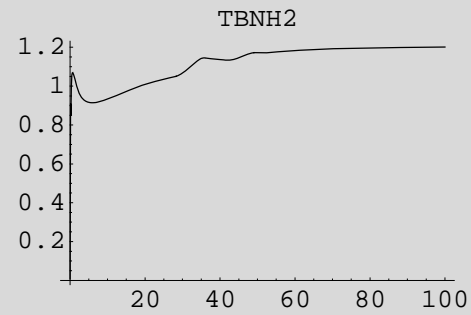
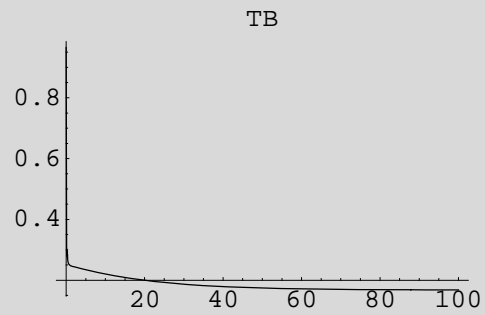






```
(*  
  Display Simulations of Partition of Transaminase B and C  
  X axis is Time (min), and Y axis is Concentration ( $\mu\text{M}$ )  
*)  
  
displayTime = 100;  
Show[  
  GraphicsArray[  
    Partition[  
      Map[Plot[# [t] /. mySolution, {t, 0, displayTime}, PlotLabel -> #, PlotRange -> All,  
        DisplayFunction -> Identity] &, TBTCdistribution], 4]  
    ]  
  ];
```

```
(* Pyr[0] = 1,000 *)
```



(* Pyr[0] = 10,000 *)

