Inventory of Supplementary Information

1. **Supplementary Text. (SupplementaryInformation.doc)**

Details of the specific equations used to simulate the networks described in the manuscript and the relevant parameter values.

2. **Figure S1 (figureS1.pdf)**

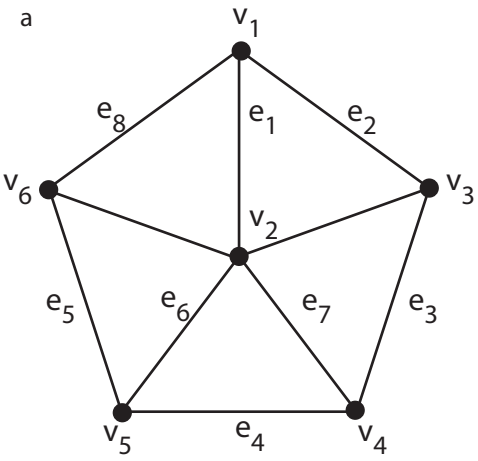
a) Example of a graph with six vertices and eight edges (top panel) and its adjacency matrix (bottom panel). b) A coloring of the graph shown in (a) (top panel). The rows and columns of the adjacency matrix are reordered such that vertices with the same color are grouped together (bottom panel).

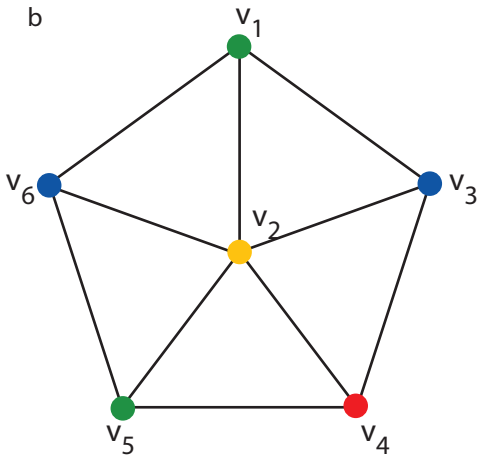
3. **Movie S1 (TravelingWaves.mpg)**

The top panel of the animation shows the time evolution of activity on a (10 x 10) grid consisting of 100 PNs. The neurons that were extracted to form a sub–network are represented by the filled circles on the grid. The activity of the sub–network forms traveling waves that are separated by time dT. The values of the color map and the scale of the traces in the bottom panel are given in Figure 7 of the manuscript.

Figure S1

figure S1





	v_1	v_2	v_3	v_4	v_5	v_6
v_1	0	1	1	0	0	1
v_2	1	0	1	1	1	1
v_3	1	1	0	1	0	0
v_4	0	1	1	0	1	0
v_5	0	1	0	1	0	1
v_6	1	1	0	0	1	0

	v_1	v_5	v_3	v_6	v_2	v_4
v_1	0	0	1	1	1	0
v_5	0	0	0	1	1	1
v_3	1	0	0	0	1	1
v_6	1	1	0	0	1	0
v_2	1	1	1	1	0	1
v_4	0	1	1	0	1	0

Supplementary Information

The model network simulations were based on a realistic model of the insect antennal lobe developed by Bazhenov et. al.(Bazhenov et al., 2001a; Bazhenov et al., 2001b).

LN model equations

Each LN was modelled as a single compartment that included voltage and Ca^{2+} dependent currents described by Hodgkin–Huxley kinetics. The membrane potential (V_{LN}) of LNs was given by the following equation,

$$C_m \frac{dV_{LN}}{dt} = -g_L(V_{LN} - E_L) - I_K - I_{Ca} - I_{KCa} - g_{KL}(V_{LN} - E_{KL}) - I_{GABA_A} - I_{nACh} - I_{ext}$$

Where the membrane capacitance, $C_m = 1.43 \times 10^{-4}\,\mu F$. The conductances of the leak currents were $g_L = 0.15\mu S$ and $g_{KL} = 0.02\mu S$. The corresponding reversal potentials were $E_L = -50mV$ and $E_{KL} = -50mV$ respectively. LNs included fast potassium (I_K), Calcium (I_{Ca}) and a calcium dependent potassium (I_{KCa}) currents as well. LNs inhibited each other and PNs via fast GABAergic synapses. Excitation from PNs was mediated by cholinergic synapses. The corresponding currents were I_{GABA_A} and I_{nACh} respectively. An external DC input was provided to all the neurons and given by I_{ext}. In addition to this external input a noise term consisting of random fluctuations of ~10% of the amplitude of the DC input was introduced into each neuron. A description of the intrinsic and synaptic currents is given below.

Fast potassium current I_K

$$I_K = g_k n^4 (V - E_K),$$

where the conductance is $g_k = 10\mu S$ and $E_K = -95mV$. The equation for the gating variable n is given by,

$$\frac{dn}{dt} = -\frac{1}{\tau_n}\left(n - n_\infty(V)\right),$$

where the steady state value, n_∞, and the time constant, τ_n, are nonlinear functions of V and given by,

$$n_\infty = \frac{\alpha_n}{\left(\alpha_n + \beta_n\right)\phi}$$

$$\tau_n = \frac{1}{\left(\alpha_n + \beta_n\right)\phi}.$$

The variable ϕ depends on the temperature and is given by $\phi = 3^{\left(\frac{22-26}{10}\right)}$ at 26°C.

$$\alpha_n = 0.02\frac{\left(15 - V_{shift}\right)}{\exp\left(\frac{10 - V_{shift}}{40}\right)}$$

$$\beta_n = 0.5\exp\left(\frac{10 - V_{shift}}{40}\right).$$

For I_K and I_{Na}, these nonlinear functions were derived from experimental recordings of ionic currents and described in (Traub et al., 1997)

<u>Ca^{2+}current I_{Ca}</u>

$$I_{Ca} = g_{Ca} m^2 h\left(V - E_{Ca}\right).$$

Where, the conductance is $g_{Ca} = 3\mu S$ and $E_{Ca} = 140mV$,

The gating variables satisfy the equations,

$$\frac{dm}{dt} = -\frac{1}{\tau_m}\left(m - m_\infty(V)\right)$$

$$\frac{dh}{dt} = -\frac{1}{\tau_h}\left(h - h_\infty(V)\right).$$

The steady state values of the gating variables are given by,

$$m_\infty(V) = \frac{1}{1+\exp\left(-\frac{V+20}{6.5}\right)}$$

$$h_\infty(V) = \frac{1}{1+\exp\left(\frac{V+25}{12}\right)}.$$

The time constants are $\tau_m = 1.5$ and

$$\tau_h = 0.3\exp\left(\frac{V-40}{13}\right) + 0.002\exp\left(-\frac{V-60}{29}\right).$$

<u>Ca^{2+} dependent potassium current I_{KCa}</u>

$$I_{KCa} = g_{KCa} m^2 h\left(V - E_{KCa}\right),$$

where $g_{KCa} = -0.3$ and $E_{KCa} = -90$. The gating variable satisfies the equation,

$$\frac{dm}{dt} = -\frac{1}{\tau_m + \tau_x}\left(m - m_\infty(V)\right)$$

while, $m_\infty(V) = \frac{[Ca^{2+}]}{[Ca^{2+}]+2}$, $\tau_m = \frac{100}{[Ca^{2+}]+2}$ and τ_x is obtained from a uniform distribution extending from -0.02 to 0.01.

The calcium concentration satisfies a simple first order equation,

$$\frac{d[Ca^{2+}]}{dt} = -AI_{Ca} - \frac{\left([Ca^{2+}] - [Ca^{2+}]_\infty\right)}{\tau},$$

where, $[Ca^{2+}]_\infty = 2.4 \times 10^{-4}\, mM$ and $\tau = 150\, ms$

<u>Projection neurons</u>

The membrane potential of projection neurons was given by

$$C_m \frac{dV_{PN}}{dt} = -g_L\left(V_{LN} - E_L\right) - I_{Na} - I_K - I_A - g_{KL}\left(V_{LN} - E_{KL}\right) - I_{GABA_A} - I_{nACh} - I_{ext}$$

$E_L = -55mV$ for PNs. The remaining passive parameters were the same for both PNs and LNs. The intrinsic currents were given by the following equations,

Sodium (I_{Na}) and potassium currents (I_K)

$$I_{Na} = g_{Na} m^3 h (V - E_{Na}),$$

$$I_K = g_K n^4 (V - E_K),$$ (Traub et al., 1997)

where $g_{Na} = 7.15\mu S,\ E_{Na} = 50mV,\ g_k = 1.43\mu S,\ E_K = -95mV.$

The gating variables m, h, and n satisfy equations of the same form as the gating variables for LN intrinsic currents. $m_\infty, h_\infty, n_\infty, \tau_m, \tau_h, \tau_n$ were nonlinear functions of V derived from experimental recordings and described in

Transient potassium A current I_A

$$I_A = g_A m^4 h (V - E_K),$$

where $g_A = 10\mu S$ and $E_K = -95mV.$

The steady state values of the gating variables are given by,

$$m_\infty(V) = \frac{1}{1 + \exp\left(-\frac{V + 60}{8.5}\right)}$$

$$h_\infty(V) = \frac{1}{1 + \exp\left(\frac{V + 78}{6}\right)}$$

The time constants were given by,

$$\tau_m = \frac{1}{\left[\exp\left(\frac{V + 35.82}{19.69}\right) + \exp\left(-\frac{V + 79.69}{12.7}\right) + 0.37\right]}$$

$$\tau_h = \frac{1}{\left[\exp\left(\frac{\mathrm{V}+46.05}{5}\right)+\exp\left(-\frac{\mathrm{V}+238.4}{37.45}\right)\right]} \text{ for } V < -63 \text{ and}$$

$\tau_h = 19$ if $V \geq -63\,\mathrm{mV}$

Synaptic currents

Inhibition mediated by fast GABAergic synapes and excitation mediated by cholinergic synapses led to currents that satisfied the following equation,

$I_{syn} = g_{syn}[O](V - E_{syn})$.

The fraction of open channels $[O]$ is calculated according to

$$\frac{d[O]}{dt} = \alpha(1-[O])[T] - \beta[O].$$

Where, $[T] = A\Theta(t_0 - t_{\max} - t)\Theta(t - t_0)$ for cholinergic synapses

and $[T] = \frac{1}{1+\exp\left(-\frac{V(t)-V_0}{\sigma}\right)}$ for GABAergic synapses.

Θ is the Heaviside step function, t_0 is the time of receptor activation, A=0.5, $t_{\max}$=0.3ms, $V_0 = -20mV$ and $\sigma = 1.5$. The rate constants were $\alpha = 10ms^{-1}$ and $\beta = 0.16ms^{-1}$ for GABA synapses and $\alpha = 10ms^{-1}$ and $\beta = 0.2ms^{-1}$ for cholinergic synapses. The peak synaptic conductances were set to $g_{GABA_A} = 0.1\mu S$ between LNs, $g_{GABA_A} = 0.03\mu S$ from LNs to PNs and $g_{nACh} = 0.6\mu S$ from PNs to LNs in the networks used to generate the dynamics seen in figure 5. LN–LN interactions were set to zero in some cases as stated in the text and figure caption.

Graph coloring

In the manuscript we use the terms graph and network interchangeably to mean a set of vertices (neurons) that are connected by a set of edges (synapses).

A graph G, is an ordered pair G = (V,E), where V is a finite set of elements called vertices, while E is a finite set of unordered vertices called edges. The graph in figure S1a consists of 6 vertices V = {v_1, v_2, v_3, v_4, v_5, v_6} and 8 edges E = { e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8} with an edge, say e_2, being an unordered pair of vertices e_2 = { v_1, v_3}.

The vertices v_i and v_j contained in the set V are considered adjacent (or neighbors) if there is an edge {v_i,v_j} in the set E connecting the two vertices. The adjacency matrix of a graph consisting of *n* vertices is an *n* x *n* matrix (figure S1a, bottom panel). The $(i,j)^{th}$ entry of this matrix is one if the vertices v_i and v_j are adjacent to each other and zero otherwise. The graphs that we used in the manuscript do not possess self edges (edges of the form $\{v_i,v_i\}$) and are undirected. The adjacency matrix of the graph therefore has zeros along the diagonal and is symmetric on either side of the diagonal.

In general, a coloring of a graph can be achieved by assigning colors to the vertices or the edges of the graph with different rules that determine what constitutes a proper coloring of the graph. The form of coloring that we employ in the manuscript is the most typical form, termed vertex coloring. A proper vertex coloring of a graph is one that assigns different colors to nodes that are directly connected to each other. In general, a graph with *n* nodes can always be colored using *n* colors while ensuring that two adjacent vertices do not share the same color. However, a key question in graph coloring problems is whether a coloring using $p < n$ colors, termed a *p*–coloring, of the graph is possible. In particular we are interested in the minimum $p < n$ below which a legal coloring of the

graph is not possible. The value is known as the chromatic number of the graph. Figure S1b shows an example of a graph that can be colored using four colors but not three (based on the example from (Chartrand, 1984)). Given a coloring of the graph, the adjacency matrix can always be written in a form that has diagonal blocks of zeros and off–diagonal non–zero elements (figure S1b, bottom panel).

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Traub, R.D., Jefferys, J.G.R., and Whittington, M.A. (1997). Simulation of gamma rhythms in networks of interneurons and pyramidal cells. Journal of Computational Neuroscience *4*, 141-150.